2.5:

The problem misstates optical loss in units of photons/sec instead of 1/sec.

a) As defined on p. 39, the photon decay rate, or optical loss rate, is

$$\text{optical loss rate} = 1/\tau_p = 4 \times 10^{12}\ 1/\text{s}$$

$$\tau_p = \frac{1}{4 \times 10^{12}\ 1/\text{s}} = 2.5 \times 10^{-13}\ \text{s}$$

---

b) From Eq. 2.24,

$$< g_{th} >= \text{threshold modal gain} = \frac{1}{v_g\ \tau_p} = \frac{\overline{n}_g}{c\ \tau_p}$$

where $v_g = c/\overline{n}_g$. Use $\overline{n}_g = 4$ for InGaAsP in-plane lasers (p. 40).

$$< g_{th} >= \frac{4}{(3 \times 10^{10} \frac{\text{cm}}{\text{s}})\ (2.5 \times 10^{-13}\ \text{s})} = 533\ \text{cm}^{-1}$$

Note: Optical and carrier loss rates can be defined in two ways:

1) Number of carriers/photons lost per unit time (units of $\frac{\text{carriers or photons}}{\text{s}}$) or

2) $1/\tau$ where τ is the carrier/photon lifetime (units of $\frac{1}{\text{s}}$)

To distinguish between the two definitions, it is necessary to look at the units of the loss rate.

2.6:

a)

$$R \approx 0.32 \qquad (p.39)$$

From Eq 2.23,

$$< g_{th} >=< \alpha_i > + \frac{1}{L} \ln\left(\frac{1}{R}\right)$$

where $< \alpha_i >= 10 \text{ cm}^{-1}$ and $L = 400\mu\text{m} = 0.04 \text{ cm}$

$$< g_{th} >= 10 \text{ cm}^{-1} + \frac{1}{0.04 \text{ cm}} \ln\left(\frac{1}{0.314}\right) = 38.5 \text{ cm}^{-1}$$

---

b)

$$\alpha_m = \frac{1}{L} \ln\left(\frac{1}{R}\right) = 28.5 \text{ cm}^{-1}$$

$$\eta_d = \frac{\eta_i \ \alpha_m}{< \alpha_i > + \alpha_m} \qquad (2.35)$$

$$\eta_d = \frac{(0.8)\ (28.5 \text{ cm}^{-1})}{10 \text{ cm}^{-1} + 28.5 \text{ cm}^{-1}} = 59.2\%$$

---

c)

$$\delta\lambda = \frac{\lambda^2}{2(\overline{n}_{ga} L_a + \overline{n}_{gp} L_p)} \qquad (2.27)$$

$$L_p = 0$$

$$\overline{n}_{ga} = \overline{n}_{gp} = 4. \qquad (p.40)$$

$$\delta\lambda = \frac{(1.55\mu\text{m})^2}{2\ (4)\ (400\mu\text{m})} = 7.51 \times 10^{-4}\mu\text{m}$$

$$\delta\lambda = 7.51\mathring{A}$$

As described on p. 39,

$$R \approx 0.32 \qquad \text{for cleaved facets}$$

This has been found to be a good value for the TE polarized modes of InP and GaAs based lasers. Note that values calculated from typical values for the modal index, $\overline{n}$ according to Eqs. 3.10 and 3.42 give slightly different values for reflection. Reflection values calculated from these equations are only approximate due to the differences in the physics of the reflection for the guided mode vs. the plane wave. More exact calculations involve a dividing a mode into its composite plane wave spectrum and considering the reflection of each plane wave in the spectrum.

We will continue to use $R \approx 0.32$ for cleaved facets throughout this book.

2.7:

a)

$$\frac{\eta_i I_{th}}{qV} = AN_{th} + BN_{th}^2 + CN_{th}^3$$

Assume that the spontaneous emission term dominates.

$$\frac{\eta_i I_{th}}{qV} = \frac{\eta_i J_{th} A}{qAt} \approx BN_{th}^2$$

Use the value for B from p. 31

$$N_{th} = \sqrt{\frac{\eta_i J_{th}}{qtB}} = \sqrt{\frac{(1.0)\ (10^3\ \mathrm{A/\ cm^2})}{(1.602 \times 10^{-19}\ \mathrm{C})\ (0.1 \times 10^{-4}\ \mathrm{cm})\ (10^{-10}\ \mathrm{cm^{-3}/\ s})}}$$

$$N_{th} = 2.5 \times 10^{18}\ \mathrm{cm^{-3}}$$

---

b)

$$P_0 = \eta_d \frac{h\nu}{q} (I - I_{th})$$

$$\eta_d = \eta_i \frac{\alpha_m}{< \alpha_i > + \alpha_m}$$

$$R \approx 0.32 \qquad (p.39)$$

$$\alpha_m = \frac{1}{L} \ln \frac{1}{R} = \frac{1}{300 \times 10^{-4}} \ln \frac{1}{0.32} = 38\ \mathrm{cm^{-1}}$$

Assume that the wavelength is defined by the GaAs bandgap:

$$\lambda = \frac{1.24\mu\mathrm{m\ eV}}{1.424\ \mathrm{eV}} = 0.87\mu\mathrm{m}$$

$$\frac{h\nu}{q} = \frac{(1.24\mu\mathrm{m\ eV})(1.6 \times 10^{-19}\ \mathrm{J/\ eV})}{(0.87\mu\mathrm{m})(1.6 \times 10^{-19}\ \mathrm{C})} = 1.424 \frac{\mathrm{J}}{\mathrm{C}}$$

For each micron of width, we have

$$I - I_{th} = (J - J_{th})A = (1\ \mathrm{kA/\ cm^2})\ (300 \times 10^{-4}\ \mathrm{cm})\ (1 \times 10^{-4}\ \mathrm{cm}) = 3\ \mathrm{mA}$$

$$< \alpha_i > = 10\ \mathrm{cm^{-1}}$$

$$\eta_i = 1$$

Plug values into the first equation of part b solution:

$$P_0 = 1 \left(\frac{38\ \mathrm{cm^{-1}}}{10\ \mathrm{cm^{-1}} + 38\ \mathrm{cm^{-1}}}\right) \left(1.424 \frac{\mathrm{J}}{\mathrm{C}}\right) (3\ \mathrm{mA}) = 3.38\ \mathrm{mW}\ \text{for each micron of width}$$

This is the power from both facets. The power from each facet is half of this:

$$P = 1.69\ \mathrm{mW}\ \text{for each micron of width}$$

c) Photon density:

$$N_p = \frac{P_0}{v_g \alpha_m h\nu V_p} \tag{2.33}$$

Assume $\Gamma = 1$, which implies that $V_p = V = t \times w \times l$.

$$v_g = \frac{c}{\bar{n}_g}$$

From p.40, $\bar{n}_g = 4.5$ for GaAs DH structures. Using this value with the values from part (b), we can solve for N_p.

$$N_p = \frac{3.47 \times 10^{-3}\ \mathrm{W}}{\frac{3\times10^{10}\ \mathrm{cm/s}}{4.5}\ (38\ \mathrm{cm}^{-1})\ (1.46\ \mathrm{eV})\ (1.6 \times 10^{-19}\ \mathrm{J/eV})\ w\ (0.1 \times 10^{-4}\ \mathrm{cm})\ (300 \times 10^{-4}\ \mathrm{cm})}$$

$$w\ N_p = 1.95 \times 10^{11}\ \mathrm{cm}^{-2}$$

To find the photon density per micron of width, use $w = 1\mu\mathrm{m} = 10^{-4}$ cm.

$$N_p = 2.0 \times 10^{15}\ \mathrm{cm}^{-3} \text{ per micron of width}$$

Carrier density: Above threshold, the carrier density is clamped at its threshold value:

$$N = N_{th} = 2.5 \times 10^{18}\ \mathrm{cm}^{-3}$$

2.11:

a)

$$< g_{th} >=< \alpha_i > + \frac{1}{L} \ln \frac{1}{R}$$

Single pass gain of 1% implies that

$$e^{-L(<g_{th}>)} = 1.01$$

$$\begin{aligned} R &= e^{-L(<g_{th}>-<\alpha_i>)} \\ &= e^{-L(<g_{th}>)} \; e^{L(<\alpha_i>)} \\ &= (1.01) \; e^{(1.5\times10^{-4}\text{ cm})(25\text{ cm}^{-1})} \\ &= 0.9938 \end{aligned}$$

$$r = \sqrt{R} = 0.9969$$

where R is the mean power reflectivity and r is the mean field reflectivity.

---

b) For $I > I_{th}$,

$$\frac{P_0}{A} = \eta_d \left(\frac{h\nu}{q}\right)(J - J_{th})$$

$$\alpha_m = \frac{1}{1.5\times10^{-4}\text{ cm}} \ln \frac{1}{0.99375} = 41.8\text{ cm}^{-1}$$

$$\eta_d = \eta_i \frac{\alpha_m}{< \alpha_i > + \alpha_m} = 0.8 \frac{41.8}{25+41.8} = 0.501$$

$$\frac{h\nu}{q} = 1.424\text{ J/C}$$

$$\frac{P_0}{A} = 0.713\,(J - 1000)\text{ W/cm}^2$$

where J is in A/cm^2.

For $I < I_{th}$,

$$P_0 = \eta_c \eta_i \eta_r \frac{h\nu}{q} I$$

$$\eta_c = 10\%$$

$$\eta_i = 80\%$$

$$\eta_r = 100\%$$

$$\frac{P_0}{A} = 0.113 \times J\text{ W/cm}^2$$

where J is in A/cm^2.

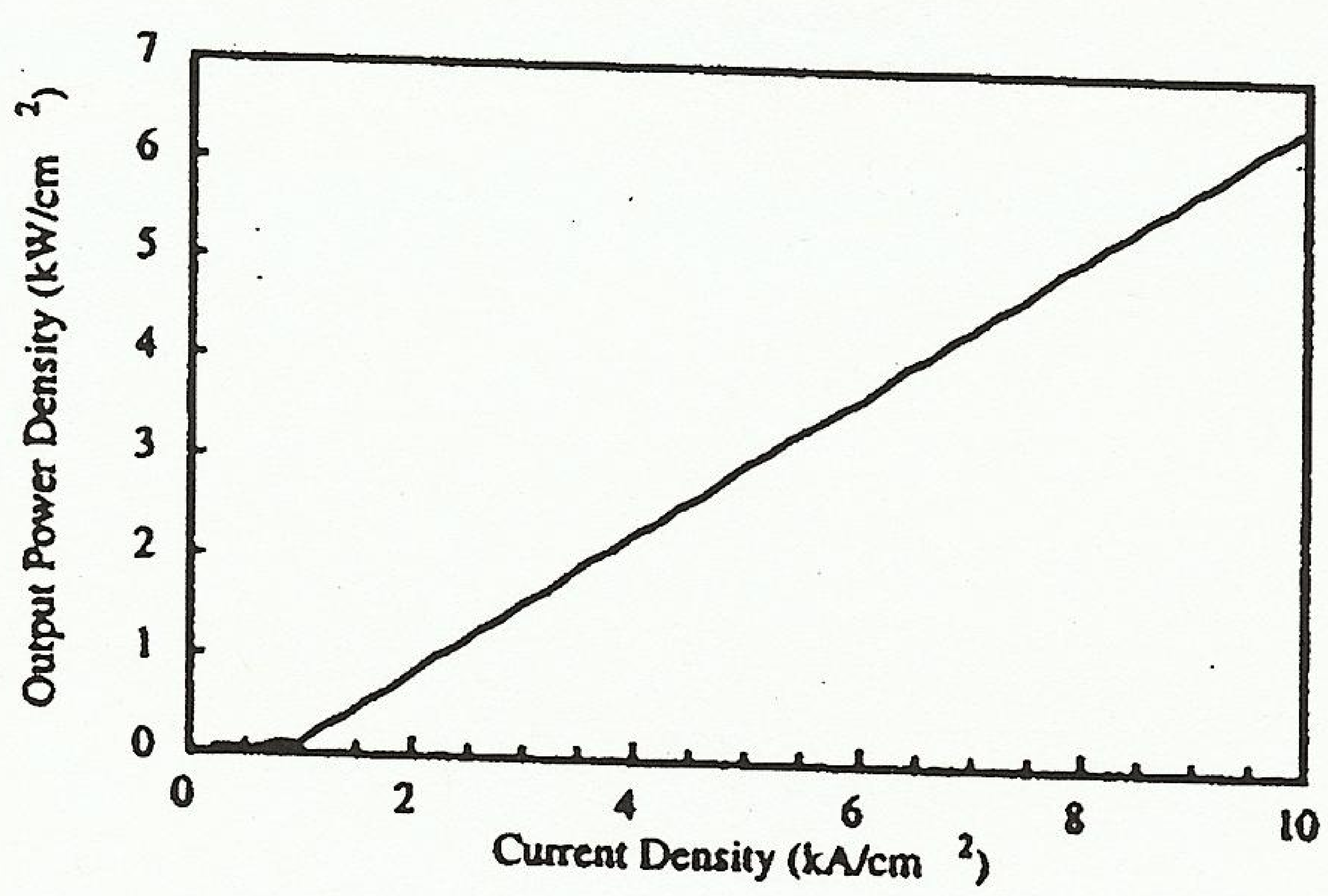

Figure 2.11b. Output power from both facets vs. current density

c)

$$\frac{\eta_i J_{th}}{qL_a} = BN_{th}^2$$

$$\Gamma g_{th} = <\alpha_i> + \alpha_m$$

$$g_{th} = a(N_{th} - N_0)$$

Combine these three equations and solve for J_{th}

$$J_{th} = \left(\frac{qL_aB}{\eta_i}\right)\left(\frac{<\alpha_i> + \alpha_m}{\Gamma a} + N_0\right)^2$$

N_0 is used as a fitting parameter for the gain versus current relation. It does not have physical significance as the transparency carrier density. For this problem, we assume that N_0 is much less than N_{th} and can, therefore, be neglected in the calculation of N_{th}: $N_0 \approx 0$. (See Fig. 2.10). Then

$$J_{th} = \frac{qL_aB}{\eta_i\Gamma^2a^2}\,(<\alpha_i> + \alpha_m)^2 = K\,(<\alpha_i> + \alpha_m)^2$$

where K is a proportionality constant.

Plug in values to solve for K:

$J = 10^3$ A/cm^2

$<\alpha_i> = 25$ cm^{-1}

$R = 0.99375$ from part (a)

$L = 1.5 \times 10^{-4}$ cm

$\alpha_m = 41.8$ cm^{-1}

$$K = 0.22 \text{ A}$$

$$J_{th} = (0.22 \text{ A})\left(25 \text{ cm}^{-1} + \frac{1}{1.5 \times 10^{-4} \text{ cm}} \ln\frac{1}{R}\right)^2$$

The problem also asks for differential efficiency:

$$\eta_d = \eta_i \frac{\alpha_m}{<\alpha_i> + \alpha_m} = \eta_i \frac{\frac{1}{L}\ln\frac{1}{R}}{<\alpha_i> + \frac{1}{L}\ln\frac{1}{R}}$$

$$\eta_d = 0.8\,\frac{\frac{1}{1.5\times10^{-4}}\ln\frac{1}{R}}{25 + \frac{1}{1.5\times10^{-4}}\ln\frac{1}{R}}$$

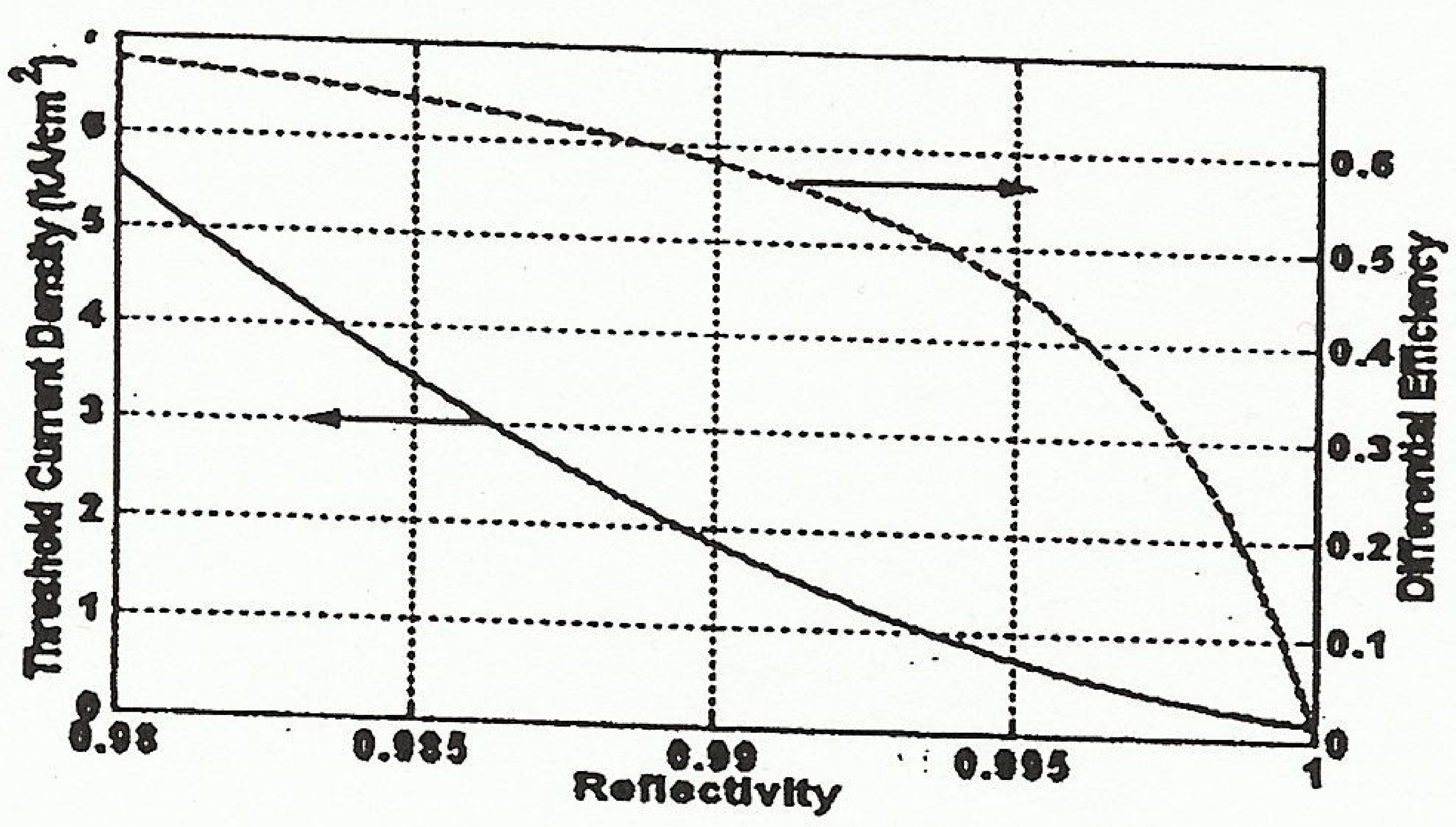

Figure 2.11c. Threshold current and differential efficiency vs. power reflectivity