3.5:

Basically, we want to plot the magnitude and phase of the following function:

$$r_{eff} = r_2 + \frac{t_2^2 r_3\, e^{-2j\beta_p L_p}}{1 + r_2 r_3\, e^{-2j\beta_p L_p}} \tag{3.34}$$

where

$$\beta_p = \frac{2\pi}{\lambda}\overline{n} - j\frac{<\alpha_i>}{2} \tag{2.19}$$

$\lambda = 1.3\mu m$

$\overline{n} = 1.6$

$<\alpha_i> = 5\ cm^{-1}$

$L_p = 200\mu m$

$r_3 = -0.9$

$r_2 = 0.5$

$t_2 = 0.3$

Since the power reflectivity of r_{eff} is always positive, the phase of r_{eff} must lie in the first or fourth quadrants.

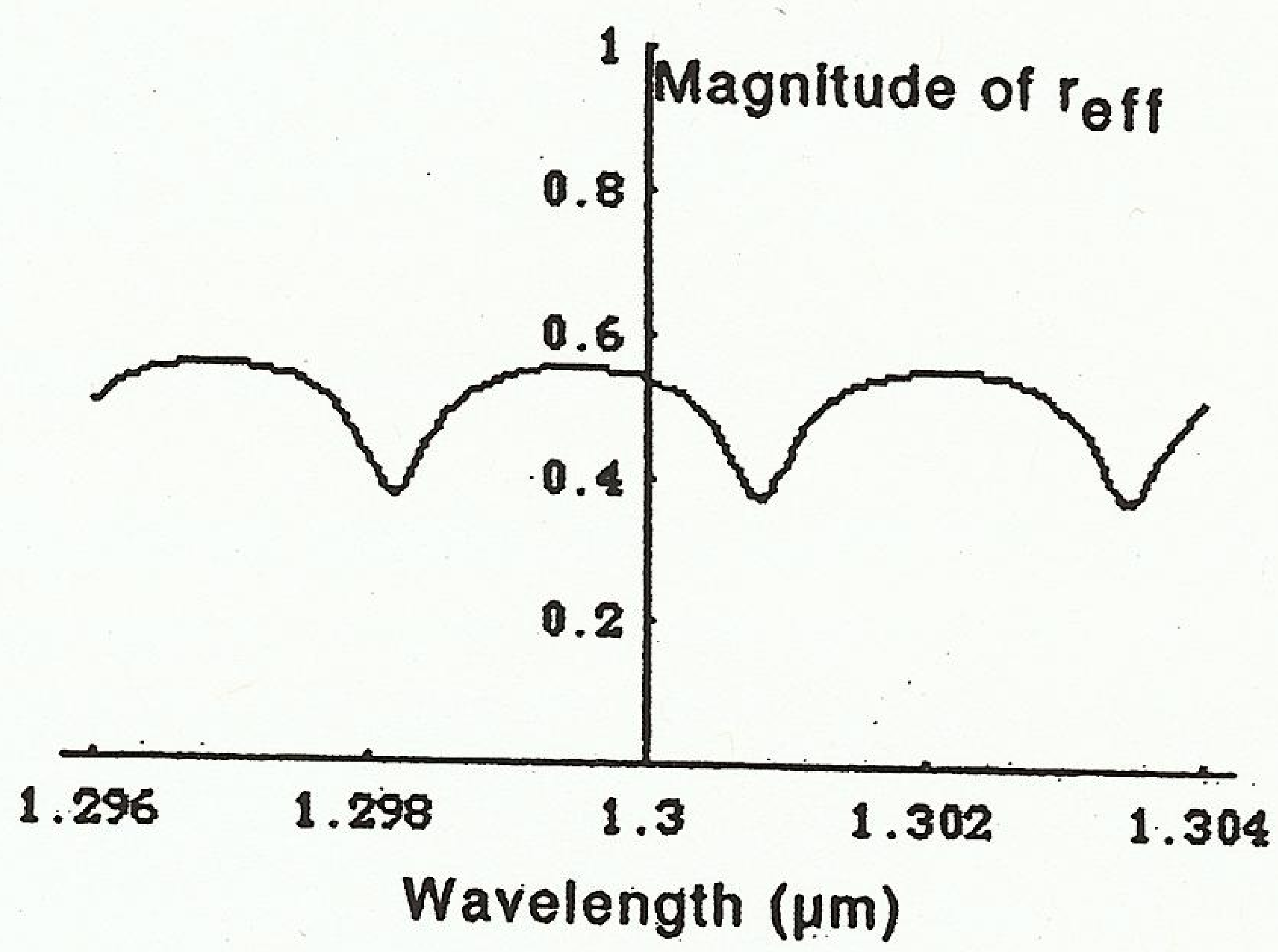

Figure 3.5. Magnitude of r_{eff}

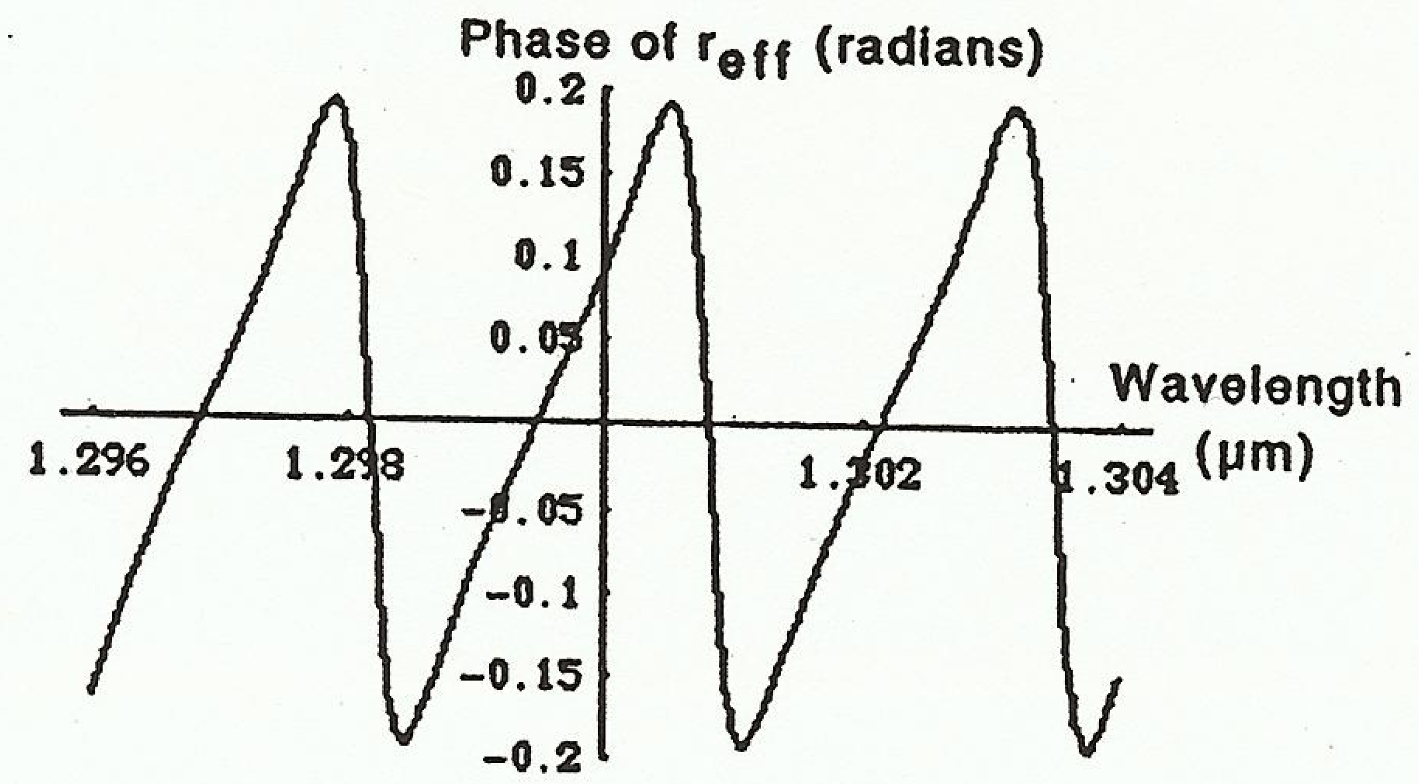

Figure 3.5. Phase of r_{eff}

3.7:

a) To be at a passive cavity resonance, the airspace must be a multiple of a half-wavelength:

$$L\bar{n} = m\frac{\lambda}{2}$$

where $n = 1.0$ for air and m is an integer.

$$L = m\frac{1.55\mu\text{m}}{2} > 1.2\mu\text{m}$$

The smallest L for which this is true occurs for $m = 2$:

$$L = 1.55\mu\text{m}$$

---

b)

$$\text{diffraction power loss} = e^{-<\alpha_i>L} = 50\%$$

First calculate the value of $<\alpha_i>$ and β.

$$<\alpha_i> = -\frac{1}{L}\ln 0.5 = 4.47 \times 10^3 \text{ cm}^{-1}$$

$$\begin{aligned}\tilde{\beta} &= \beta + \frac{j}{2}(<g> - <\alpha_i>) \\ &= \frac{2\pi\bar{n}}{\lambda} - \frac{j}{2} <\alpha_i> \\ &= \frac{2\pi\bar{n}}{1.55\mu\text{m}} - \frac{j}{2}0.447\mu\text{m}^{-1} \\ &= (4.05 - j0.224)\mu\text{m}^{-1}\end{aligned}$$

$r_1 = r_2 = \sqrt{R} = \sqrt{0.32} = -0.566$ (See note in solution to problem 2.6 about facet reflections.)

Assume lossless reflections:

$t_1 = t_2 = \sqrt{1 - r_1^2} = 0.825$

$$S_{S11} = -r_1 + \frac{t_1^2 r_2\, e^{-2j\tilde{\beta}L}}{1 - r_1 r_2\, e^{-2j\tilde{\beta}L}} = 0.566 + \frac{0.825^2(-0.566)(0.5)}{1-(-0.566)^2(0.5)} = 0.3367$$

$$S_{S21} = \frac{t_1 t_2\, e^{-j\tilde{\beta}L}}{1 - r_1 r_2\, e^{-2j\tilde{\beta}L}} = \frac{0.825^2\sqrt{0.5}}{1-(-0.566)^2 0.5} = 0.5724$$

By symmetry, $r_1 = r_2$ and $t_1 = t_2$. Therefore,

$$S_{S22} = S_{S11} = 0.33$$

$$S_{S12} = S_{S21} = 0.57$$

$$\sigma = \frac{S_{S21}S_{S12}}{S_{S11}S_{S22}} = \frac{0.5724^2}{0.3367^2} = 2.9$$

$r_3 = -r_1 = 0.566$

$$R_1' = S_{S22} r_3 \, e^{-2j\tilde{\beta}_p L_p}$$

For resonance, the exponential is =1.

$$R_1' = (0.3367)(0.566)(1) = 0.1905$$

$$r_{eff} = S_{S11}\left(1 + \frac{\sigma R'}{1-R'}\right) = (0.3324)\left(1 + \frac{3\,(0.1863)}{1-0.1863}\right) = 0.5744$$

In summary,

$$\begin{aligned} S_{S22} = S_{S11} &= 0.33 \\ S_{S12} = S_{S21} &= 0.57 \\ R_1' &= 0.19 \\ \sigma &= 2.9 \\ r_{eff} &= 0.57 \end{aligned}$$

---

c)

$$\tilde{\beta}_{gap} = \frac{2\pi}{\lambda} - j(0.224)\mu\text{m}^{-1}$$

$$S_{S11} = S_{S22} = -r + \frac{t^2 r \, e^{-2j\tilde{\beta}_{gap}L}}{1 - r^2 \, e^{-2j\tilde{\beta}_{gap}L}}$$

$$S_{S12} = S_{S21} = \frac{t^2 \, e^{-j\tilde{\beta}_{gap}L}}{1 - r^2 \, e^{-2j\tilde{\beta}_{gap}L}}$$

$$\text{Plot :} \quad r_{eff} = S_{S11}\left(1 + \frac{\sigma R'}{1-R'}\right)$$

At $\lambda = 1.55\mu m$, $\tilde{\beta}_p = \frac{2\pi\bar{n}}{1.55\mu m}$. At resonance, $\tilde{\beta}_p L_p = k\pi$, where k is an integer. This implies

$$L_p = \frac{1.55\mu\text{m}}{2\bar{n}} k$$

Choose $k = 500$. Then $L_p = 109\mu m$.

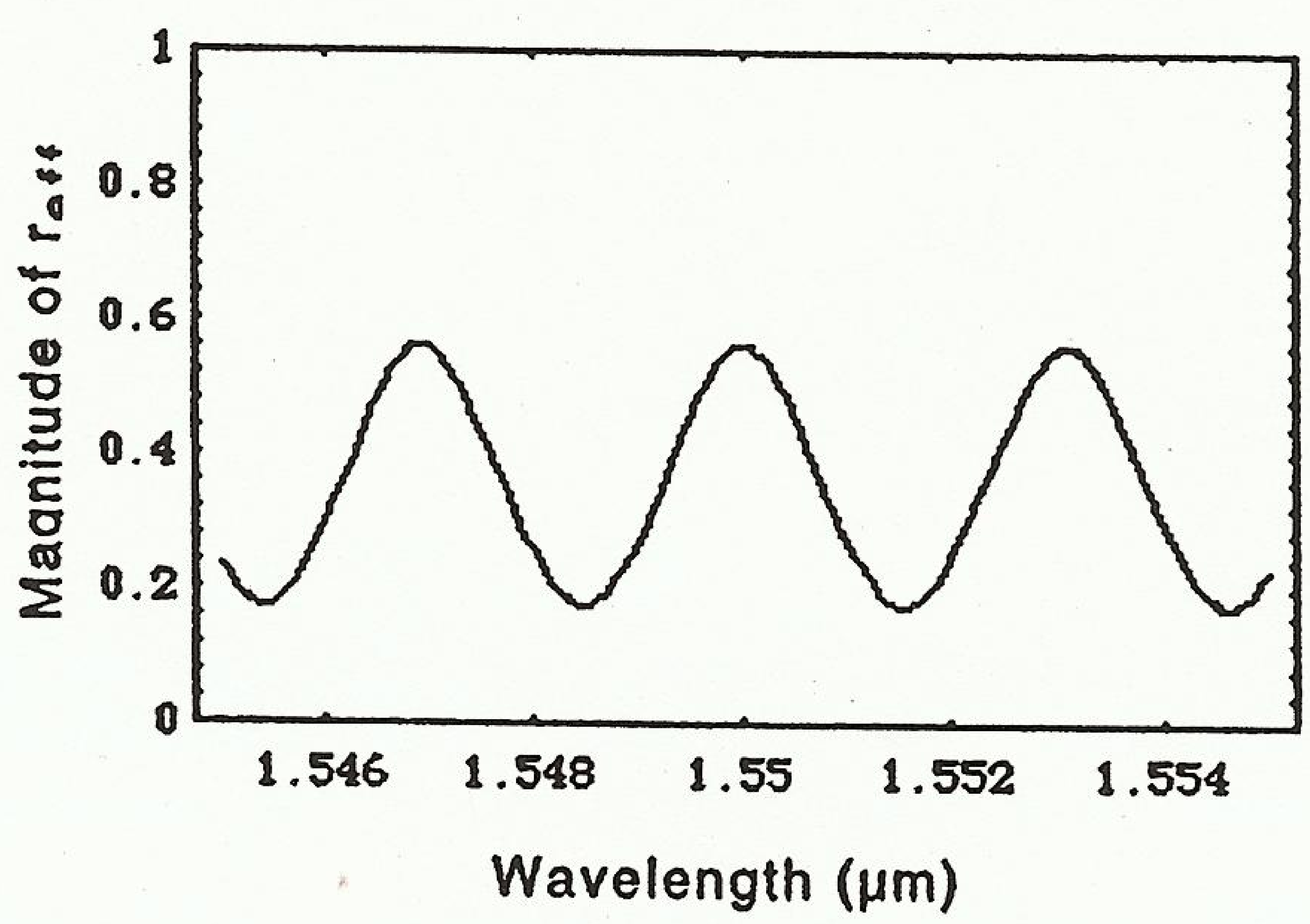

Figure 3.7c. Magnitude of r_{eff}