Minding Your BER’s and Q’s

Bit-Error-Rate and Q Factor Measurement: Theory and Practice

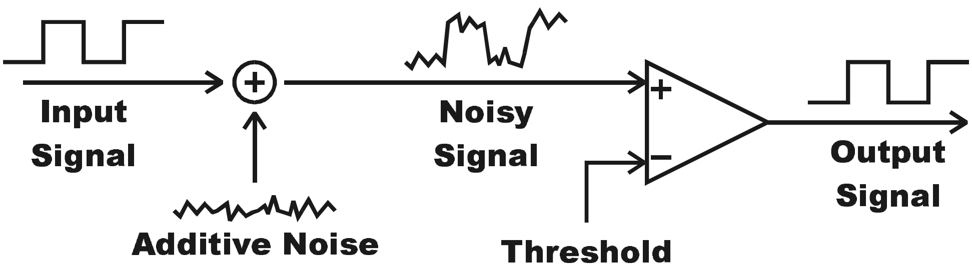
Mark Summerfield, Thursday November 4, 1999

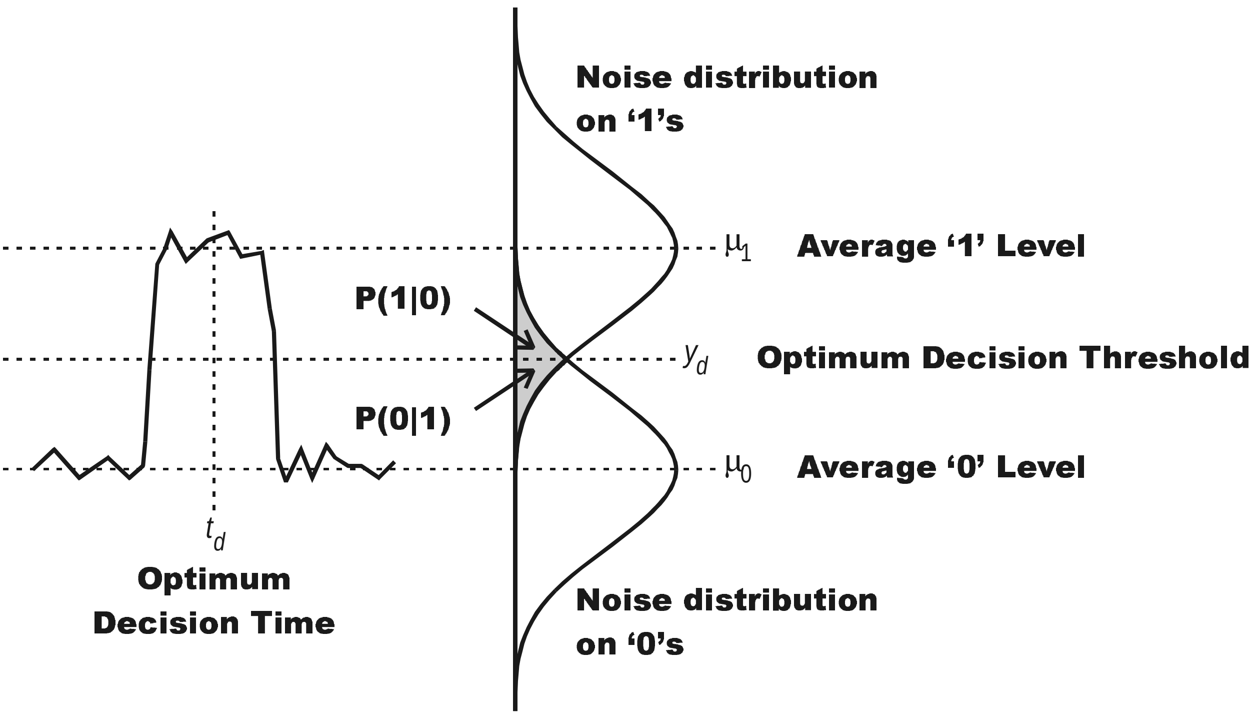
Outline

- “Standard” theory of bit-error measurement in optical systems
- Experimental methods for measuring system BER performance
- Interpreting of the results of BER measurements
- Principles and purpose of “Q-factor” measurement using a BER Test Set

What Causes Bit-Errors?

- Bit-errors are the result of incorrect decisions being made in a receiver due to the presence of noise on a digital signal
  - ◊ consider two-level (OOK) modulation only today





Bit Error Rate

- The bit-error-rate (or bit-error-ratio) is

$$BER = p(1)P(0\,|\,1) + p(0)P(1\,|\,0)$$

- If the same number of ‘1’s as ‘0’s are sent

$$BER = \frac{1}{2}\left[P(0\,|\,1) + P(1\,|\,0)\right]$$

- For Gaussian noise

$$p_1(y) = \frac{1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{(y_d-\mu_1)^2}{2\sigma_1^2}\right)$$

$$p_0(y) = \frac{1}{\sqrt{2\pi}\sigma_0}\exp\left(-\frac{(y_d-\mu_0)^2}{2\sigma_0^2}\right)$$

$$P(1\,|\,0) = P(y > y_d \,|\, y \sim p_0) = \frac{1}{2}\mathrm{erfc}\left(\frac{y_d-\mu_0}{\sqrt{2}\sigma_0}\right)$$

$$P(0\,|\,1) = P(y < y_d \,|\, y \sim p_1) = \frac{1}{2}\mathrm{erfc}\left(\frac{\mu_1-y_d}{\sqrt{2}\sigma_1}\right)$$

Optimum Threshold and "Q"

- For Gaussian noise, the BER is given by

$$BER = \frac{1}{4}\left[\operatorname{erfc}\left(\frac{\mu_1 - y_d}{\sqrt{2}\sigma_1}\right) + \operatorname{erfc}\left(\frac{y_d - \mu_0}{\sqrt{2}\sigma_0}\right)\right]$$

- The optimum setting of y_d is that which gives minimum BER, i.e. when

$$p_1(y_d) = p_0(y_d)$$

  ◊ this equation must be solved numerically

  ◊ a common (and accurate) approximation is

$$P(1\,|\,0) = P(0\,|\,1) \quad \Rightarrow \quad y_d = \frac{\sigma_0\mu_1 + \sigma_1\mu_0}{\sigma_0 + \sigma_1}$$

  ◊ for which the BER is

$$BER = \frac{1}{2}\operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{1}{\sqrt{2\pi}Q}\exp\left(-\frac{Q^2}{2}\right)$$

  ◊ where $Q = \dfrac{\mu_1 - \mu_0}{\sigma_0 + \sigma_1}$

WARNING: There is more than one common definition of erfc(*x*)!

What is "Q"?

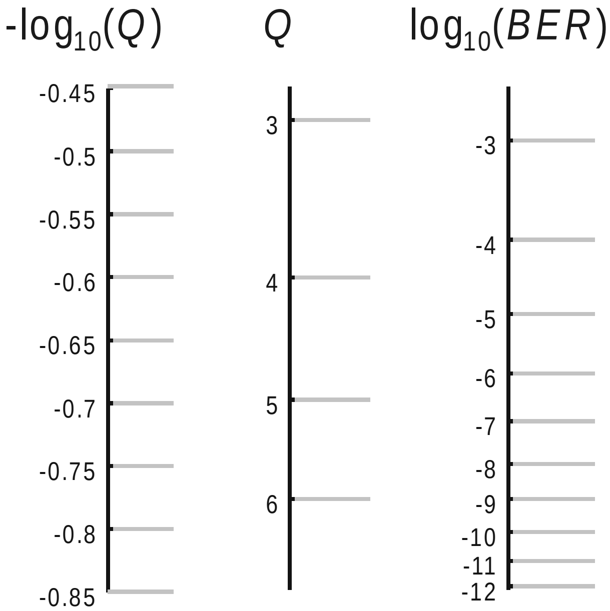
- Q is a measure of the "quality" of any signal
  - ◊ Q is defined for any signal for which the mean levels μ_1 and μ_0, and the noise powers σ_1^2 and σ_0^2 can be sensibly defined, even if the noise is not Gaussian
  - ◊ Q is sometimes referred to as the "signal-to-noise ratio in voltage (or current) units"

    In my view this is not a useful definition: *Q* is *Q*
- *If* the noise is known to be Gaussian, Q fully determines the BER
- In many cases of interest $Q \propto (\mu_1 - \mu_0)$
  - ◊ if you plot Q against signal amplitude, you get a straight line
  - ◊ thus to get a straight line for BER vs signal amplitude, you can convert BER to Q

$$Q = \sqrt{2}\left\{\log\left[\frac{1}{2}\operatorname{erfc}(\cdot)\right]\right\}^{-1}(x), \text{ where } x = \log(BER)$$

BER “Graph Paper”

- In practice we tend to measure signal amplitude on a logarithmic scale (i.e. in decibel units), thus we need log(*Q*) on the y-axis



- Using this scaled axis, the BER curve will clearly be a straight line if
  - ◊ the noise is (approximately) Gaussian; and
  - ◊ the s.d. of the noise (square-root of noise power) is *independent* of the signal amplitude
- However, this is now a log(*P*)-log(*Q*) graph...

Bit Error Rate Measurements

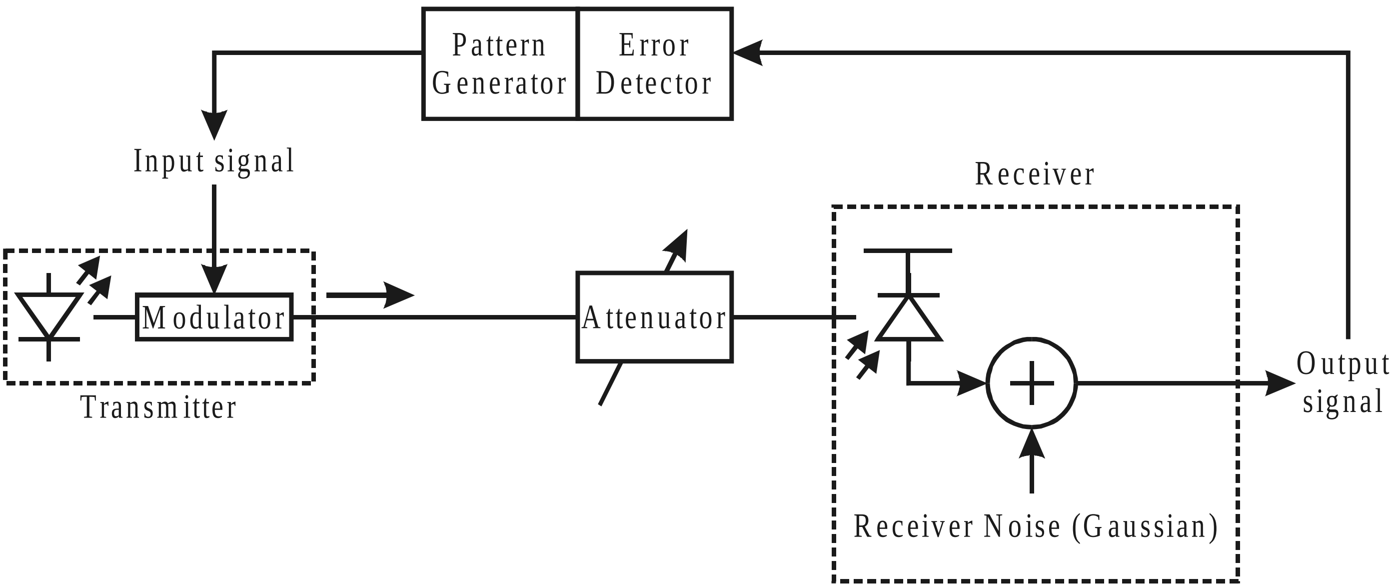
- "Traditionally" (i.e. in electrical cable or wireless systems) BER is measured as a function of the signal-to-noise ratio
  - ◊ means of comparing different modulation schemes
  - ◊ dominant noise contributions may not be in the receiver (source noise or channel noise)
  - ◊ usually, SNR is proportional to *Q*
- In optical systems, we measure BER as a function of mean *received optical power* (ROP)
  - ◊ the quantity measured by a power meter
  - ◊ if the dominant noise source is thermal noise contributed by the receiver electronics, and the extinction ratio is high (i.e. $P_1 >> P_0$), then

$$Q \propto ROP$$
$$SNR \propto (ROP)^2$$
$$\text{i.e. } SNR\,(\text{dB}) \propto ROP\,(\text{dBm})$$

  - ◊ typically the case for "back-to-back" measurement

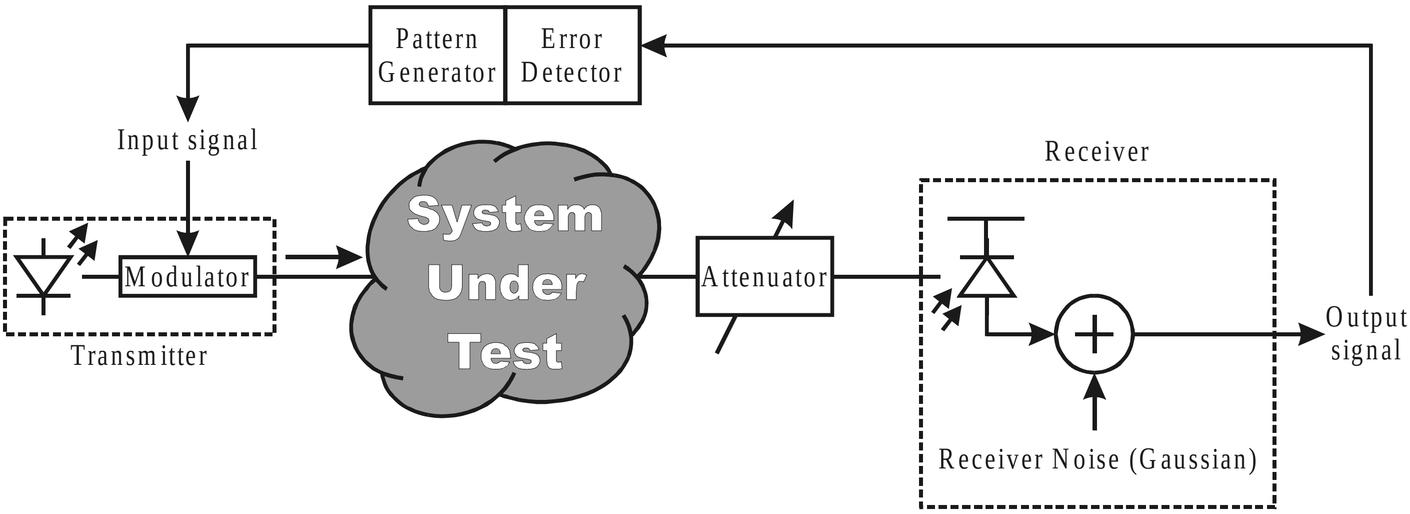
The "Back-To-Back" Measurement

- The back-to-back measurement characterises the receiver
  - ◊ we assume that the transmitted signal quality is high (negligible noise and distortion)
  - ◊ shot noise is negligible at datacomm rates
  - ◊ ROP is controlled by attenuating the output of the transmitter
  - ◊ for each value of ROP, the decision threshold is optimised, and the BER measured
  - ◊ receiver sensitivity is the ROP required to achieve a specific BER (typ. 10^{-9}, 10^{-10} or 10^{-12})



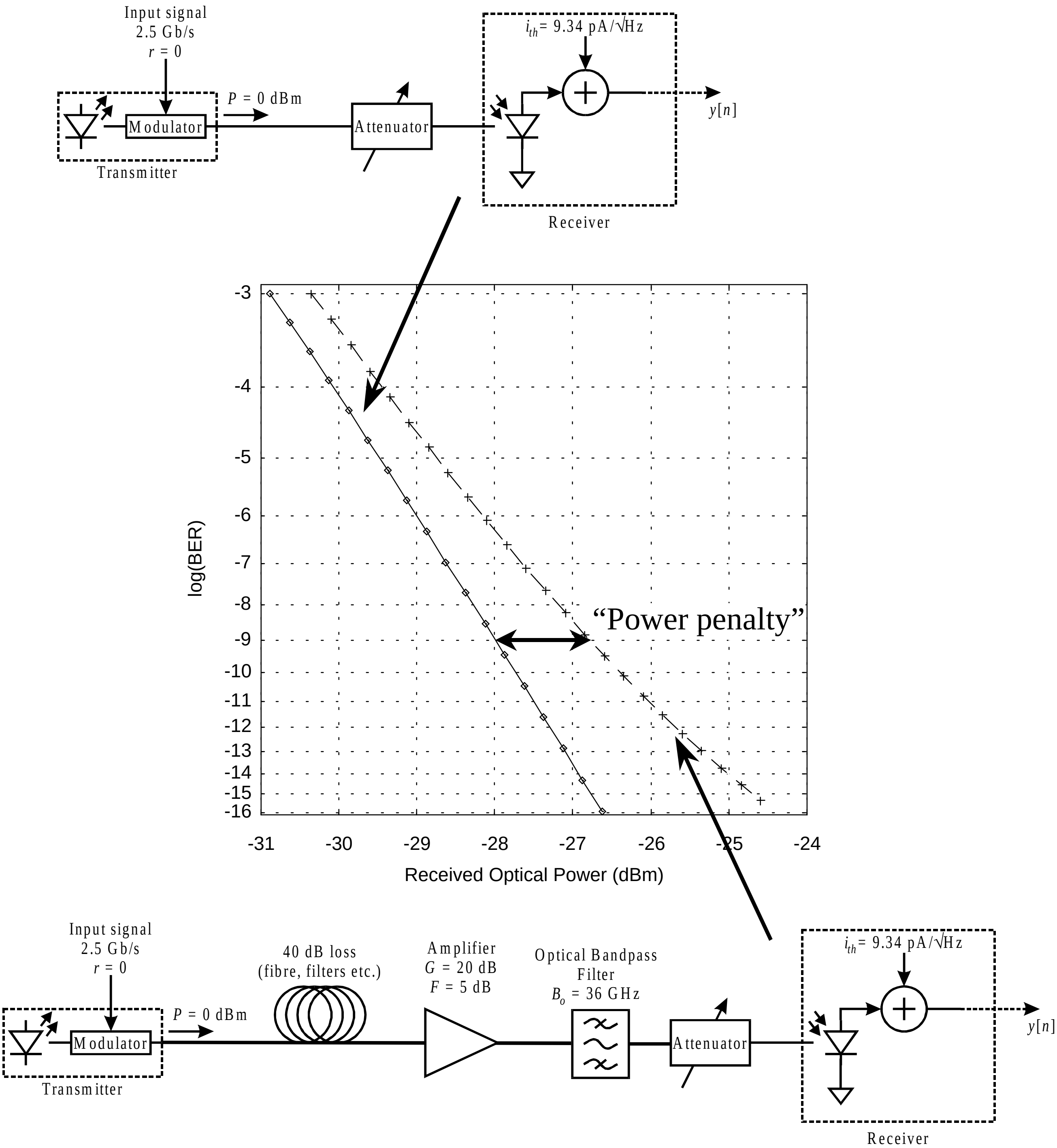
The "System" Measurement

- We now add the system under test and repeat the measurement

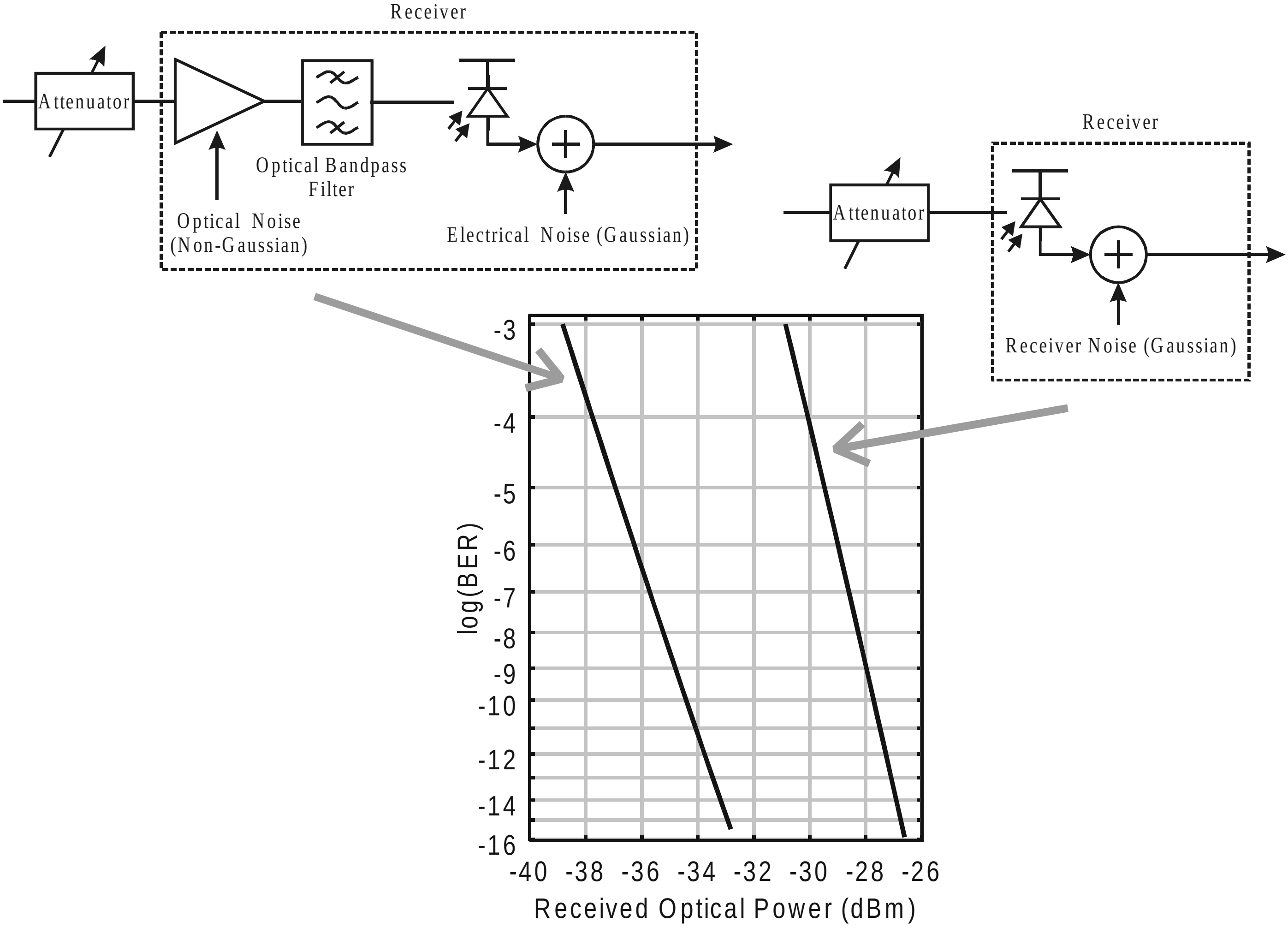


- If the system adds noise, or otherwise degrades or modifies the signal, we expect to see the effects in the BER curve
  - ◊ note that attenuation of the optical signal within the system does not, *by itself*, cause any degradation, since BER is measured as a function of ROP
  - ◊ however, attenuation followed by a process which adds noise (e.g. amplification) will result in a reduction in *Q*

Example: An Optically-Amplified Link



Example: Optically Preamplified Receiver



- The *gradient* of the two curves is different
  - ◊ due to signal-dependent (sig.-spont. beat) noise in the preamplified receiver
- Non-Gaussian noise produces a straight line
  - ◊ optimum decision threshold is different, though
- Similar effects observed with APD receivers

Identifying Causes of Signal Degradation

- A BER measurement cannot tell you what physical mechanism resulted in a particular degradation of the received signal
  - ◊ in general, this depends upon what is in the system under test
  - ◊ however, BER measurements can be very helpful in determining what *type* of degradation is occurring
  - ◊ other measurements (e.g. eye patterns) should also be performed for corroboration
- A general approach to identifying sources of degradation: if you suspect that you know the cause, *and* you can isolate or eliminate it, then do so, and repeat the BER measurement
  - ◊ e.g. if you suspect crosstalk in a WDM transmission system, switch off all the channels except the one you are measuring
- Interpreting BER curves...

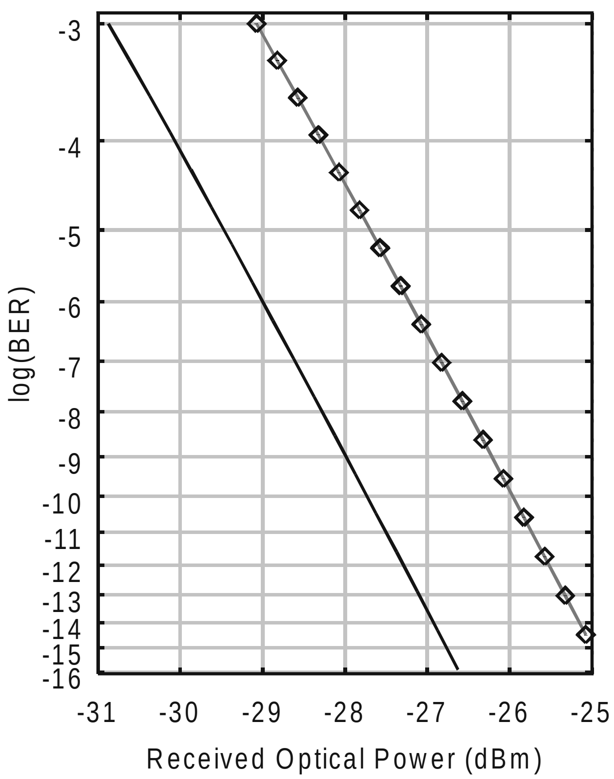
Extinction Ratio Degradation

- The *extinction ratio* (ER) of an optical OOK signal is the ratio of the average power in a '0' to the average power in a '1'
  - ◊ ideally ER is infinite, i.e. $P_0 = 0$
- ER degradation by itself causes a *shift* in the BER curve
  - ◊ you can think of the power in '0's as being "wasted" optical power - it carries no information
  - ◊ straightforward to show that the power penalty is

$$PP = 10\log_{10}\left(\frac{1+r}{1-r}\right),$$

$$\text{where } r = \frac{P_0}{P_1}$$

  - ◊ e.g. $r = 0.2$, $PP = 1.76$ dB



Additive Optical Noise

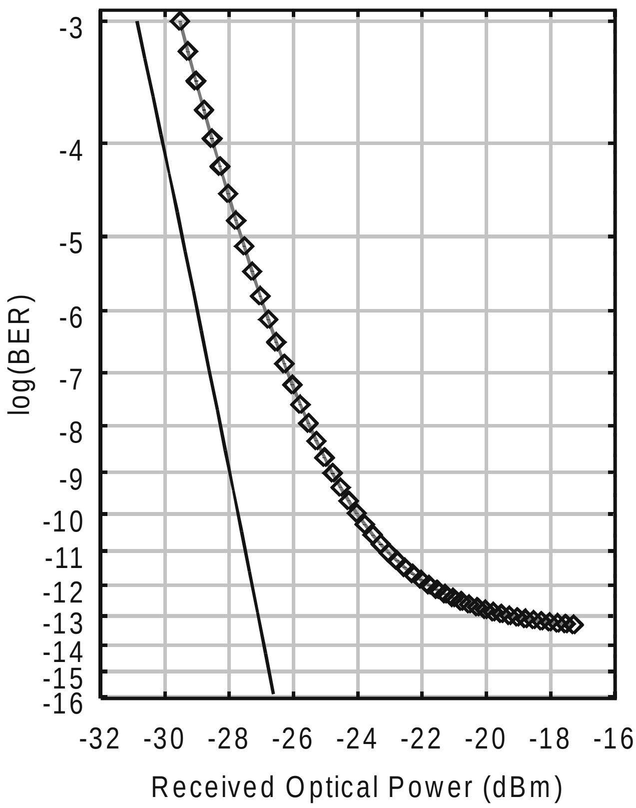
- The example of an optically-amplified link before illustrated the effect of added Amplified Spontaneous Emission (ASE) noise
  - ◊ There are two contributions:

    the average ASE noise power is included in the ROP measurement, and acts like an ER degradation

    the optical noise *cannot* be overcome by increasing the ROP, and results in an *error-rate floor*

- More extreme example:
  - ◊ OSNR (measured in 0.1nm bandwidth) of 12 dB

    with a 36 GHz optical filter this results in about the same amount of ER degradation (r = 0.19) as the previous example



Intersymbol Interference

- Intersymbol interference (ISI) in a single-channel system may result from a number of sources
  - ◊ e.g. dispersion, fibre nonlinearities, component nonlinearities
- ISI may result in a power penalty and/or an error-rate floor
  - ◊ BER measurements alone may not be sufficient to identify the nature of the problem
    - eye patterns may help; measurements at other points within the system may be required
- A common signature of ISI is *pattern dependence* of the BER curve
  - ◊ compare results for 2^{31}-1 PRBS down to 2^7-1 PRBS, as well as "0101…" pattern
  - ◊ *never* dismiss differences as unimportant, and *never* publish results based on short patterns unless you have a clear justification

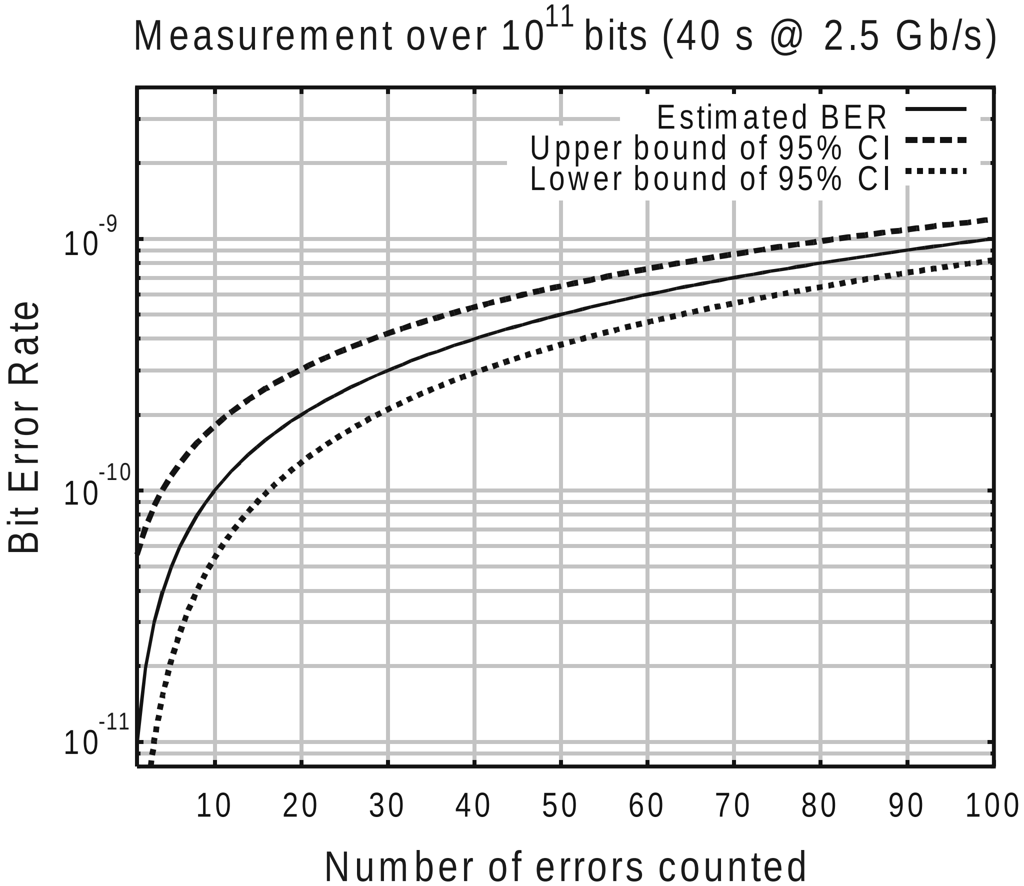
Statistical Significance

- BER is the probability that any one given bit is received in error
- In a BER measurement, we *estimate* this by receiving a large number of bits N_b, and counting the number of errors n_e

$$BER \approx \frac{n_e}{N_b}$$

  - ◊ this estimator is itself a statistical quantity
  - ◊ the number of errors actually observed is, assuming ***independent errors***, binomial-distributed
- How many errors do you need to count before you can be confident that your BER estimate is accurate?
  - ◊ Under the assumption of independent errors, and thus binomial-distributed counts, we can compute confidence intervals for the actual error rate

Confidence Intervals



- To measure the BER to a reasonable degree of accuracy [log(BER) to ~2 sig. figs. with 95% confidence] requires the observation of 100 errors
  - ◊ approx. 40 s at 2.5 Gb/s (10^{11} bits) for 10^{-9} BER
  - ◊ approx. 11 hours (10^{15} bits) for 10^{-12} BER !!!

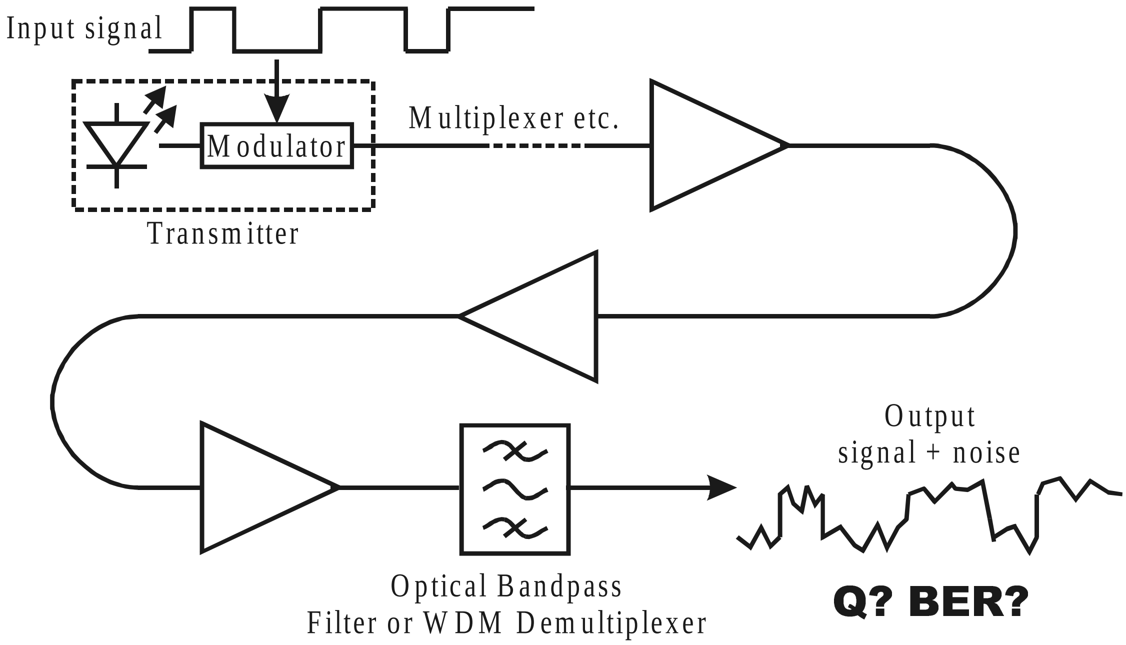
Error Bursts

- Some systems exhibit *correlated errors*, i.e. bit-errors are *not* independent
  - ◊ suppose that if one bit is received in error, it is *more likely* than average that the subsequent bit will also be in error
  - ◊ a common cause of this is an external influence which temporarily increases the probability of error, e.g. environmental fluctuations
- You might be able to measure a low error rate for an extended period, but then see a sudden burst which pushes the BER up
  - ◊ do you know the cause of the bursts?
  - ◊ can you justify recording only those results for which error bursts do not occur?
  - ◊ if not, then you must measure over a statistically-significant number of *bursts*
  - ◊ the “error interval” counters on the BER test set can help you in these circumstances

Why Try to Measure Q?

- Before optical amplifiers, the *power budget* was the most important parameter in optical link design
  - ◊ if you don't have enough optical power available at the receiver, you cannot achieve a low BER
  - ◊ the *system margin* is the difference between the actual ROP available at the receiver, and the ROP required to achieve the desired performance (e.g. BER < 10^{-9})
- With optical amplifiers, it is usually possible to obtain higher ROP (at the expense of some added ASE noise)
  - ◊ furthermore, it is of interest to know how the signal actually degrades over each span of an optically-amplified link
  - ◊ what we would like to do is measure the quality of the signal *at the output power level from the span* and determine a "Q-margin"
  - ◊ typically, the BER is too low to measure

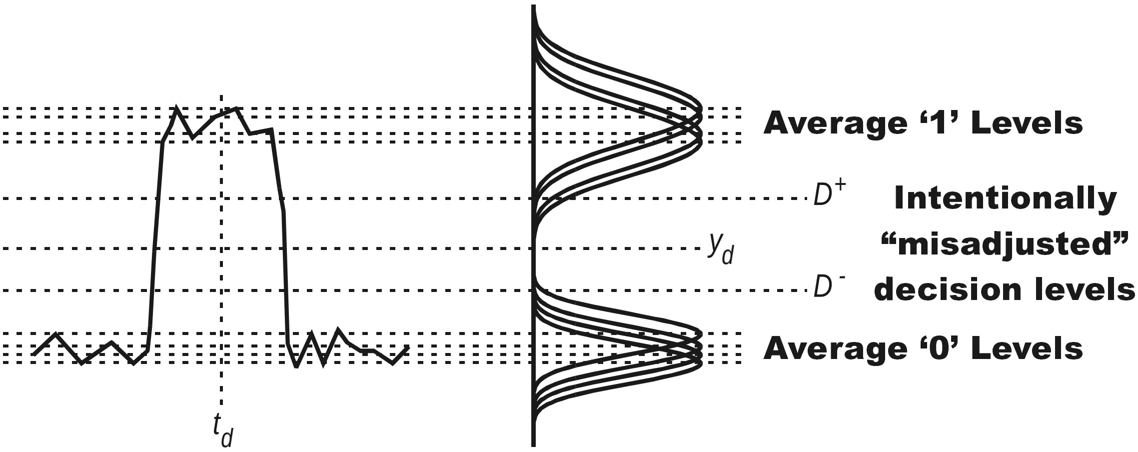
The Problem...



- Determine a useful measure of Q when
  - ◊ ROP is well above the receiver sensitivity limit
  - ◊ sources of impairment other than Gaussian noise may be present
- Measure optical SNR, e.g. on an OSA?
  - ◊ only the effect of ASE noise can be computed
- Eye pattern/histogram, e.g. on CSA?
  - ◊ effects of dispersion, ISI etc. hard to distinguish
  - ◊ mean levels uncertain, tends to overestimate noise

The Solution...

- Suppose the distributions can be modeled as a superposition of Gaussian distributions
  - ◊ pattern-dependent signal levels due to ISI; or
  - ◊ model for a non-Gaussian noise distribution



- Try to look only at the "tails" of the distributions
  - ◊ "misadjust" the decision threshold over ranges where measureable errors occur (e.g. 10^{-5} to 10^{-9})
    - for "D^+" errors are predominantly '1's detected as '0's
    - for "D^-" errors are predominantly '0's detected as '1's
  - ◊ this effectively measures the area in the tail of the distribution as a function of decision threshold
  - ◊ for Gaussian distributions, we expect the resulting data to follow the complementary error function

Calculating Q (and BER)

- For '1's detected as '0's (D^+)

$$BER = \frac{1}{4}\operatorname{erfc}\left(\frac{\mu_1^{eff} - D^+}{\sqrt{2}\sigma_1^{eff}}\right)$$

$$\log(2 \cdot BER) = \log\left[\frac{1}{2}\operatorname{erfc}\left(\frac{\mu_1^{eff} - D^+}{\sqrt{2}\sigma_1^{eff}}\right)\right]$$

$$D^+ = \mu_1^{eff} - \sigma_1^{eff} f\left[\log(2 \cdot BER)\right]$$

◊ where $f(x) = \sqrt{2}\left\{\log\left[\frac{1}{2}\operatorname{erfc}(\cdot)\right]\right\}^{-1}(x)$

- Similarly

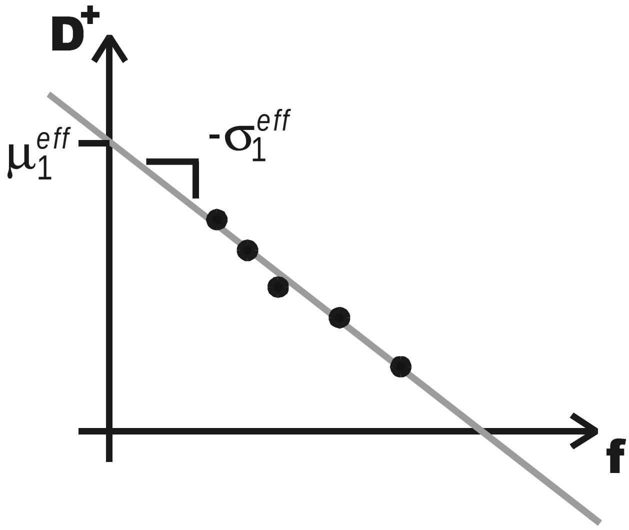
$$D^- = \mu_0^{eff} + \sigma_0^{eff} f\left[\log(2 \cdot BER)\right]$$

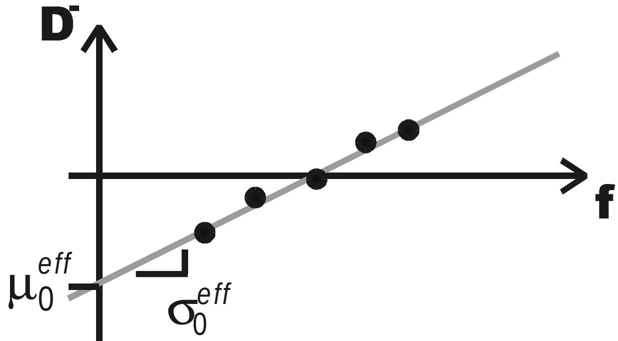
- Thus, if the assumption of Gaussian noise is valid, graphs of D^+ and D^- vs *f* should be straight lines with

$$\begin{aligned}\text{gradient } &= \left|\sigma_{0,1}^{eff}\right| \\ \text{intercept} &= \mu_{0,1}\end{aligned}$$

Procedure

- Measure and plot graphs of *D* vs *f*





  ◊ the straight lines should be fitted by linear regression

    some references suggest a correlation coefficient greater than 0.98 justifies the Gaussian assumption

- The optimum threshold setting, Q factor and BER can then be computed

$$D_{opt} = \frac{\mu_1^{eff}\sigma_0^{eff} + \mu_0^{eff}\sigma_1^{eff}}{\sigma_0^{eff} + \sigma_1^{eff}} \qquad Q = \frac{\mu_1^{eff} - \mu_0^{eff}}{\sigma_0^{eff} + \sigma_1^{eff}}$$

$$BER = \frac{1}{2}\operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{1}{\sqrt{2\pi}Q}\exp\left(-\frac{Q^2}{2}\right)$$

References and Resources

[1] Marcuse, D., "Calculation of Bit-Error Probability for a Lightwave System with Optical Amplifiers and Post-Detection Gaussian Noise," *J. Lightwave Technol.*, **9** (4), April 1991, pp. 505-513.

All BER curves presented in this talk we calculated using the method described in this paper. If you are interested in using the computer program (Win95) in your work, you can come and talk to me about it.

[2] Bergano, N.S., Kerfoot, F.W and Davidson, C.R., "Margin Measurements in Optical Amplifier Systems," *IEEE Photon. Tech. Letts.*, **5** (3), March 1993, pp. 304-306.

Note that this paper uses a *different definition of the complementary error function* than the one I have used here.

[3] Mood, A.M., Graybill, F.A. and Boes, D.C., *Introduction to the Theory of Statistics*, 3rd Ed., McGraw-Hill (New York: 1974).

Or any equivalent statistics text - I used this as a reference for the confidence intervals on the binomial distribution only because it was the first book I picked off the shelf in the library!

[4] Hewlett-Packard, *Lightwave Transmission Seminar*, 1996.

Be warned that this document should be read critically - it is not very rigorous! Be especially wary of the discussion of "noise-loaded Q measurements."

I also have a computer program (Win95) which performs the axis scaling for BER plots, as well as a Gnuplot line which labels the y-axis appropriately. See me for copies.