

 6.1:

a)

$$\Delta\beta = \frac{k_0^2}{2\beta} \frac{\int \Delta\epsilon |U|^2 dA}{\int |U|^2 dA} \quad (6.9)$$

$$k_0 = 2\pi/\lambda_0$$

$$\beta = 2\pi\bar{n}/\lambda_0$$

$$\Delta\epsilon = 2n\Delta n \quad \text{where } n \text{ is the index of the guiding region.}$$

$$\Delta\beta = \left(\frac{2\pi}{\lambda_0}\right) \frac{n\Delta n}{\bar{n}} \frac{\int \int_0^{150 \text{ nm}} |U|^2 dA}{\int \int_{-\infty}^{\infty} |U|^2 dA}$$

Assuming that the mode doesn't change significantly due to the index perturbation, the fraction containing the integrals is the confinement factor of the original mode ($\Gamma = 0.5$ given by problem) multiplied by the overlap of the part of the guide that changed ($\frac{150 \text{ nm}}{300 \text{ nm}}$).

$$\Delta\beta = \frac{2\pi n}{\lambda_0 \bar{n}} \Delta n \Gamma_{xy} = \frac{2\pi}{1.55 \mu\text{m}} (0.02) \frac{1}{4} = 203 \text{ cm}^{-1} \quad (6.12, 6.10)$$

(Note that this assumes that $n \approx \bar{n}$ in the waveguide region.)