7.3:

a) $V = k_0 d\sqrt{n_{guide}^2 - n_{cladding}^2} = \frac{2\pi}{1.55\mu m}(0.4\mu m)\sqrt{3.4^2 - 3.2^2} = 1.86$ (A3.12)

$a = 0$

$b = 0.40$ from Fig. A3.2

Solve Eq. A3.12 for $\overline{n}$

$\overline{n} = \sqrt{n_c^2 + b(n_g^2 - n_c^2)} = \sqrt{3.2^2 + 0.40(3.4^2 - 3.2^2)} = 3.281$

$$\begin{aligned}\theta_i &= \tan^{-1}\left(\frac{\beta}{k_{ix}}\right)\\ &= \tan^{-1}\left(\frac{k_0\overline{n}}{k_0\sqrt{n_g^2 - \overline{n}^2}}\right)\\ &= \tan^{-1}\left(\frac{3.281}{\sqrt{3.4^2 - 3.281^2}}\right)\\ &= 74.7^o\end{aligned}$$

---

b) For the same θ_i, $\overline{n}$,a, and b, we have, from Fig A3.2 , for m=1,

$$V = 5.8$$

$$d = \frac{V}{k_0\sqrt{n_g^2 - n_c^2}} = \frac{5.8}{\frac{2\pi}{1.55\mu m}\sqrt{3.4^2 - 3.2^2}} = 1.24\mu m \qquad (A3.12)$$

---

c)

$$a = \frac{n_{c2}^2 - n_{c1}^2}{n_g^2 - n_{c2}^2} = \frac{3.2^2 - 3^2}{3.4^2 - 3.2^2} = 0.939 \approx 1.0 \qquad (A3.12)$$

$$b = 0.40$$

From Fig. A3.2,

$$V = 2.2$$

$$d = \frac{V}{k_0\sqrt{n_g^2 - n_c^2}} = \frac{2.2}{\frac{2\pi}{1.55\mu m}\sqrt{3.4^2 - 3.2^2}} = 0.47\mu m \qquad (A3.12)$$

7.4:

As described by Fig. 7.13, modes occur at resonances of the layer structure. The resonances can be found by using the transmission matrices of chapter 3 (see Table 3.3).

Assuming lossless material, the propagation matrices are given by

$$T_p = \begin{bmatrix} e^{j\beta_t L_t} & 0 \\ 0 & e^{-j\beta_t L_t} \end{bmatrix}$$

where

$\beta_t = k_x = \sqrt{k_0^2 - k_z^2} = \frac{2\pi}{\lambda_0}\sqrt{n^2 - \overline{n}^2}$

L_t =distance of propagation (i.e. thickness of layer).

The reflection matrices are given by

$$T_{r_{12}} = \frac{1}{t_{12}} \begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix}$$

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t_{12} = \sqrt{1 - r_{12}^2}$$

The reflection values shown in Fig. 7.13,

$$r_A = r_B = \frac{T_{21}}{T_{11}}$$

$$T = T_{pa} T_{r_{a1}} T_{p1} T_{r_{12}} T_{p2} T_{r_{21}} T_{p1} T_{r_{12}}$$

Modes of the waveguide structure occur for

$$r_A^2 = 1$$

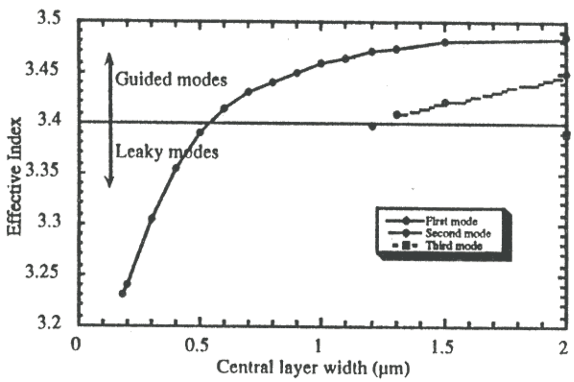

Figure 7.4. Effective index vs. central layer thickness

7.5:

For perfectly reflecting boundaries placed at 30μm, $r = -1$. From Table 1.1, n(GaAs at 1.0μm)= 3.52 and n($Al_{.2}Ga_{.8}As$ at 1.0μm)= 3.39. This layer structure is shown in Fig. 7.11.

Solve for guided modes:

Guided modes can be solved for as done in Appendix 3. Using Eq. A3.12 and Fig. A3.2, we find that
$V = \frac{2\pi}{1.0\mu m}(0.3\mu m)\sqrt{3.52^2 - 3.39^2} = 1.79$
So there is one guided mode.
$b = 0.36$
$\overline{n} = \sqrt{3.39^2 + (0.36)(3.52^2 - 3.39^2)} = 3.437$
$\beta(m = 1) = \frac{2\pi}{1.0\mu m} 3.437 = 21.598$

Solve for radiation modes:

$$k = \sqrt{k_x^2 + \beta^2}$$
$$k_x = \sqrt{k^2 - \beta^2} = \frac{2\pi}{\lambda}\sqrt{n^2 - \overline{n}^2}$$

In order to satisfy the characteristic equation (round trip gain =1), we have

$$r_1\, e^{k_{1x}d_1}\, e^{k_{2x}d_2}\, e^{k_{3x}d_3} r_2\, e^{k_{1x}d_1}\, e^{k_{2x}d_2}\, e^{k_{3x}d_3} = 1$$

$r_1 = r_2 = -1$
$d_1 = d_3 = 30\mu m$
$d_2 = 0.3\mu m$
$k_1 = k_3 = \frac{2\pi}{1.0\mu m}\sqrt{3.39^2 - \overline{n}^2}$
$k_2 = \frac{2\pi}{1.0\mu m}\sqrt{3.52^2 - \overline{n}^2}$
The phase of the characteristic equation can be simplified to

$$m\,\frac{\lambda}{2} = (30\mu m)\sqrt{3.39^2 - \overline{n}^2} + (0.3\mu m)\sqrt{3.52^2 - \overline{n}^2} + (30\mu m)\sqrt{3.39^2 - \overline{n}^2} \qquad \text{for } m \geq 2$$

Solve for β using perturbation theory described in Chapter 6.

Simplify this expression by replacing the waveguide region with the same width of cladding material. This simplifies the expression for β significantly and allows us to write a closed form solution for β.

$$m\,\frac{\lambda}{2} \approx (60.3\mu m)\,\frac{\lambda}{2\pi}\sqrt{\left(\frac{2\pi n_1}{\lambda}\right)^2 - \beta^2}$$

$$\beta(m) \approx \sqrt{\left(\frac{2\pi n_1}{\lambda}\right)^2 - \left(\frac{m\pi}{60\mu m}\right)^2}$$

The perturbation due to the waveguide can then be added to the value of β for each mode:

$$\Delta\beta = \frac{\int \Delta\epsilon k_0^2 |U|^2 dA}{2\beta \int |U|^2 dA} \qquad (6.9)$$

For odd numbered modes,

$$U(x) = A\cos(k_{x1m}x)$$

where A is a normalization coefficient and k_{x1m} is the value of the x component of the k vector for mode number m: $k_{x1m} = \sqrt{k_1^2 - \beta_m^2}$

Similarly, for even numbered modes,

$$U(x) = A\sin(k_{x1m}x)$$

The index perturbation is $\Delta\epsilon = 3.52^2 - 3.39^2 = 0.90$
$k_0 = \frac{2\pi}{1.0\mu m} = 6.28\mu m^{-1}$

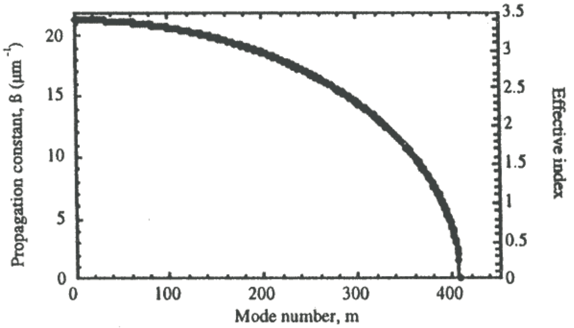

Figure 7.5. β and $\overline{n}$ vs. radiation mode number

Note: If better accuracy is desired, the values of β can be solved for numerically or graphically using the transcendental equations described by Eq. 7.26.

7.6:

Use Eq. 7.36 to implement the WKB method:

$$2\int_{x=x_A}^{x=x_B} k_x(x)\,dx = (2m+1)\pi \tag{7.36}$$

Use symmetry to simplify this equation further

$$4\int_{0}^{x=x_B} k_x(x)\,dx = (2m+1)\pi \tag{7.36}$$

$$k_x(x) = k_0\sqrt{n^2(x) - \overline{n}^2}$$

$$n(x) = \begin{cases} \Delta n\left(1 - \frac{2x}{d}\right) + n_{min} & 0 \le x \le \frac{d}{2} \\ \Delta n\left(1 + \frac{2x}{d}\right) + n_{min} & -\frac{d}{2} \le x \le 0 \\ n_{min} & \text{otherwise} \end{cases}$$

Rearrange this equation to solve for x_B (where $n(x) = \overline{n}$):

$$x_B = -x_A = \frac{d}{2}\left(1 - \frac{\overline{n} - n_{min}}{n_{max} - n_{min}}\right)$$

Use computer to numerically plot right side of equation vs. $\overline{n}$. Values of $\overline{n}$ can then be chosen for each mode (i.e. $m = 1, 2, 3, \cdots$).

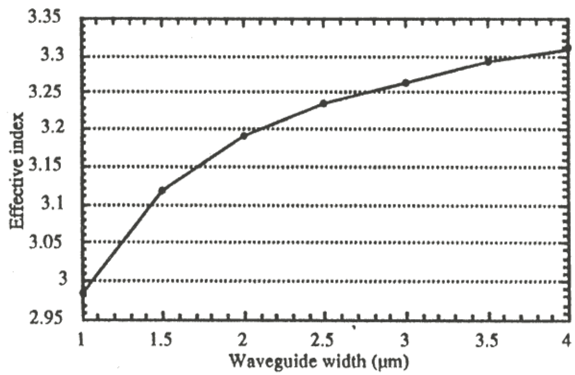

Figure 7.6. Effective index vs. waveguide width

The WKB approximation may not be accurate for $\overline{n}$ near n_{max} or n_{min}.

Plot ray path:

$$\frac{\Delta x}{\Delta z} = \pm \tan\theta_i$$

$$\theta_i = \cos^{-1}\left(\frac{\overline{n}}{n(x)}\right)$$

$d = 2\mu\text{m}$

$\overline{n} = 3.19$ from part (a)

At the center of the waveguide,

$$x = 0$$

$$\theta_i = \cos^{-1}\left(\frac{3.19}{3.5}\right) = 24.2^o$$

So in this case, we can't use the small angle approximation.

At the turnaround point,

$$x = x_B$$

$$n(x) = \overline{n}$$

$$\theta_i = \cos^{-1}\left(\frac{3.19}{3.19}\right) = 0^o$$

$$x_B = 0.124\mu\text{m}$$

To calculate the ray path, step along the path starting at

$$x_0 = 0$$

$$z_0 = 0$$

Using the recursive relation

$$x_{m+1} = x_m + \Delta z \tan\left[\cos^{-1}\left(\frac{\overline{n}}{n(x_m)}\right)\right]$$

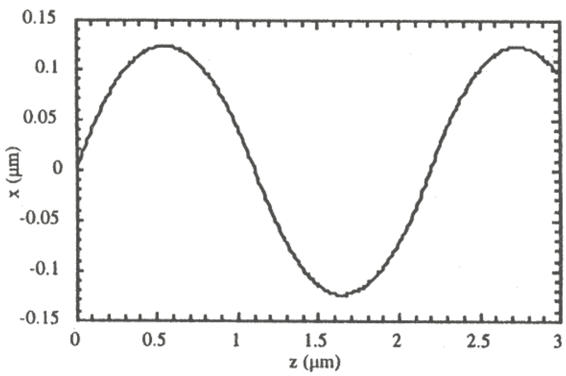

Figure 7.6. Ray path for the case of $d = 2$

Note that the curve is parabolic in shape (i.e. not a sine wave).