**5.15:**

Using normalized fields for convenience such that

$$< E(t)E(t)^* >= P_1$$

we can write the autocorrelation function in Eq. 5.144 as

$$< E(t)E(t-\tau)^* >= P_1\, e^{j\omega\tau}\, e^{-|\tau|/\tau_{coh}} \tag{5.144}$$

Now, if the power in the delayed leg is attenuated by $\alpha$, such that $P_2 = \alpha P_1$, we can write the power at the detector as

$$\begin{aligned}
P_{det} &=< \left[E(t) + \sqrt{\alpha}E(t-\tau)\right]\ \left[E(t) + \sqrt{\alpha}E(t-\tau)\right]^* > \\
&=< E(t)E(t)^* > +\alpha < E(t-\tau)E(t-\tau)^* > +\sqrt{\alpha} < E(t)E(t-\tau)^* > +\sqrt{\alpha} < E(t-\tau)E(t)^* > \\
&= P_1 + \alpha P_1 + \sqrt{\alpha}P_1\, e^{j\omega\tau}\, e^{-|\tau|/\tau_{coh}} + \sqrt{\alpha}P_1\, e^{-j\omega\tau}\, e^{-|\tau|/\tau_{coh}} \\
&= P_1 + P_2 + 2\sqrt{P_1P_2}\cos(\omega\tau)\, e^{-|\tau|/\tau_{coh}}
\end{aligned}$$

Then, fringes are visible as the value of $\cos(\omega\tau)$ is varied:

$$P_{max} = P_1 + P_2 + 2\sqrt{P_1P_2}\, e^{-|\tau|/\tau_{coh}}$$

$$P_{min} = P_1 + P_2 - 2\sqrt{P_1P_2}\, e^{-|\tau|/\tau_{coh}}$$

$$P_{max} - P_{min} = 4\sqrt{P_1P_2}\, e^{-|\tau|/\tau_{coh}}$$

$$P_{max} + P_{min} = 2(P_1 + P_2)$$

Therefore,

$$e^{-|\tau|/\tau_{coh}} = \frac{P_1 + P_2}{2\sqrt{P_1P_2}}\, \frac{P_{max} - P_{min}}{P_{max} + P_{min}} \tag{5.145}$$

---

The problem asks for the coherence time for the case when $\tau = 3$ ns:

$$e^{-|\tau|/\tau_{coh}} = 0.2$$

$$\tau_{coh} = \frac{-3}{\ln(0.2)}\ \text{ns} = 1.864\ \text{ns}$$

The linewidth is then given by Eq. 5.149:

$$\Delta\nu_{FW} = \frac{1}{\pi\tau_{coh}} = 171\ \text{MHz} \tag{5.149}$$

**5.16:**

From Fig. 5.26, estimate $\mu_2$ near $P_{02} = 2.5$ mW:

$$\mu_2 \equiv \frac{\Delta P_{p-p}}{2P} \approx \frac{0.2 \text{ mW}}{2.5 \text{ mW}} = 0.08$$

Using the effective mirror model with VCSELs, for all practical purposes, we can assume lossless mirrors (The transmission is reduced by $e^{-\alpha L_{eff}}$ according to the discussion surrounding Eq. 3.64. However, $\alpha L_{eff} < 0.005$ represents a negligible reduction in transmission for VCSELs.)

Eq. 5.175 is derived assuming lossless mirrors and can be used to extract $\kappa_f$ from $\mu_2$. If all of the light is coupled out of mirror 2, then $F_1 = 0$ and since $F_1 + F_2 = 1$ for lossless mirrors, we have shown that $F_2 = 1$. The problem also says to assume that we are well above threshold; so we can neglect the third term in defining $\mu_2$. With $F_2 = 1$, we can write Eq. 5.175 as:

$$\mu_2 = 2\kappa_f \tau_p \left(\frac{\eta_i}{\eta_d} - 1\right)$$

Solve for $\kappa_f$:

$$\begin{aligned}\kappa_f &= \frac{\mu_2}{2\tau_p \left(\frac{\eta_i}{\eta_d} - 1\right)} \\ &= \frac{0.08}{2(\cancel{2.77 \text{ ps}})\left(\frac{0.8}{0.494} - 1\right)} \quad ; \ \tau_p \simeq 2.20\,ps \\ &= \cancel{2.331} \times 10^{10} \text{ s}^{-1} \\ &\quad 2.935\end{aligned}$$

$\tau_p$ ≈2.2

---

$$\kappa_f = \frac{t_2^2 \sqrt{f_{ext}}}{r_2 \tau_L}$$

Solve for $f_{ext}$:

$$f_{ext} = \left(\frac{\kappa_f r_2 \tau_L}{t_2^2}\right)^2$$

From Table 5.1,

$r_2 = \sqrt{R_2} = 0.995$

$r_1 = 1$

$t_2^2 = 1 - 0.995^2 = 0.00998$ assuming lossless mirrors

$\tau_L = \frac{2L}{v_g} = \frac{2(10^{-4} \text{ cm})}{\frac{3}{4.2} \times 10^{10} \text{ cm/s}} = 2.8 \times 10^{-14}$ s

$$\begin{aligned}f_{ext} &= \left(\frac{(\overset{2.935}{\cancel{2.331}} \times 10^{10} \text{ s}^{-1})(0.995)(2.8 \times 10^{-14} \text{ s})}{0.00998}\right)^2 \\ &= \overset{6.706}{\cancel{4.23}} \times 10^{-3}\end{aligned}$$

$$\begin{aligned}f_{ext}(\text{dB}) &= 10 \log(f_{ext}) \\ &= \cancel{-23.7 \text{ dB}} \\ &\quad -21.73 \text{ dB}\end{aligned}$$

**5.17**:

From Fig. 5.26 and Eq. 5.175, $\cos(2\beta L_p)$ goes through one complete period for a change in current of 0.4 mA. Assuming that the change of $\beta$ with current is approximately linear, we can write

$$\frac{d(2\beta L_p)}{dI} = \pm\frac{2\pi}{0.4\text{ mA}}$$

$$\beta \equiv \frac{2\pi\overline{n}}{\lambda_0}$$

Neglecting the change in index due to current and temperature, we can write the first equation above as

$$2L_p(2\pi\overline{n})\frac{d\frac{1}{\lambda_0}}{dI} = \pm\frac{2\pi}{0.4\text{ mA}}$$

which can be rearranged:

$$L_p = \pm\frac{1}{(0.4\text{ mA})\ \overline{n}\ 2\ \frac{d\left(\frac{1}{\lambda_0}\right)}{dI}}$$

and simplified

$$L_p = \pm\frac{-\lambda_0^2}{(0.8\text{ mA})\overline{n}\left(\frac{d\lambda_0}{dI}\right)}$$

All variables are known except $\frac{d\lambda_0}{dI}$. Assuming that the wavelength change with current is due solely to the change in temperature, we can write

$$\frac{d\lambda_0}{dI} = \frac{d\lambda_0}{dT}\frac{dT}{dP_d}\frac{dP_d}{dI}$$

where $T$ and $P_d$ denote temperature and dissipated power, respectively.

$$\frac{d\lambda_0}{dT} = 0.08\frac{\text{nm}}{^\circ\text{C}} \qquad \text{given in problem}$$

$$\frac{dT}{dP_d} = 3\frac{^\circ\text{C}}{\text{mW}} \qquad \text{given in problem}$$

$$P_d = P_{in} - P_{out} = IV - P_0$$

$$\frac{dP_d}{dI} = \frac{dP_{in}}{dI} - \frac{dP_{out}}{dI} = 3V - \frac{2.5\text{ mW} - 0\text{ mW}}{6.5\text{ mA} - 1.75\text{ mA}} = 2.47\frac{\text{mW}}{\text{mA}}$$

$$\frac{d\lambda_0}{dI} = 0.59\frac{\text{nm}}{\text{mA}}$$

Then, plugging back into the equation for $L_p$, we have

$$L_p = \frac{(980\text{ nm})^2}{(0.8\text{ mA})(4.2)\left(0.59\frac{\text{nm}}{\text{mA}}\right)}$$

$$L_p = \text{substrate thickness} = 480\mu\text{m}$$

**5.18:**

**a)**

$$\kappa_f = \frac{v_g t_2^2 \sqrt{f_{ext}}}{2Lr_2} \qquad \text{(p. 247)}$$

$$\kappa_f = \frac{v_g(1-r_2^2)(r_3 t_c)}{2Lr_2}$$

$\overline{n}_g$ is not specified by the problem. Assume that laser is made in the InGaAsP material system and the mode has $\overline{n}_g = 4.0$ as mentioned on p. 40.

Assume a facet reflectivity $r_2^2 = 0.32$.

$$\kappa_f = \frac{\frac{3.0\times10^{10}\ \text{cm/s}}{4.0}(1-0.32)(\sqrt{0.04}\sqrt{0.25})}{2(0.03\ \text{cm})\sqrt{0.32}} = 1.50\times10^{10}\ \text{s}^{-1}$$

$$\tau_{ext} = \frac{2n_{fiber}L_p}{c} = \frac{2(1.45)L_p}{3\times10^{10}\ \text{cm/s}} = (0.967\times10^{-5}\ \text{s/km})L_p \qquad \text{(p. 250)}$$

$$C = \kappa_f \tau_{ext}\sqrt{1+\alpha^2} = (1.45\times10^5\ \text{km}^{-1})L_p\sqrt{1+\alpha^2} \qquad (5.180)$$

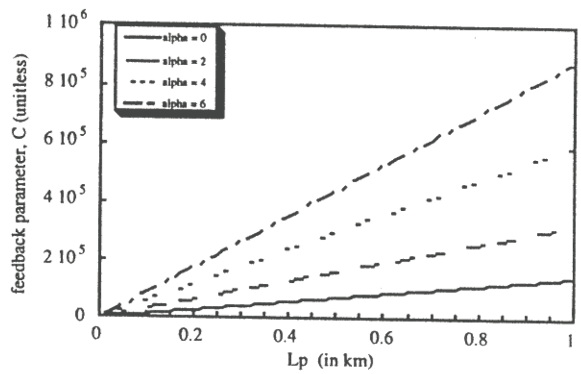


Figure 5.18a. Feedback coefficient, $C$, vs. fiber length, $L_p$.

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**b)**

$$\frac{\Delta\nu}{\Delta\nu_0} = \frac{1}{(1+C\ \cos(2\beta L_p+\phi_\alpha))^2} \qquad (5.181)$$

Since $\cos(2\beta L_p+\phi_\alpha)$ can be adjusted to any value between -1 and +1, we can break this equation into 2 cases:

Case 1: $C<1$:

minimum $\frac{\Delta\nu}{\Delta\nu_0} = \frac{1}{(1+C)^2}$

maximum $\frac{\Delta\nu}{\Delta\nu_0} = \frac{1}{(1-C)^2}$

Case 2: $C\geq1$:

minimum $\frac{\Delta\nu}{\Delta\nu_0} = \frac{1}{(1+C)^2}$

maximum $\frac{\Delta\nu}{\Delta\nu_0} = \infty$

For $C\geq1$, $\cos(2\beta L_p+\phi_\alpha)$ can adjusted to be equal to $1/C$, which yields an infinite bandwidth. In this case, the laser has split into two lasing modes, which implies that that $\Delta\nu$ is meaningless for this case.

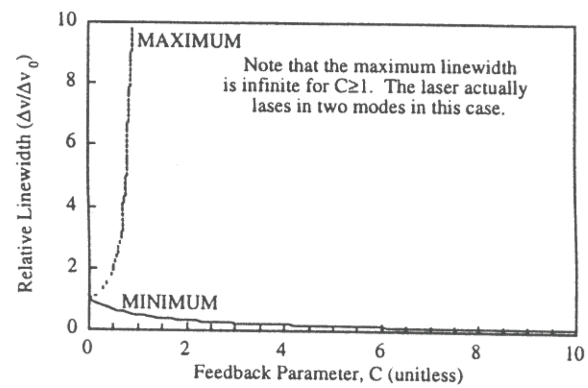


Figure 5.18b. Laser linewidth vs. feedback coefficient, $C$.