4.2:

a) We have two descriptions for g_{21}.

$$g_{21} = g_{max}(f_2 - f_1) \qquad 4.33$$

$$g_{21} = \frac{\lambda_0^2}{8\pi n^2 \tau_{sp}^{21}} h\rho_r(E_{21})(f_2 - f_1) \qquad A6.25$$

Setting these expressions equal, we can solve for τ_{sp}^{21}.

$$\tau_{sp}^{21} = \frac{\lambda_0^2}{8\pi n^2 g_{max}} h\rho_r(E_{21})$$

$g_{max} = 10^4\ \text{cm}^{-1}$.
From Table 1.1:

	E_g	n
$Al_{.2}Ga_{.8}As$	1.673 eV	
$GaAs$	1.424 eV	3.62

Find the C1–HH1 band edge energy by solving for the quantum well states in the conduction and valence bands. $\Delta E_g = 0.249 eV$ from Table 1.1. Assuming that 60% of the band offset occurs within the conduction band as in the previous problem (see also p. 10), we have $\Delta E_C = 166$ meV and $\Delta E_V = 83$ meV. Using Eqs. A1.14, A1.17, and Fig. A1.4, we can derive the energy levels of the quantum wells as we have done in previous problems, such as 4.1:

$E_{C1}^{\infty} - E_C = 56.12$ meV
$E_V - E_{HH1}^{\infty} = 9.89$ meV

$E_{C1} - E_C = 29$ meV
$E_{C2} - E_C = 107$ meV
$E_V - E_{HH1} = 6.8$ meV
$E_V - E_{HH2} = 27$ meV

$$E_{21} = 1.424\ \text{eV} + 0.029\ \text{eV} + 0.007\ \text{eV} = 1.46\ \text{eV} \qquad (\text{C1} - \text{HH1 band edge})$$

$$E_{21} = 1.424\ \text{eV} + 0.107\ \text{eV} + 0.027\ \text{eV} = 1.56\ \text{eV} \qquad (\text{C2} - \text{HH2 band edge})$$

$$\lambda_0 = \frac{1.24\ \text{eV}\,\mu\text{m}}{1.46\ \text{eV}} = 0.849\ \mu\text{m}$$

$$\begin{aligned}\rho_r(E_{21}) &= \frac{m_r^*}{\pi\hbar^2 d} \\ &= \frac{(0.057)(9.1\times10^{-31}\ \text{kg})}{\pi(1.05\times10^{-34}\ \text{J}\,s)^2\ (10^{-8}\ \text{m})} \\ &= 1.50\times10^{44}\ \text{m}^{-3}\ J^{-1}\end{aligned}$$

$$\begin{aligned}\tau_{sp}^{21} &= \frac{(8.49\times10^{-7}\ \text{m})^2\ (6.62\times10^{-34}\ \text{J}\,s)\ (1.50\times10^{44}\ \text{m}^{-3}\ J^{1})}{8\pi(3.62)^2\ (10^6\ \text{m}^{-1})} \\ &= 2.17\times10^{-10}\ \text{s} \\ &= 0.217\ \text{ns}\end{aligned}$$

b)

$$g_{21} = \sum g_{21}^{sub}$$

$$g_{21}^{sub}(E_{21}) = g_{max}(E_{21})(f_2 - f_1)$$

where $g_{max} = 10^4$ cm^{-1} at $E_{21} = 1.46$ eV. For C–HH transitions g_{max} has a $\frac{1}{E_{21}}$ dependence according to Eq. 4.37 ($h\nu$ in denominator). If we assume $\rho_r(E_{21})$ and $|M_T(E_{21})|^2$ do not vary with transition energy, $g_{max} \propto \frac{1}{E_{21}}$.

$$g_{21}^{sub}(E_{21}) = (10^4\ \text{cm}^{-1})\ \frac{E_{g_{ii}}\ \text{eV}}{E_{21}}\ (f_2 - f_1)$$
$$= (10^4\ \text{cm}^{-1})\ \frac{E_{g_{ii}}\ \text{eV}}{E_{21}}\ \left(1 - \frac{1}{1 + e^{(E_1 - E_{FV})/kT}}\right)$$

where we let $E_{FC} \rightarrow \infty \Rightarrow f_2 = 1$

We must consider C1–HH1 and C2–HH2 transitions. All others are forbidden.

$$g_{21} = g_{21}^{sub}(1,1) + g_{21}^{sub}(2,2)$$

For C1–HH1 transitions (i.e. (1,1)), $E_1 = E_{HH1} - (E_{21} - E_{g11})\ \frac{m_r}{m_v}$ (4.29)

For C2–HH2 transitions (i.e. (2,2)), $E_1 = E_{HH2} - (E_{21} - E_{g22})\ \frac{m_r}{m_v}$

For case (i), $E_{FV} = E_{HH1}$

For the C1–HH1 transition,

$$E_1 - E_{FV} = -(E_{21} - E_{g11})\frac{m_r}{m_v}$$
$$= -(E_{21} - 1.46)\frac{0.057}{0.38}$$
$$= (-E_{21} + 1.46)0.15$$

For the C2–HH2 transition,

$$E_1 - E_{FV} = (E_{HH2} - E_{HH1}) - (E_{21} - E_{g22})\frac{m_r}{m_v}$$
$$= ((E_v - 0.027) - (E_v - 0.007))\ \text{eV} - (E_{21} - 1.56\ \text{eV})\frac{0.057}{0.38}$$
$$= (-E_{21} + 1.43\ \text{eV})0.15$$

For case (ii), $E_{FV} = E_{HH2}$

For the C1–HH1 transition,

$$E_1 - E_{FV} = (E_{HH1} - E_{HH2}) - (E_{21} - E_{g11})\frac{m_r}{m_v}$$
$$= ((E_v - 0.007) - (E_v - 0.027))\ \text{eV} - (E_{21} - 1.46\ \text{eV})\frac{0.057}{0.38}$$
$$= (-E_{21} + 1.59\ \text{eV})0.15$$

For the C2–HH2 transition,

$$E_1 - E_{FV} = -(E_{21} - E_{g22})\frac{m_r}{m_v}$$
$$= -(E_{21} - 1.56)\frac{0.057}{0.38}$$
$$= (-E_{21} + 1.56)0.15$$

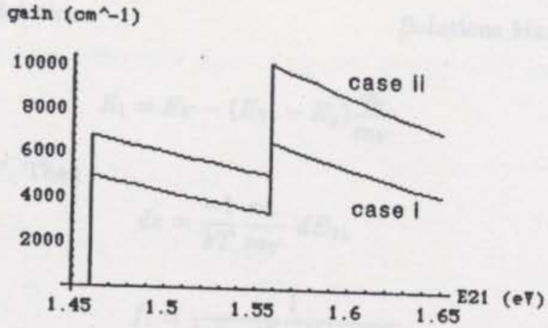

Figure 4.2b. b) QW gain as a function of photon energy.

c)

$$R_{sp}^{21} = A_{21}\rho_r(E_{21})f_2(1-f_1) \tag{A6.31}$$

Use substitution: $A_{21} = 1/\tau_{sp}^{21}$.

A maximum occurs when ($f_2 = 1, f_1 = 0$).

$$\begin{aligned} R_{sp,max}^{21} &= \frac{\rho_r(E_{21})}{\tau_{sp}^{21}} \\ &= \frac{1.50 \times 10^{44}\ \mathrm{m^{-3}\ J^{-1}}}{2.17 \times 10^{-10}\ \mathrm{s}} \\ &= 6.91 \times 10^{53}\ \mathrm{m^{-3}\ J^{-1}\ s^{-1}} \\ &= 1.11 \times 10^{29}\ \mathrm{cm^{-3}\ eV^{-1}\ s^{-1}} \end{aligned}$$

d)

$$V = (10^{-6}\ \mathrm{cm})\ (2 \times 10^{-4}\ \mathrm{cm})\ (200 \times 10^{-4}\ \mathrm{cm}) = 4 \times 10^{-12}\ \mathrm{cm^3}$$

$$R_{sp} = \int R_{sp}^{21}\, dE_{21} = \frac{I_{rad}}{qV}$$

$$I_{rad} = qV \int R_{sp}^{21}\, dE_{21}$$

$$R_{sp}^{21} = R_{sp,max}^{21}\ f_2(1-f_1)$$

Note: The assumptions stated in the problem allow us to assume $R_{sp,max}^{21}$ is independent of energy or, equivalently, wavelength. This is not normally the case.

We must integrate for both C1-HH1 and C2-HH2 transitions. We again assume $f_2 = 1$ and $f_1 = \frac{1}{1+e^{\frac{E_1-E_{FV}}{kT}}}$.

$$I_{rad} = qV R_{sp,max}^{21} \int (1-f_1)\, dE_{21}$$

$$(1-f_1) = \frac{1}{1+e^{-\frac{E_1-E_{FV}}{kT}}}$$

$$E_1 = E_V - (E_{21} - E_g)\frac{m_r}{m_V} \tag{4.29}$$

Define $x = (E_1 - E_{FV})/kT$. Then

$$dx = \frac{-1}{kT}\frac{m_r}{m_V}\, dE_{21}$$

$$f_1 = \frac{1}{1+\ e^{(E_1 - E_{FV})/kT}}$$
$$= \frac{1}{1+\ e^{x}}$$

$$1 - f_1 = 1 - \frac{1}{1+\ e^{x}}$$
$$= \frac{e^x}{1+\ e^{x}}$$
$$= \frac{1}{1+\ e^{-x}}$$

$$\int (1-f_1)\ dE_{21} = \int_{E_g}^{\infty} \frac{dE_{21}}{1+\ e^{-x}}$$
$$= \frac{m_V}{m_r}\ kT \int_{E_{21}=E_g}^{E_{21}=\infty} \frac{1}{1+\ e^{-x}}\ dx$$
$$= -\frac{m_V}{m_r}\ kT\ [\ln(1+\ e^x)|_{E_{21}=E_g}^{E_{21}=\infty}$$

where $E_g = E_{g11}$ for the C1–HH1 transitions and $E_g = E_{g22}$ for the C2–HH2 transitions.

---

case (1): $E_{FV} = E_{HH1}$:

C1–HH1:

Using the definition above for E_1, with $E_V = E_{HH1}$ for the C1–HH1 transition, we have

$x = (E_1 - E_{FV})/kT$

$= (E_1 - E_{HH1})/kT$

$= (E_{HH1} - (E_{21} - E_g)\frac{m_r}{m_V} - E_{HH1})/kT$

$= -(E_{21} - E_g)\frac{m_r}{m_V}\frac{1}{kT}$

$x(E_{21} = E_{g11}) = 0$

$x(E_{21} = \infty) = -\infty$

Then solving for the integral of the fermi function as described above, we have

$$\int (1-f_1)\ dE_{21} = -\frac{m_V}{m_r}\ kT\ [\ln(1+\ e^x)|_{E_{21}=E_g}^{E_{21}=\infty}$$
$$= -\frac{m_V}{m_r}\ kT\ [\ln(1+\ e^x)|_{x=0}^{x=-\infty}$$
$$= -\frac{m_V}{m_r}\ kT\ [\ln(1) - \ln(2)]$$
$$= \frac{m_V}{m_r}\ kT\ \ln(2)$$
$$= \frac{m_V}{m_r}\ kT\ 0.693$$

C2–HH2:

Using the definition above for E_1, with $E_V = E_{HH1}$ for the C2–HH2 transition, we have

$$\begin{aligned}
x &= (E_1 - E_{FV})/kT \\
&= (E_1 - E_{HH1})/kT \\
&= (E_{HH2} - (E_{21} - E_g)\frac{m_r}{m_V} - E_{HH1})/kT \\
&= (E_{HH2} - E_{HH1})/kT - (E_{21} - E_g)\frac{m_r}{m_V}\frac{1}{kT} \\
&= -0.77 - (E_{21} - E_g)\frac{m_r}{m_V}\frac{1}{kT}
\end{aligned}$$

$x(E_{21} = E_{g22}) = -0.77$

$x(E_{21} = \infty) = -\infty$

Then solving for the integral of the fermi function as described above, we have

$$\begin{aligned}
\int (1 - f_1)\, dE_{21} &= -\frac{m_V}{m_r}\, kT\ [\ln(1 + e^x)|_{E_{21}=E_g}^{E_{21}=\infty} \\
&= -\frac{m_V}{m_r}\, kT\ [\ln(1 + e^x)|_{x=-0.77}^{x=-\infty} \\
&= -\frac{m_V}{m_r}\, kT\ [\ln(1) - \ln(1.46)] \\
&= \frac{m_V}{m_r}\, kT\ 0.380
\end{aligned}$$

$$I_{rad} = (1.6 \times 10^{-19})\ (4 \times 10^{-12}\ \text{cm}^3)\ (1.11 \times 10^{29}\ \text{cm}^{-3}\, eV^{-1}\, s^{-1}) \left(\frac{0.38}{0.057}\ (0.026\ \text{eV})\ (0.693 + 0.380)\right)$$

$= 13.2$ mA

---

case (2): $E_{FV} = E_{HH2}$:

C1–HH1:

Using the definition above for E_1, with $E_V = E_{HH2}$ for the C1–HH1 transition, we have

$$\begin{aligned}
x &= (E_1 - E_{FV})/kT \\
&= (E_1 - E_{HH2})/kT \\
&= (E_{HH1} - (E_{21} - E_g)\frac{m_r}{m_V} - E_{HH2})/kT \\
&= (E_{HH1} - E_{HH2})/kT - (E_{21} - E_g)\frac{m_r}{m_V}\frac{1}{kT} \\
&= 0.77 - (E_{21} - E_g)\frac{m_r}{m_V}\frac{1}{kT}
\end{aligned}$$

$x(E_{21} = E_{g11}) = 0.77$

$x(E_{21} = \infty) = -\infty$

Then solving for the integral of the fermi function as described above, we have

$$\begin{aligned}
\int (1 - f_1)\, dE_{21} &= -\frac{m_V}{m_r}\, kT\ [\ln(1 + e^x)|_{E_{21}=E_g}^{E_{21}=\infty} \\
&= -\frac{m_V}{m_r}\, kT\ [\ln(1 + e^x)|_{x=0.77}^{x=-\infty} \\
&= -\frac{m_V}{m_r}\, kT\ [\ln(1) - \ln(3.16)] \\
&= \frac{m_V}{m_r}\, kT\ 1.151
\end{aligned}$$

4.3:

Show

$$f_1(E_{HH1}) = (1 - f_2(E_{C1}))^{\frac{1}{D}}$$

From Appendix 2,

$$N = \frac{m_c^* kT}{\pi \hbar^2 d} \ln\left(1 + e^{-\frac{E_{C1}-E_{FC}}{kT}}\right) \qquad (A2.7)$$

Similarly,

$$P = \frac{m_V^* kT}{\pi \hbar^2 d} \ln\left(1 + e^{\frac{-(E_{FV}-E_{HH1})}{kT}}\right)$$

(consider only C1 and HH1)

By charge neutrality, $N = P$. This implies that

$$m_c^* \ln\left(\frac{1}{1 - f_2(E_{C1})}\right) = m_V^* \ln\left(\frac{1}{f_1(E_{HH1})}\right)$$

and

$$f_1(E_{HH1}) = (1 - f_2(E_{C1}))^{\frac{1}{D}}$$

where $D = \frac{m_V^*}{m_c^*}$

a) Fraction of empty states at $E_{HH1} = 1 - f_1(E_{HH1})$.

b) Stimulated absorption fermi factor $= (1 - f_2(E_{C1}))f_1(E_{HH1})$.

Stimulated emission fermi factor $= f_2(E_{C1})(1 - f_1(E_{HH1}))$

c) $g_{21} = g_{max}(f_2(E_{C1}) - f_1(E_{HH1}))$

Normalized gain $= \frac{g_{21}}{g_{max}} = f_2(E_{C1}) - f_1(E_{HH1})$

d) $N = \frac{m_c^* kT}{\pi \hbar^2 d} \ln\left(1 + e^{-\frac{E_{C1}-E_{FC}}{kT}}\right) = \frac{m_c^* kT}{\pi \hbar^2 d} \ln\left(\frac{1}{1 - f_2(E_{C1})}\right)$

$$\text{Normalized carrier density} = \frac{N(f_2(E_{C1}))}{N(f_2 = 0.5)} = \frac{\ln(\frac{1}{1-f_2(E_{C1})})}{\ln(2)}$$

a)

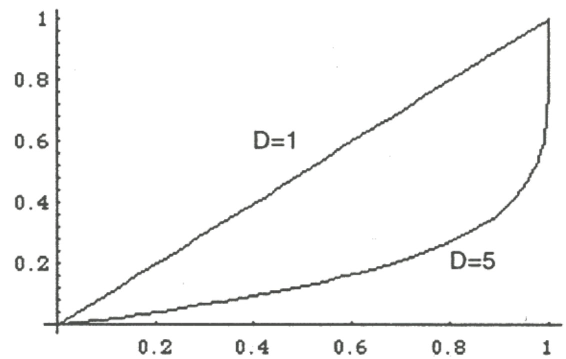

Figure 4.3a. Fraction of empty states at E_{HH1}.

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b)

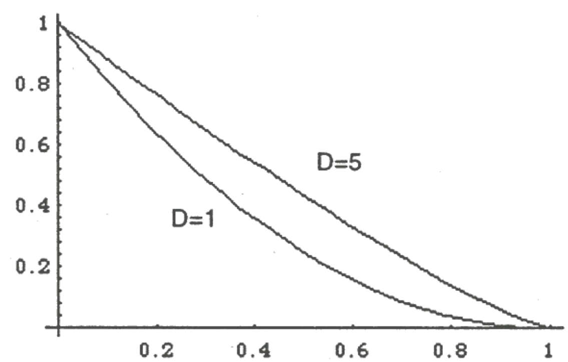

Figure 4.3b. Stimulated absorption.

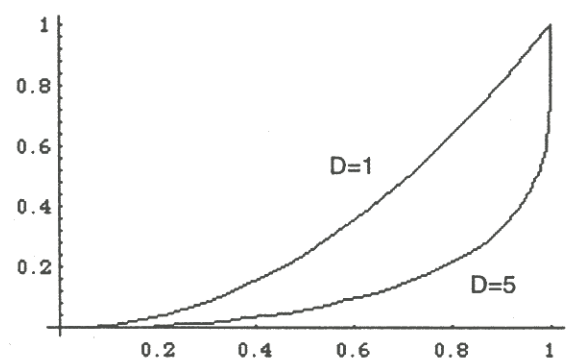

Figure 4.3b. Stimulated emission.

c)

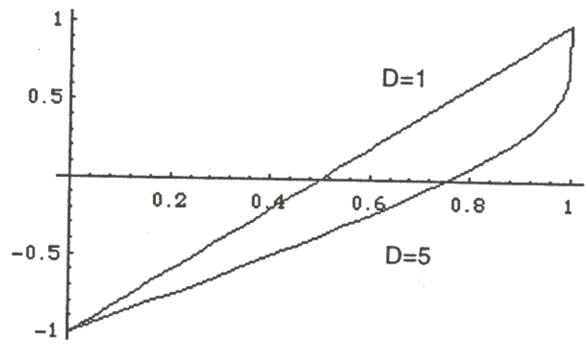

Figure 4.3c. Normalized gain.

d)

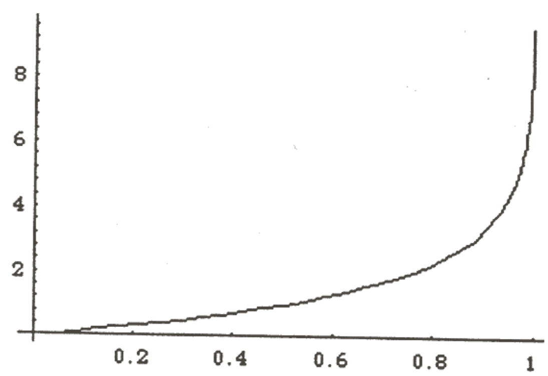

Figure 4.3d. Normalized carrier density.

e) Transparency occurs when

$g = 0$

or equivalently, when $f_2 - f_1 = 0$

or equivalently, when stimulated absorption = stimulated emission.

Plots (b) and (c) show that the symmetric ($D = 1$) band structure reaches transparency first. The reason is that we require neutrality ($N = P$) to hold and in the asymmetric case, there is a much higher DOS in the valence band than the conduction band. Hence, one must push E_{FC} well into the conduction band before $E_{FC} - E_{FV} = E_g (= E_{C1} - E_{HH1})$ and $f_2 - f_1 = 0$.

f) The symmetric structure reaches transparency at $f_2 = 0.5$ while the asymmetric structure reaches transparency at $f_2 = 0.75$ according to plot (c). This corresponds to a normalized carrier density of 1 for the symmetric case and 2 for the asymmetric case according to plot (d). If current density $\propto BN^2$, the ratio of the asymmetric current density to the symmetric current density will be 4:1.

---

g) We start with a definition that we will need. By eqs. 4.37, 4.25, Table 4.3, and $D = \frac{m^*_{HH}}{m^*_c}$.

$$
\begin{aligned}
g_{max} &= \frac{\pi q^2 \hbar}{n\epsilon_0 c m_0^2}\frac{1}{E_{21}}|M_T(E_{21})|^2\,\rho_r(E_{21}) \\
&= \frac{\pi q^2 \hbar}{n\epsilon_0 c m_0^2}\frac{1}{E_{21}}|M_T(E_{21})|^2\frac{1}{\pi\hbar^2 d}\frac{m^*_c m^*_V}{m^*_c + m^*_V} \\
&= \frac{\pi q^2 \hbar}{n\epsilon_0 c m_0^2}\frac{1}{E_{21}}|M_T(E_{21})|^2\frac{1}{\pi\hbar^2 d}\frac{m^*_c}{\frac{1}{D}+1} \\
&= \frac{\pi q^2 \hbar}{n\epsilon_0 c m_0^2}\frac{1}{E_{21}}|M_T(E_{21})|^2\frac{1}{\pi\hbar^2 d}\frac{m^*_c D}{D+1}
\end{aligned}
$$

Note that π in denominator of Eq. 4.37 should be n.

First, we calculate $\frac{dg_{21}}{df_2}$.

$$g_{21} = g_{max}(f_2 - f_1)$$

$$f_1 = (1-f_2)^{\frac{1}{D}}$$

$$\frac{dg_{21}}{df_2} = g_{max}\left(1 + \frac{1}{D}(1-f_2)^{\frac{1}{D}-1}\right)$$

Next, we calculate $\frac{dN}{df_2}$.

$$N = \frac{m^*_c kT}{\pi\hbar^2 d}\ln\left(\frac{1}{1-f_2}\right)$$

$$\frac{dN}{df_2} = \frac{m^*_c kT}{\pi\hbar^2 d}\frac{1}{1-f_2}$$

$$
\begin{aligned}
\frac{dg_{21}}{dN} &= \frac{\frac{dg_{21}}{df_{21}}}{\frac{dN}{df_{21}}} \\
&= \frac{\pi q^2 \hbar}{n\epsilon_0 c m_0^2 kT}\frac{1}{E_{21}}|M_T(E_{21})|^2\left(\frac{D}{D+1}\right)\left((1-f_2) + \frac{1}{D}(1-f_2)^{\frac{1}{D}}\right) \\
&\propto \left(\frac{D}{D+1}\right)\left((1-f_2) + \frac{1}{D}(1-f_2)^{\frac{1}{D}}\right)
\end{aligned}
$$

Evaluating at transparency:

$$
\begin{array}{lll}
D = 1: & f_2 = 0.5 & \frac{dg_{21}}{dN} \propto 0.5 \\
D = 5: & f_2 = 0.75 & \frac{dg_{21}}{dN} \propto 0.33
\end{array}
$$

$$\text{ratio} = \frac{0.5}{0.33} = 1.52$$

Therefore, the symmetric case has 50% higher differential gain than the asymmetric case.

---

h) From the above discussion, we clearly want a symmetric band structure. Strain lowers the asymmetry parameter D. As a result, we expect lower N_{tr} and higher differential gain. This leads to lower threshold current and higher speed performance.