6.9:

Since we know $\kappa L_g = 1$, we can write the characteristic equation as a function of one variable, $\tilde{\delta} L_g$. To do this, rewrite the grating reflection and transmission formulae using $\tilde{\delta} L_g$ and κL_g:

$$\begin{aligned}\tilde{r}_g &= -j\frac{\kappa L_g \tanh(\sqrt{(\kappa L_g)^2 - (\tilde{\delta} L_g)^2})}{\sqrt{(\kappa L_g)^2 - (\tilde{\delta} L_g)^2} + j\tilde{\delta} L_g \tanh(\sqrt{(\kappa L_g)^2 - (\tilde{\delta} L_g)^2})} \\ &= -j\frac{\tanh(\sqrt{1 - (\tilde{\delta} L_g)^2})}{\sqrt{1 - (\tilde{\delta} L_g)^2} + j\tilde{\delta} L_g \tanh(\sqrt{1 - (\tilde{\delta} L_g)^2})}\end{aligned} \tag{6.37}$$

$$\begin{aligned}\tilde{t}_g &= \frac{\sqrt{(\kappa L_g)^2 - (\tilde{\delta} L_g)^2}\,\mathrm{sech}(\sqrt{(\kappa L_g)^2 - (\tilde{\delta} L_g)^2})}{\sqrt{(\kappa L_g)^2 - (\tilde{\delta} L_g)^2} + j\tilde{\delta} L_g \tanh(\sqrt{(\kappa L_g)^2 - (\tilde{\delta} L_g)^2})} e^{-j\beta_0 L_g} \\ &= \frac{\sqrt{1 - (\tilde{\delta} L_g)^2}\,\mathrm{sech}(\sqrt{1 - (\tilde{\delta} L_g)^2})}{\sqrt{1 - (\tilde{\delta} L_g)^2} + j\tilde{\delta} L_g \tanh(\sqrt{1 - (\tilde{\delta} L_g)^2})}\end{aligned} \tag{6.40}$$

We have assumed that $e^{-j\beta_0 L_g} = 1$ in the expression for $\tilde{t}_g$ because we assume that this phase term is accounted for in the phase of r_2.

We want to find the following as a function of ϕ_2:

1) $\delta L_g = real(\tilde{\delta} L_g)$

2) $(\Gamma g_{th} - <\alpha_i>) L_g = 2 \times imag(\tilde{\delta} L_g)$

3) $\Gamma g_{th} = \frac{2\times imag(\tilde{\delta} L_g)}{L_g} + <\alpha_i> = \frac{2\times imag(\tilde{\delta} L_g)}{0.03\ \mathrm{cm}} + 15\ \mathrm{cm}^{-1}$

The characteristic equation for this device is given by

$$\frac{r_1 r_2 t_g^2}{(1 - r_1 r_g)(1 - r_2 r_g)} = 1 \tag{6.49}$$

or

$$1 - r_1 r_g - r_2 r_g + r_1 r_2 (r_g^2 - t_g^2) = 0$$

This characteristic equation can be solved numerically using the expressions for the grating reflection and transmission that are given above.

Be warned: Getting these solutions to converge is not easy. The discontinuities in the normalized threshold currents indicate that in each case, we may not have chosen the correct mode. The mode spacing is approximately $\Delta(\delta L_g) \approx 3$ as can be seen by the discontinuities in the δL_g parts of these graphs.

The threshold modal gain is not plotted because it contains only redundant information (see above).

---

a) $r_1 = 0$

$|r_2| = 0.566$

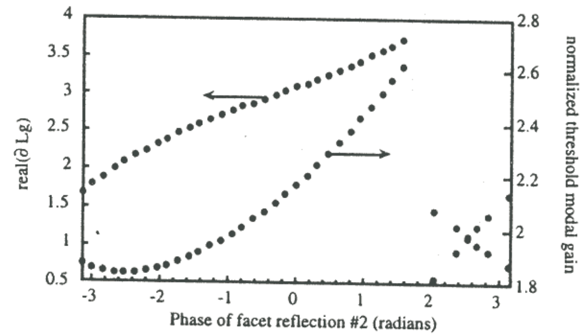

Figure 6.9a. Wavelength deviation and normalized threshold modal gain vs. ϕ_2.

---

b) $r_1 = 0.566$

$|r_2| = 0.566$

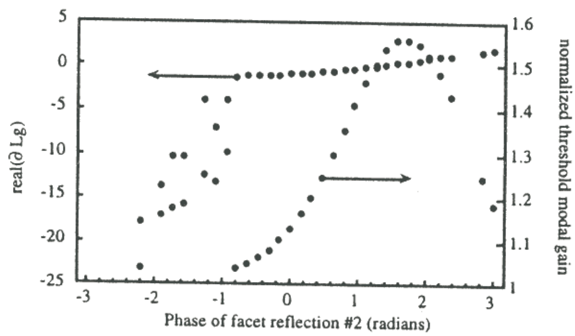

Figure 6.9b. Wavelength deviation and normalized threshold modal gain vs. ϕ_2.

---

c) $r_1 = 0.566\ e^{j\pi/2}$

$|r_2| = 0.566$

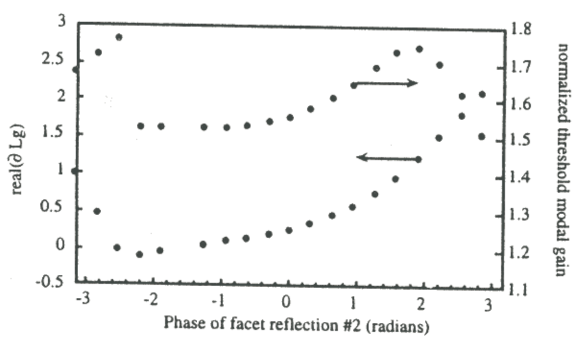

Figure 6.9c. Wavelength deviation and normalized threshold modal gain vs. ϕ_2.

6.10:

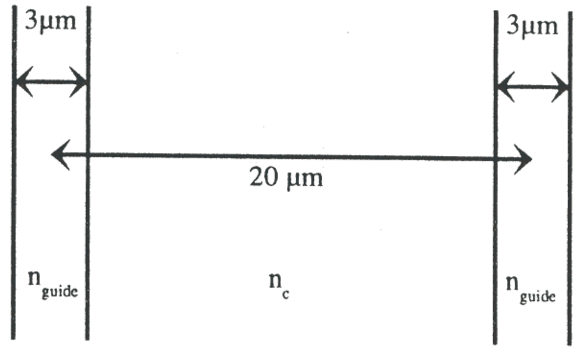

Figure 6.10. Index perturbation profile.

$n_c = 3.50$

$\Delta n = 0.04$

$n_{guide} = 3.54$

$\Gamma_x = 0.4$

$\overline{n} = \Gamma_x n_{guide} + (1 - \Gamma_x) n_c = 3.516$

---

a)

$$\begin{aligned}\kappa_{21} = \kappa_{12} &= \frac{k_0^2}{2\beta} \frac{\int \Delta\epsilon U_1^* U_2 \, dA}{\int |U_1|^2 \, dA} \\ &= \frac{4\pi^2}{\lambda_0} \frac{1}{2} \frac{\lambda_0}{2\pi\overline{n}} (2n_c \Delta n) \frac{\int U_1^* U_2 \, dA}{\int |U_1|^2 \, dA} \\ &= \frac{\pi}{(1.55\mu\text{m})3.516} (2(3.5)(0.04))(0.40 * 10\%) \\ &= 64.6 \text{ cm}^{-1}\end{aligned} \tag{6.61}$$

Note that the overlap integral between the modes of each guide was estimated instead of being calculated exactly. This estimate assumes that the field profile of the mode is roughly constant at 100% and 10% within the primary and coupled guides, respectively.

---

b) To find the length for complete coupling, use Eq. 6.70:

$$\sin^2(\kappa_{12} L_c) = 1 \tag{6.70}$$

$$\kappa_{12} L_c = \frac{\pi}{2}$$

$$L_c = \frac{\pi}{2\kappa_{12}} = \frac{\pi}{2(64.6 \text{ cm}^{-1})} = 0.0243 \text{ cm} \tag{6.71}$$

$$L_c = 243\mu\text{m}$$

Approximate index as a linear function of wavelength. Using the two data points given by the problem, we can write

$n_1(\lambda) = 3.81 - \frac{\lambda}{5}$

$n_2(\lambda) = 4.12 - \frac{2\lambda}{5}$

The propagation constants can then be written as a function of wavelength:

$\beta_{1,2}(\lambda) = \frac{2\pi}{\lambda} n_{1,2}(\lambda)$

$\Delta\beta(\lambda) = \frac{2\pi}{\lambda}(n_2(\lambda) - n_1(\lambda)) = \frac{2\pi}{\lambda}(0.31 - \frac{\lambda}{5})$

$s = \sqrt{\left(\frac{\Delta\beta}{2}\right)^2 + \kappa^2}$

Find the coupling length for a wavelength of 1.55μm.

$$L = \frac{\pi}{2\kappa_{12}} = \frac{\pi}{2(0.01\mu\text{m}^{-1})} = 157\mu\text{m} \tag{6.71}$$

Use Eq. 6.66 to find the coupling between guides for other wavelengths:

$$\frac{|a_2(L)|}{|a_1(0)|} = \frac{\kappa_{21}}{s}\sin(sL)$$

The bandwidth of the device can be found by finding the wavelengths at which the coupling between the guides is reduced by 3dB:

$$\frac{|a_2(L)|}{|a_1(0)|} = \frac{\kappa_{21}}{s}\sin(sL) = \frac{1}{\sqrt{2}}$$

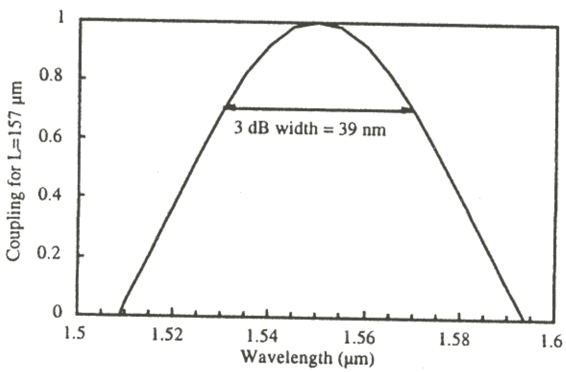

Figure 6.11. Coupling vs. wavelength.

$$\Delta\lambda = 39 \text{ nm}$$

~~17.22 GHz~~

~~17.22 THz~~

4.92 THz

6.12:

Use Eq. 6.84 to calculate the proportion of power coupled from one waveguide to the next:

$$\frac{P_2(0^+)}{P_1(0^-)} = \left| t \int U_2^* U_1 \, dA \right|^2 \tag{6.84}$$

From Table 1.1, the values for index in the materials of this waveguide are

InGaAsP(1.3μm): $n_g = 3.40$

InP: $n_c = 3.17$

$\lambda = 1.55\mu$m

$$t = \sqrt{\frac{4\overline{n}_1\overline{n}_2}{(\overline{n}_1+\overline{n}_2)^2}} \tag{6.82}$$

Calculate the effective indices for the two waveguides using equations from Appendix 3:

Smaller waveguide:

$$\begin{aligned} V &= \frac{2\pi}{\lambda} d\sqrt{n_g^2 - n_c^2} \\ &= \frac{2\pi}{1.55\mu\text{m}}(0.200\mu\text{m})\sqrt{3.40^2 - 3.17^2} \qquad (A3.12) \\ &= 0.996 \end{aligned}$$

$a = 0$

Find b from Fig. A3.2: $b = 0.19$

$$\overline{n}_1 = \sqrt{b(n_g^2 - n_c^2) + n_c^2} = \sqrt{0.19(3.40^2 - 3.17^2) + 3.17^2} = 3.215 \tag{A3.12}$$

$$k_x = \sqrt{k_0^2(n_g^2 - \overline{n}^2)} = \sqrt{\left(\frac{2\pi}{1.55\mu\text{m}}\right)^2 (3.40^2 - 3.215^2)} = 4.48\mu\text{m}^{-1} \tag{A3.7}$$

$$\gamma_x = \sqrt{k_0^2(\overline{n}^2 - n_c^2)} = \sqrt{\left(\frac{2\pi}{1.55\mu\text{m}}\right)^2 (3.215^2 - 3.17^2)} = 2.17\mu\text{m}^{-1} \tag{A3.7}$$

From Eq. A3.8,

$$U_{g1}(x) = A_1 \cos((4.48\mu\text{m}^{-1})x) \qquad (|x| < 100 \text{ nm})$$

$$U_{c1}(x) = B_1\, e^{\pm(2.17\mu\text{m}^{-1})x} \qquad (|x| \geq 100 \text{ nm})$$

Match boundary conditions at the edge of the waveguide:

$$\begin{aligned} B_1 &= A_1 \frac{\cos((4.48\mu\text{m}^{-1})(0.1\mu\text{m}))}{e^{\pm(2.17\mu\text{m}^{-1})(0.1\mu\text{m})}} \\ &= 1.12 A_1 \end{aligned}$$

Larger waveguide:

$V = 1.993$

$a = 0$

$b = .43$

$\overline{n}_2 = 3.26$

$k_x = 3.91\mu\text{m}^{-1}$

$\gamma_x = 3.08\mu\text{m}^{-1}$

$$U_{g1}(x) = A_1 \cos((3.91\mu\text{m}^{-1})x) \qquad (|x| < 100 \text{ nm})$$

$$U_{c1}(x) = B_1 \; e^{\pm(3.08\mu\text{m}^{-1})x} \qquad (|x| \geq 100 \text{ nm})$$

$$B_1 = 1.31 A_1$$

$$t = 0.99998 \approx 1.$$

Do the overlap integral between the fields numerically:

$$\frac{P_2(0^+)}{P_1(0^-)} = \left| t \int U_2^* U_1 \, dA \right|^2 \approx 0.968 \tag{6.84}$$

$$\text{Coupling (power) loss} = 3.2\%$$

Since U_1 and U_2 are real, the coupling loss is the same for both directions.