6.3:

$\underline{L_{eff}}$:

For large L, $\tanh(\kappa L) \approx 1$, which means that the result of the previous problem can be approximated as

$$L_{eff} \approx \frac{1}{2\kappa}$$

$\underline{L_p}$:

$$E(z) = A(z)\ e^{-j\beta_0 z} + B(z)\ e^{j\beta_0 z} \tag{6.25}$$

$$A(z) = A_1\ e^{-\sigma z} + A_2\ e^{\sigma z}$$
$$B(z) = B_1\ e^{-\sigma z} + B_2\ e^{\sigma z} \tag{6.28, 6.29}$$

For large L, the return power is approximately zero at the end of the grating. This gives us one boundary condition:

$$B(L) = 0 = B_1\ e^{-\sigma L} + B_2\ e^{\sigma L}$$

$$B_2 = -B_1\ e^{-2\sigma L}$$

Then, by Eq. 6.30,

$$A_1 = \frac{j(\sigma + j\delta)}{\kappa} B_1$$

and

$$A_2 = \frac{j(\sigma - j\delta)}{\kappa} B_1\ e^{-2\sigma L}$$

Condensing all of these equations, we can write

$$E(z) = \left(\frac{j(\sigma + j\delta)}{\kappa} B_1\ e^{-\sigma z} + \frac{j(\sigma - j\delta)}{\kappa} B_1\ e^{\sigma(z-2L)}\right)\ e^{-j\beta_0 z} + \left(B_1\ e^{-\sigma z} + B_1\ e^{\sigma(z-2L)}\right)\ e^{j\beta_0 z}$$

Simplify this expression using three substitutions:

$e^{-2\sigma L} \approx 0$ for large L.

$\delta = 0$ at the Bragg wavelength.

$\sigma = \kappa$ at the Bragg wavelength.

Then

$$E(z) = B_1\ e^{-\kappa z}\left(j\ e^{-j\beta_0 z} + e^{j\beta_0 z}\right)$$

Therefore, the field decay is proportional to $e^{-\kappa z}$ and the power decay is proportional to $e^{-2\kappa z}$. The defintion of the penetration depth, L_p, is the length at which the field decays to $\frac{1}{e}$ of its peak value:

$$e^{-2\kappa L_p} = e^{-1}$$

Thus, we see that, for large L,

$$L_p = \frac{1}{2\kappa} \approx L_{eff}$$

6.4:

Plot Γg_{th} vs. δ

$\lambda = 1.55\mu$m

$<\alpha_i> = 15$ cm^{-1}

$\kappa = 50$ cm^{-1}

Mode is aligned with DBR at 1.55μm.

Consider $\phi = 0$, 90°, and 180°

and $\kappa L_g = 0.5$ and 1

From chapter 2, we know that

$$\Gamma g_{th} = <\alpha_i> + \frac{1}{L_a} \ln\left(\frac{1}{|r_1 r_{eff}|}\right) \tag{2.23}$$

The only unknown on the right side of this equation is r_{eff}:

$$r_{eff} = r_g + \frac{t_g^2 r_2}{1 - r_g r_2} \tag{6.46}$$

$$\tilde{r}_g = -j\frac{\tilde{\kappa}_{-1}\tanh(\tilde{\sigma}L_g)}{\tilde{\sigma} + j\tilde{\delta}\tanh(\tilde{\sigma}L_g)} \tag{6.37}$$

$$\tilde{t}_g = \frac{\tilde{\sigma}\mathrm{sech}(\tilde{\sigma}L_g)}{\tilde{\sigma} + j\tilde{\delta}\tanh(\tilde{\sigma}L_g)} e^{-j\beta_0 L_g} \tag{6.40}$$

where

The phase term of t_g is accounted for in the expression for r_2. So $e^{-j\beta_0 L_g} = 1$.

$r_1 = \sqrt{0.32}$

$r_2 = \sqrt{0.32}\, e^{j\phi_2}$

$\tilde{\sigma} = \sqrt{\kappa^2 - \tilde{\delta}^2}$ (6.32)

$\tilde{\delta} = \tilde{\beta} - \beta_0 = \frac{2\pi\bar{n}}{\lambda} - \frac{2\pi\bar{n}}{\lambda_0} + j\frac{<g>_{xy} - <\alpha_i>}{2}$ (6.33)

$<g>_{xy} = 0$ in DBR

$<\alpha_i> = 15$ cm^{-1}

$\rightarrow \quad \tilde{\delta} = \delta - j\frac{<\alpha_i>}{2}$

$\tilde{\kappa}_{-1} = \kappa = 50$ cm^{-1} since there is no perturbation of the loss or gain.

$L_g = \frac{\kappa L_g}{\kappa}$

All of these equations can be combined to create an expression for threshold modal gain, Γg_{th} in terms of the variables κL_g, ϕ_2, and δ.

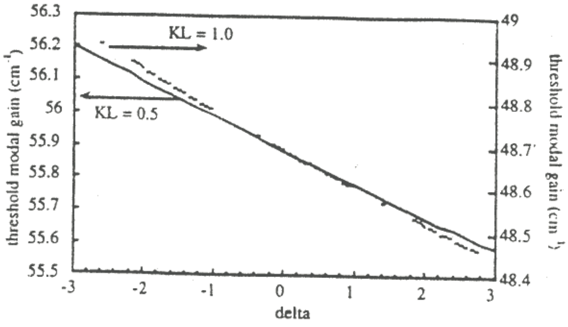

Figure 6.4. Threshold modal gain vs. detuning for $\phi_2 = 0^o$.

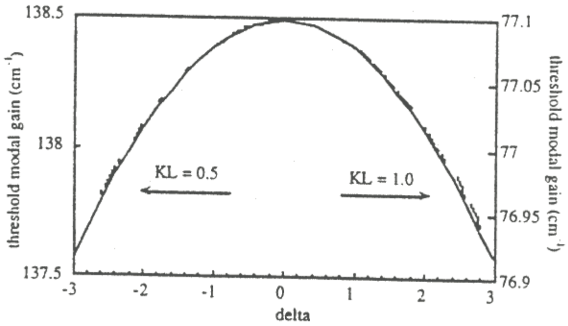

Figure 6.4. Threshold modal gain vs. detuning for $\phi_2 = 90^o$.

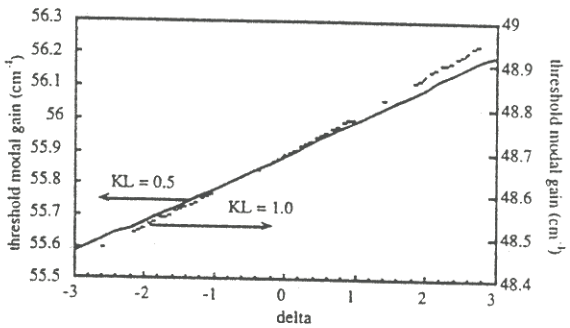

Figure 6.4. Threshold modal gain vs. detuning for $\phi_2 = 180^o$.

6.6:

$$\tilde{r}_g = -j\frac{\tilde{\kappa}_{-1}\tanh(\tilde{\sigma}L)}{\tilde{\sigma} + j\tilde{\delta}\tanh(\tilde{\sigma}L)} = |\tilde{r}_g|\, e^{j\phi} \qquad (6.37)$$

The characteristic equation for a $\frac{\lambda}{4}$ shifted DFB laser is

$$\tilde{r}_g^2\, e^{-2j\tilde{\beta}L} = 1 \qquad \text{(corrected 3.68)}$$

where

$L = \frac{\lambda}{2\overline{n}}$ (see Fig. 3.17)

$\tilde{\beta} = \frac{2\pi\overline{n}}{\lambda} + j\frac{g-\alpha}{2}$

Thus, the threshold condition is

$$\tilde{r}_g^2\, e^{2\lambda(g-\alpha)/\overline{n}} = 1$$

or

$$|\tilde{r}_g|^2\, e^{2j\phi}\, e^{2\lambda(g-\alpha)/\overline{n}} = 1$$

6.7:

a) The characteristic equation is

$$\tilde{\sigma}_{th} = -j\tilde{\delta}_{th} \tanh(\tilde{\sigma}_{th} L_g) \tag{6.41}$$

$\tilde{\sigma}_{th} = \sqrt{\kappa^2 - \tilde{\delta}_{th}^2}$

$\kappa = \frac{\kappa L}{L} = \frac{1}{400\mu\text{m}} = 25\ \text{cm}^{-1}$

$\tilde{\delta}_{th} = \beta_{th} - \beta_0 + j\frac{<g>_{xy} - <\alpha_i>_{xy}}{2} = \Delta\beta + \frac{j<g>_{xy}}{2}$

$< \alpha_i >= 20\ \text{cm}^{-1}$

Solve the characteristic equation given above for the two unknowns, $\Delta\beta$ and $< g >_{xy}$. Solve numerically:

First mode:

$$\Delta\beta = \pm 65\ \text{cm}^{-1}$$

$$< g >_{xy} = 103\ \text{cm}^{-1}$$

$$\delta L = \Delta\beta L = \pm 2.6$$

Second mode:

$$\Delta\beta = \pm 141\ \text{cm}^{-1}$$

$$< g >_{xy} = 152\ \text{cm}^{-1}$$

$$\delta L = \Delta\beta L = \pm 5.64$$

---

b) For a DFB with a $\frac{\lambda}{4}$ shift, the characteristic equation is (neglecting gain across the short cavity length, $\frac{\lambda}{2\bar{n}}$)

$$\tilde{r}_g^2 = 1 \tag{Problem6.6}$$

$$\tilde{r}_g = \frac{\kappa \tanh\left(\frac{\tilde{\sigma}L}{2}\right)}{\tilde{\sigma} + j\tilde{\delta}\tanh\left(\frac{\tilde{\sigma}L}{2}\right)} \tag{6.37}$$

Solving as in part (a), we get

First mode:

$$\Delta\beta = 0$$

$$< g >_{xy} = 96\ \text{cm}^{-1}$$

$$\delta L = \Delta\beta L = 0$$

Second mode:

$$\Delta\beta = \pm 113\ \text{cm}^{-1}$$

$$< g >_{xy} = 128\ \text{cm}^{-1}$$

$$\delta L = \Delta\beta L = \pm 4.52$$

c) The mode supression ratio is given by

$$MSR = \left(\frac{\Delta\alpha + \Delta g}{\delta} + 1\right) \frac{\alpha_m(\lambda_0)}{\alpha_m(\lambda_1)} \frac{F_1(\lambda_0)}{F_1(\lambda_1)} \qquad (3.73, 3.74)$$

$\frac{F_1(\lambda_0)}{F_1(\lambda_1)} \approx 1$

$\alpha_m =< g >_{xy} - < \alpha_i >$

$\Delta\alpha = \alpha_m(\lambda_1) - \alpha_m(\lambda_0)$

$\Delta g \approx 0$ (since $\Delta\lambda$ is small, material gain is approximately constant over this wavelength range)

$\delta = \eta_r \beta_{sp} < g >_{xy} \frac{I_{th}}{I - I_{th}}$ (3.77)

We can use the threshold value of $< g >_{xy}$ because the gain is controlled by the carrier density, which is clamped at threshold.

Assume that

$\eta_r = 1$

$\beta_{sp} = 10^{-4}$

$I = 2I_{th}$

---

case(a):

$\delta = 10^{-4}(103 \text{ cm}^{-1}) = 0.0103 \text{ cm}^{-1}$

$\alpha_m(\lambda_0) = 103 - 20 = 83 \text{ cm}^{-1}$

$\alpha_m(\lambda_1) = 152 - 20 = 132 \text{ cm}^{-1}$

$\Delta\alpha = 132 - 83 = 49 \text{ cm}^{-1}$

$MSR = 2991$

$$MSR(\text{ dB}) = 10 \log(MSR) = 34.8 \text{ dB}$$

---

case(b):

$\delta = 10^{-4}(96 \text{ cm}^{-1}) = 0.0096 \text{ cm}^{-1}$

$\alpha_m(\lambda_0) = 96 - 20 = 76 \text{ cm}^{-1}$

$\alpha_m(\lambda_1) = 128 - 20 = 108 \text{ cm}^{-1}$

$\Delta\alpha = 108 - 76 = 32 \text{ cm}^{-1}$

$MSR = 2346$

$$MSR(\text{ dB}) = 10 \log(MSR) = 33.7 \text{ dB}$$