4.6:

Find carrier density profile for both case (i) and case (ii):

The carrier diffusion equation is

$$D_{np}\frac{d^2N(x)}{dx^2} = -\frac{I(x)}{qV} + \frac{N(x)}{\tau_{np}} \tag{4.94}$$

$$\Rightarrow \frac{d^2N(x)}{dx^2} - \frac{N(x)}{D_{np}\tau_{np}} = -\frac{I(x)}{qVD_{np}}$$

Define x=0 as the (lateral) center of the stripe.

$$I(x) = \begin{cases} I_0 & |x| \le |\frac{W}{2}| \\ 0 & |x| \ge |\frac{W}{2}| \end{cases}$$

Define $L \equiv \sqrt{D_{np}\tau_{np}}$ and $G \equiv \frac{I_0}{qVD_{np}}$.

Then the carrier diffusion equation can be written as

$$\frac{d^2N(x)}{dx^2} - \frac{N(x)}{L^2} = -G$$

For $|x| < |\frac{W}{2}|$ (within the stripe width):

The carrier diffusion equation has a solution of

$$\Rightarrow N(x) = A\cosh\left(\frac{x}{L}\right) + GL^2 (\text{symmetric about } x = 0)$$

where A is a constant to be determined by matching boundary conditions.

For $|x| > |\frac{W}{2}|$ (outside the stripe width):

The carrier diffusion equation reduces to

$$\frac{d^2N(x)}{dx^2} - \frac{N(x)}{L^2} = 0$$

and has a solution

$$N(x) = B\, e^{-|x|/L}$$

where B is a constant to be determined by matching boundary conditions.

$e^{+|x|/L}$ is not allowed because the carrier density does not go to ∞ as $|x| \to \infty$.

Case (i)

Match boundary conditions at $x = \frac{w}{2}$:

1) $N(x)$ is continuous.

2) $\frac{dN}{dx}$ is continous.

Matching the boundary conditions and solving for A and B from the two equations yields

$$A = \frac{-1}{2} GL^2 \, e^{-w/2L}$$

$$B = -A\left(e^{w/L} - 1\right) = \frac{GL^2}{2}\left(e^{w/2L} - e^{-w/2L}\right)$$

So the carrier density profile is

$$N(x) = \begin{cases} GL^2\left[1 + \frac{-1}{2}\left(e^{(x-\frac{w}{2})/L} + e^{(-x-\frac{w}{2})/L}\right)\right] & |x| \le \frac{W}{2} \\ \frac{GL^2}{2}\left(e^{(\frac{w}{2}-|x|)/L} - e^{(-\frac{w}{2}-|x|)/L}\right) & |x| > \frac{W}{2} \end{cases}$$

Case (ii)

In this case, the solution to the carrier density equation within the stripe is the same as for Case (i). However, the boundary conditions are different. Match boundary condition at $x = \frac{w}{2}$:

1) $-qv_s N = qD_{np}\frac{dN}{dx}$

Define $v_{SD} \equiv \sqrt{\frac{D_{np}}{\tau_{np}}}$.

Matching the boundary condition and solving for A yields a carrier density profile of

$$N(x) = GL^2\left[1 - \frac{e^{x/L} + e^{-x/L}}{\left(1 + \frac{v_{SD}}{v_S}\right)e^{w/2L} + \left(1 - \frac{v_{SD}}{v_S}\right)e^{-w/2L}}\right] \qquad |x| \le \frac{W}{2}$$

Compare carrier profile for cases (i) and (ii):

When $v_{SD} = v_S$, $N(x)$ is the same for the two cases. If v_{SD} is increased, then $N(x)$ increases. This implies that the parasitic recombination current decreases by increasing v_{SD}. For $v_{SD} < v_S$, $N(x)$ is higher for case (i), and therefore the active layer should not be etched (as in case (i)). For $v_{SD} > v_S$, $N(x)$ is higher for case (ii), and therefore the active layer should be etched (as in case (ii)).

Calculate $N(x)$ for $J = 166.4$ A/cm^2:

Consider stripes of width 2μm, 10μm, and 20μm. Plot four curves for each stripe width:

1) case (i)
2) case (ii) with $v_S = 1 \times 10^4$ cm/s
3) case (ii) with $v_S = 1 \times 10^5$ cm/s
4) case (ii) with $v_S = 5 \times 10^5$ cm/s

Given:

$D_{np} = 20$ cm^2/s

$A = C = 0$ and $B = \frac{J_{th}}{qdN_{th}}$ from Fig. 4.14

From Fig. 4.14, $J_{th} = 166.4$ A/cm^2, $N_{th} = 3.62 \times 10^{18}$ cm^{-3}, $L = 250\mu$m, and $d = 80\text{Å}$.

Then solve for parameters needed in carrier density profile:

$\tau_{np} = \frac{1}{BN_{th}} = \frac{qdN_{th}^2}{J_{th}N_{th}} = 2.78$ ns

$v_{SD} = \sqrt{\frac{D_{np}}{\tau_{np}}} = 8.47 \times 10^4$ cm/s

$GL^2 = \frac{J\tau_{np}}{qD_{np}} = 3.62 \times 10^{18}$ cm^{-3}

$L = \sqrt{D_{np}\tau_{np}} = 2.36 \times 10^{-4}$ cm $= 2.36\mu$m

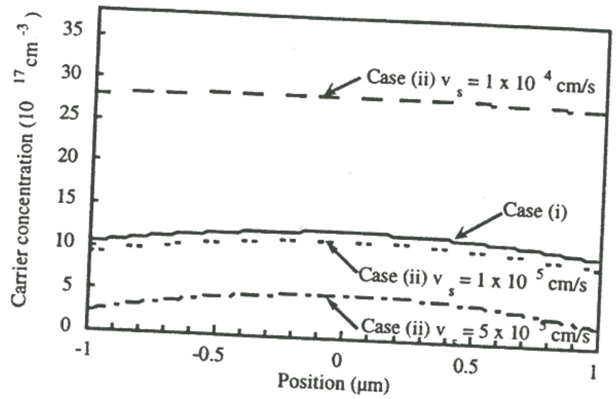

Figure 4.6. Carrier density profile for $w = 2\ \mu$m

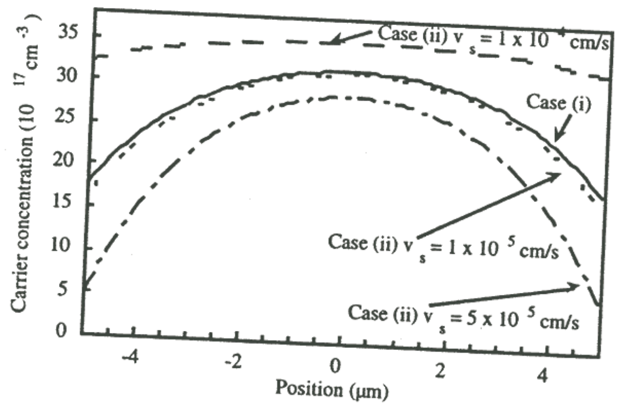

Figure 4.6. Carrier density profile for $w = 10\ \mu$m

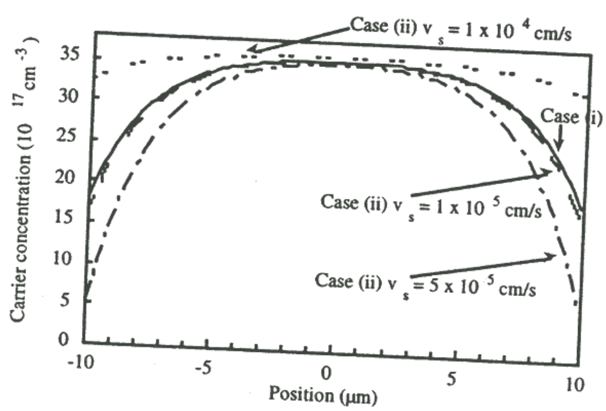

Figure 4.6. Carrier density profile for $w = 20\ \mu$m

Calculate J_{th}.

The threshold carrier density is $N_{th}(x = 0) = 3.62 \times 10^{18}$ cm^{-3} as given in Fig. 4.14. Consider cases (i) and (ii) as defined by the problem:

case (i):

$$N(0) = GL^2(1 - e^{-\frac{W}{2L}})$$

$$GL^2 = \frac{J_0 L^2}{qdD}$$

$$\Rightarrow N(0) = \frac{J_0 L^2}{qdD}\,(1 - e^{-\frac{W}{2L}})$$

$$\Rightarrow J_0 = J_{th} = \frac{qdDN(0)}{L^2(1 - e^{-\frac{W}{2L}})}$$
$$= \frac{(1.6 \times 10^{-19}\ C)(80 \times 10^{-8}\ \text{cm})(20\ cm^2/s)(3.62 \times 10^{18}\ \text{cm}^{-3})}{(2.36 \times 10^{-4}\ \text{cm})^2\ (1 - e^{-\frac{W}{2(2.36\mu m)}})}$$
$$= \frac{166.4\ \frac{A}{cm^2}}{(1 - e^{-\frac{W}{4.72\mu m}})}$$

$$w = 2\mu\text{m} \quad \Rightarrow \quad J_{th} = 482\ \frac{A}{cm^2}$$
$$w = 10\mu\text{m} \quad \Rightarrow \quad J_{th} = 189\ \frac{A}{cm^2}$$
$$w = 20\mu\text{m} \quad \Rightarrow \quad J_{th} = 169\ \frac{A}{cm^2}$$

case (ii):

$$N(0) = GL^2\left(1 - \frac{v_S}{v_{SD}\sinh\left(\frac{W}{2L}\right) + v_S\cosh\left(\frac{W}{2L}\right)}\right)$$
$$= \frac{J_0 L^2}{qdD}\left(1 - \frac{v_S}{v_{SD}\sinh\left(\frac{W}{2L}\right) + v_S\cosh\left(\frac{W}{2L}\right)}\right)$$
$$J_0 = J_{th} = \frac{N(0)qdD}{L^2}\frac{1}{\left(1 - \frac{v_S}{v_{SD}\sinh\left(\frac{W}{2L}\right) + v_S\cosh\left(\frac{W}{2L}\right)}\right)}$$
$$= \frac{166\ \frac{A}{cm^2}}{\left(1 - \frac{v_S}{v_{SD}\sinh\left(\frac{W}{2L}\right) + v_S\cosh\left(\frac{W}{2L}\right)}\right)}$$

$J_{th}\ \frac{A}{cm^2}$	$w = 2\mu m$	$w = 10\mu m$	$w = 20\mu m$
$v_S = 10^4\ \frac{cm}{s}$	210	170	167
$v_S = 10^5\ \frac{cm}{s}$	526	191	169
$v_S = 5 \times 10^5\ \frac{cm}{s}$	1170	209	171

Now, calculate the approximate threshold using Eq. 4.67:

$$
\begin{aligned}
J_{sr} &= qv_S N\left(\frac{a_S}{V}d\right) \\
&= qv_S Nd\left(\frac{2}{w}+\frac{2}{L}\right) \\
&= (1.6\times10^{-19}\ \mathrm{C})v_S(3.62\times10^{18}\ \mathrm{cm}^{-3})(80\times10^{-4}\mu\mathrm{m})2\left(\frac{1}{w}+\frac{1}{250\mu\mathrm{m}}\right) \\
&= (9.27\times10^{-3}\ \mathrm{C\ cm}^{-3}\mu\mathrm{m})v_S\left(\frac{1}{w}+\frac{1}{250\mu\mathrm{m}}\right)
\end{aligned}
$$

$$J_{th} = J_0 + J_{sr} = (166.4\ \mathrm{A/cm}^2) + J_{sr}$$

$J_{sr}\ \frac{A}{cm^2}$ $J_{th}\ \frac{A}{cm^2}$	$w = 2\mu$m	$w = 10\mu$m	$w = 20\mu$m
$v_S = 10^4\ \frac{cm}{s}$	46.7 213	9.83 176	5.0 171
$v_S = 10^5\ \frac{cm}{s}$	467 633	96.4 263	50.0 216
$v_S = 5\times10^5\ \frac{cm}{s}$	2335 2502	482 648	250 417

---

a) The inclusion of carrier diffusion allows a non-uniform carrier density to exist and decreases the estimated surface recombination.

---

b) It is important to consider carrier diffusion when $v_S \geq v_{SD}$.

---

c) As previously mentioned, $N(x)$ profiles for $|x| < \frac{w}{2}$ are the same for the two structures under consideration when $v_S = v_{SD}$. In this case, the threshold carrier densities are the same since $qD\frac{dN}{dx}$ is the same when evaluated at $x = \frac{w}{2}$ in the two cases. For $v_{SD} < v_S$, structure (i) yields a lower threshold while for $v_{SD} > v_S$, structure (ii) yields a lower threshold. This was argued earlier based upon the lateral current densities at $x = \frac{w}{2}$.

---

d) For an InGaAs QW, $v_s \approx 1 - 2\times10^5\ \frac{cm}{s}$.

Using the value that we calculated for v_{SD} $(= 8.48\times10^4\ \frac{cm}{s})$, one will obtain almost the same threshold whether or not one etches through the active region since $v_{SD} \approx v_S$.

---

e) For a GaAs QW, $v_s \approx 4 - 5\times10^5\ \frac{cm}{s}$.

One should not etch through the active region in this case since $v_s >> v_{SD}$.

Since the carrier density decreases towards the edges, the gain decreases towards the edges. This leads to a higher threshold current in reality than we calculated here in order to achieve the necessary modal gain (Γg).

4.7:

Using equations from Appendix 12, we have for the D=5 case:

$\mu = \frac{m_C^*}{m_V^*} = \frac{1}{D} = 0.2$

$\overline{k}_1 = \overline{k}_2 = -\mu\overline{k}_3 = -0.2\overline{k}_3$

$\overline{k}_4 = -(1+2\mu)\overline{k}_3 = -1.4\overline{k}_3$

$$\begin{aligned} k_3^2 &= \frac{k_g^2}{(1+2\mu)(1+\mu)} \\ &= \frac{2m_C^* E_g}{(1+2\mu)(1+\mu)\hbar^2} \\ &= \frac{(2)(0.067)(9.11\times10^{-31}\text{ kg})(1.424\text{ eV})(1.6\times10^{-19})}{(1.4)(1.2)(1.054\times10^{-34}\text{ J }s)^2} \\ &= 1.5\times10^{18}\text{ m}^{-2} \end{aligned}$$

$k_3 = 1.22\times10^{9}\text{ m}^{-1}$

$$\begin{aligned} \Delta E_1 &= \Delta E_2 \\ &= \mu\Delta E_3 \\ &= \frac{\mu^2}{(1+2\mu)(1+\mu)}\,E_g \\ &= \left(\frac{(0.2)^2}{(1.4)(1.2)}\right)(1.424\text{ eV}) \\ &= 0.034\text{ eV} \end{aligned}$$

$$\begin{aligned} \Delta E_3 &= \frac{0.034\text{ eV}}{0.2} \\ &= 0.17\text{ eV} \end{aligned}$$

$$\begin{aligned} E_T &= \Delta E_4 \\ &= \frac{1+2\mu}{1+\mu}\,E_g \\ &= \frac{1.4}{1.2}\,(1.424\text{ eV}) \\ &= 1.661\text{ eV} \end{aligned}$$

Similarly, for the D=1 case:

$\mu = \frac{1}{D} = 1.0$

$\overline{k}_1 = \overline{k}_2 = -\mu\overline{k}_3 = -1.0\overline{k}_3$

$\overline{k}_4 = -(1+2\mu)\overline{k}_3 = -3.0\overline{k}_3$

$$\begin{aligned} k_3^2 &= \frac{(2)(0.067)(9.11\times10^{-31}\text{ kg})(1.424\text{ eV})(1.6\times10^{-19})}{(3)(2)(1.054\times10^{-34}\text{ J }s)^2} \\ &= 4.2\times10^{17}\text{ m}^{-2} \end{aligned}$$

$k_3 = 6.50\times10^{8}\text{ m}^{-1}$

$$\begin{aligned}\Delta E_1 &= \Delta E_2 \\ &= \mu \Delta E_3 \\ &= \left(\frac{(1.0)^2}{(3)(2)}\right) (1.424 \text{ eV}) \\ &= 0.237 \text{ eV}\end{aligned}$$

$$\begin{aligned}\Delta E_3 &= \frac{0.237 \text{ eV}}{1.0} \\ &= 0.237 \text{ eV}\end{aligned}$$

$$\begin{aligned}E_T &= \Delta E_4 \\ &= \frac{3}{2} (1.424 \text{ eV}) \\ &= 2.14 \text{ eV}\end{aligned}$$

Clearly, the effect of strain (as demonstrated by the D=1 case) is to increase the threshold energy as compared with the unstrained case (D=5).

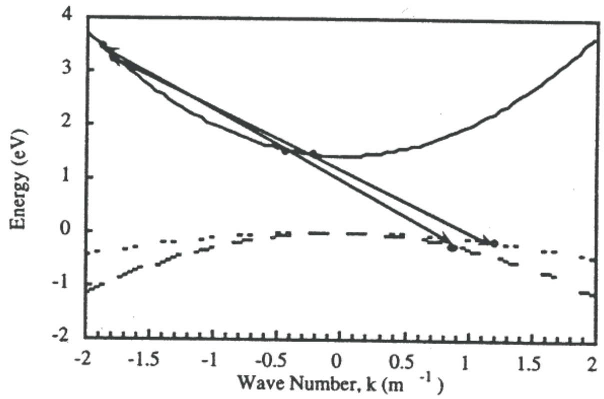

Figure 4.7. CCCH Auger transition diagram

To compare Auger transition rates, recall that

$$P_{1\to3} \approx \frac{N^2 P}{N_c^2 N_v} \; e^{-\frac{E_T - E_g}{kT}}$$

For the unstrained (D=5) and strained (D=1) cases, both the carrier concentrations and the bandgaps are assumed equal. Both have the same m_c as well. However, m_V^* and E_T are different. Hence,

$$P_{1\to3} \propto \frac{1}{(m_V^*)^{3/2}} \; e^{\frac{-E_T}{kT}} (\text{since } N_v \propto (m_V^*)^{3/2})$$

$$D = 5: \quad P_{1\to3} \propto \frac{1}{5^{3/2}} \; e^{\frac{-1.661}{0.026}} = 1.61 \times 10^{-29}$$

$$D = 1: \quad P_{1\to3} \propto \frac{1}{1^{3/2}} \; e^{\frac{-2.14}{0.026}} = 1.80 \times 10^{-36}$$

$$\Rightarrow \frac{P_{1\to3}(D=5)}{P_{1\to3}(D=1)} = 9 \times 10^6$$

The more symmetric band structure achieved with strain clearly reduces the Auger transition rate.