8.3:

Given:

$\lambda = 1.55\mu m$

output power = 10 mW

$R_s = \frac{100\mu m^2}{Lw} 10\ \Omega$

V_j = quasi Fermi level separation = $\frac{h\nu}{q} = 0.8V$

The material gain is described by Fig. 4.25 and decreases by one percent for each degree of heating of the active region: $g = (1 - \frac{\Delta T}{°C}0.01)g_0 \ln\left(\frac{J}{J_{tr}}\right)$

$g_0 = 583\ \text{cm}^{-1}$

$J_{tr} = 81\ A/cm^2$

$\eta_i = 0.70(1 - \frac{\Delta T}{°C}0.01)$ (η_i decreases by one percent for each degree of heating of the active region.)

$<\alpha_i>= 15\ \text{cm}^{-1}$

$\kappa = 50\ \text{cm}^{-1}$

$Z_T \approx \frac{\ln(4h/w)}{\pi\epsilon l}$ (2.65)

$\epsilon = 0.3$ W/(cm °C)

$h = 100\mu m$

Using Γ_x, we can find the normalized thickness of the SCH region, V, as described in Appendix 3. Then, assuming indicies of refraction of $n_g = 3.40$ for InGaAsP(1.3μm) and $n_c = 3.17$ for InP, we can solve for the effective index for the waveguide width. The lateral normalized thickness, V, and the lateral confinement factor, Γ_y, can then be described (see below in part (a)).

$0.06 = \Gamma_x = \frac{V^2}{V^2+2}$

$\rightarrow \quad V = 2.80,\ b = 0.57,\ \bar{n} = \sqrt{3.17^2 + 0.57(3.40^2 - 3.17^2)} = 3.30$

---

a)

$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{in} = IV = I(IR_s + V_j)$$

$$\eta_d' = F_1\eta_i\frac{\alpha_m}{\alpha_m + <\alpha_i>}e^{(L_g - L_{eff})(\Gamma g_{th} - <\alpha_i>}$$

$$P_{out} = \eta_d'\frac{h\nu}{q}(I - I_{th})$$

$$\Gamma g_{th} = \alpha_m + <\alpha_i>$$

$$\Gamma \approx \Gamma_x\Gamma_y$$

$$\Gamma_y \approx \frac{V^2}{2 + V^2}$$

$$V = d\frac{2\pi}{\lambda}\sqrt{n_g^2 - n_c^2} = d\frac{2\pi}{1.55\mu m}\sqrt{3.40^2 - 3.17^2}$$

$$\eta_i J_{th} = J_{tr}\ e^{\frac{g_{th}}{g_0(1-\frac{\Delta T}{°C}0.01)}}$$

$$L_g = \frac{L_a}{2}$$

$$L_{eff} = \frac{1}{2\kappa}\tanh(\kappa L_g) \tag{3.59}$$

$$r_g = \tanh(\kappa L_g) \tag{3.53}$$

$$\alpha_m = \frac{1}{2L_{eff}}\ln\left(\frac{1}{r_g^2}\right)$$

$$I_{operation} = \frac{P_{out}}{\eta_d' \frac{h\nu}{q}} + J_{th}Lw$$

$$\Delta T = (P_{in} - P_{out})Z_T$$

These equations are the ones that are used to solve for the wallplug efficiency, $\eta = \frac{P_{out}}{P_{in}}$. The algebra is tedious, but straight forward. Unfortunately, the temperature dependence of the gain and internal efficiency lead to a set of recursive functions. In other words, to solve for either ΔT or P_{in}, the other must be found first. We have gotten around this difficulty by iterating in ΔT. Start with $\Delta T = 0$ and solve for P_{in}. Use this value of P_{in} to find a new value of ΔT, which is used to find a new value of P_{in}. This method converges well for the values that we used in this problem. Once ΔT has been found, it is a simple matter to numerically find the optimum values for the length and width in order to obtain the highest wallplug efficiency.

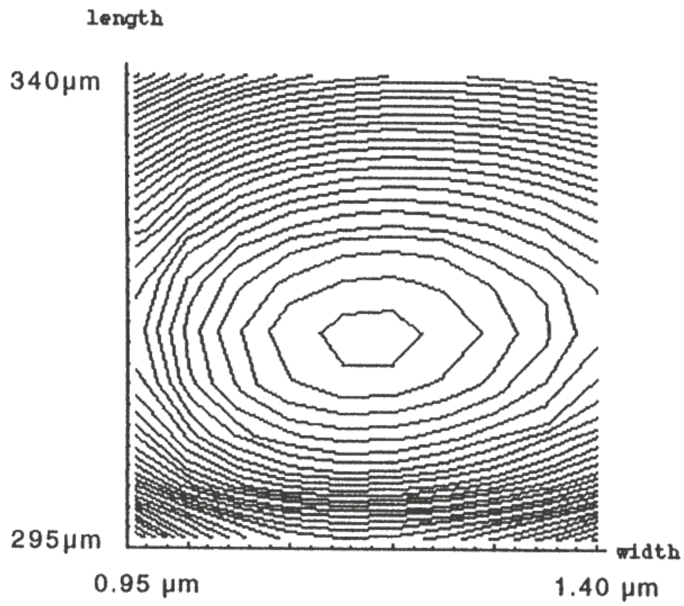

Figure 8.3a. Contour plot of wallplug efficiency as length and width are varied.

```
ListContourPlot[dTmatrix, Contours -> 30,
     Frame->False,
     Axes->True,
     AxesLabel->{"width","length"},
     ContourShading->False];
```

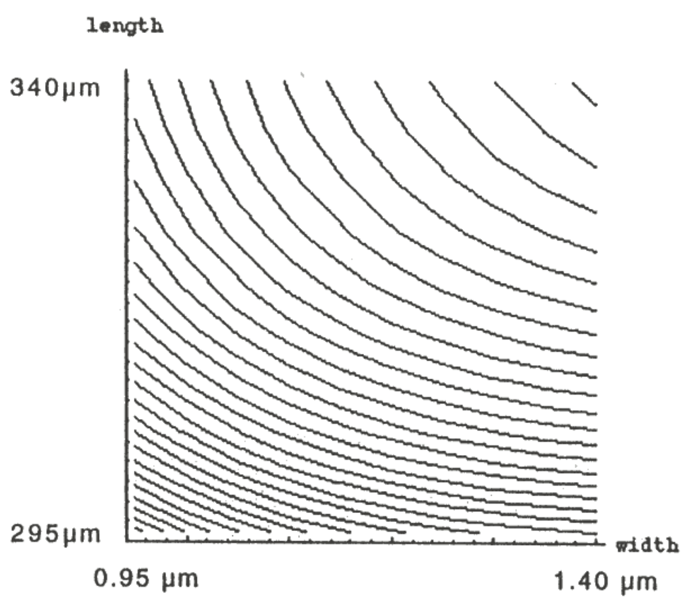

Figure 8.3a. Contour plot of temperature rise, ΔT, as length & width are varied.

The relevant values for the optimized laser design are:

$$\text{Length} = 315.\mu\text{m}$$

$$\text{Width} = 1.15\mu\text{m}$$

Temperature change = 3.5 °C

Wallplug efficiency = 0.36

Operating current = 31.2 mA

Input power = 27.7 mW

b) The IV curve is easy to plot using a continuation of the program above, but changing the value of the output power at each point. Note that since the heating effects are small, the LI curve can also be approximated using the slope efficiency, which can be derived from above. That approximation would give an error in the threshold current of 0.5 mA.

```
l = 0.0315;
w = 0.000115;
ztlw = N[zt[l,w]];

steps=100;

LIcurve = Table[0,{steps},{3}];

For[i=1,i<=steps,i++,
     LIcurve[[i,1]] = N[i 10^-4];   (* output power *)
     pout = LIcurve[[i,1]];

     For[dTold=10;dT=0;k=1,(Abs[dT-dTold]>0.1)&&(k<=40),k++,
          dTold=dT;
          dT= N[(pin[l,w,dT] - LIcurve[[i,1]])*ztlw];
         ];

     LIcurve[[i,2]]=dT;
     LIcurve[[i,3]]=iop[l,w,dT];
     ];

senddatatofile[LIcurve,"8.3b.mathematica.dat"];

LI = Join[{ {0,0}},
     Transpose[N[{Transpose[LIcurve] [[3]],Transpose[LIcurve] [[1]]}]]];
ListPlot[LI];
```

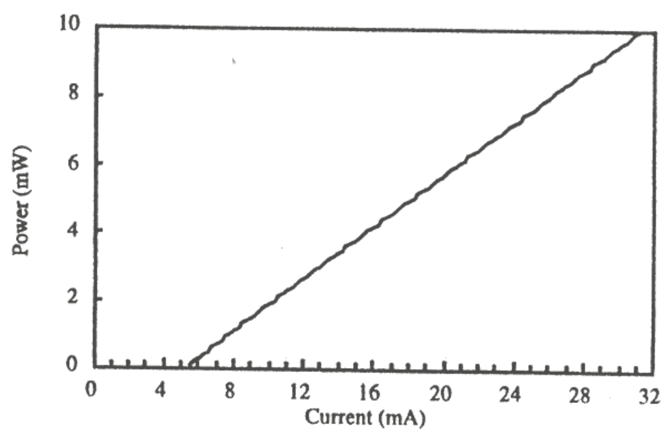

Figure 8.3b. LI curve for optimized laser.

```
dTI = Join[{ {0,0}},
     Transpose[N[{Transpose[LIcurve][[3]],Transpose[LIcurve][[2]]}]]];
ListPlot[dTI];
```

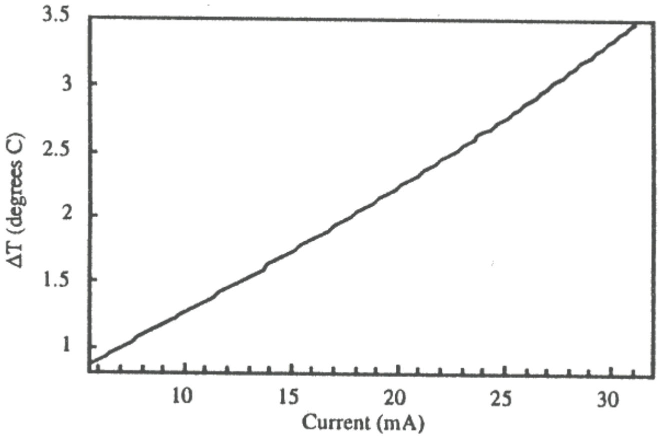

Figure 8.3b. ΔT vs. current for optimized laser.

```
thresholdI = LI[[2,1]]-(LI[[3,1]]-LI[[2,1]]);
thresholddT = dTI[[2,2]]-(dTI[[3,2]]-dTI[[2,2]]);

Print["Slope efficiency = ",
     hvoverq*etadprime[1,w,LIcurve[[1,2]]]," mW/mA"];
Print["Threshold current = ",1000*thresholdI," mA."];
Print["Threshold dT = ",thresholddT," degrees C"];
```

Slope efficiency = 0.40 mW/mA

Threshold current = 5.28 mA

Threshold ΔT = 0.85 °C

8.4:

Given:

$\lambda = 1.55\mu m$

$F = 15$

$P_0 = 5$ mW

$\Gamma_{xy} = 0.06$

$\eta_i = 0.70$

$n_{sp} = 1.2$

$\overline{n}_{g1} = 4$

$< \alpha_i >_{top} = 15$ cm^{-1}

$< \alpha_i >_{bot} = 5$ cm^{-1}

We want 100% coupling, $L_a = 300\mu m$, and $J < 8$ kA/cm^2.

---

a)

$$15 = F = \frac{\overline{n}_{g1}}{\overline{n}_{g1} - \overline{n}_{g2}} \qquad \text{(p. 376)}$$

$$\overline{n}_{g2} = 4 - \frac{4}{15} = 3.733$$

$$\Lambda = \frac{\lambda}{(\overline{n}_{g1} - \overline{n}_{g2})} \times \frac{\Delta \overline{n}_g}{\Delta \overline{n}}$$

$$= \frac{1.55\mu m}{4 - 3.733} \qquad (8.45)$$

$$= 5.812\mu m \times 3 = 17.4\ \mu m$$

---

c) MSR increases as L_c (the coupling length) increases. (See Eq. 8.49: As $L_c \to L_{tot}$, MSR$\to \infty$.) So we want the coupling length to be as large as possible such that the device can still reach an output power of 5 mW for $J < 8$ kA/cm^2 as required by the problem statement.

$$P_{out} = 5 \text{ mW} = \eta_{d1} \frac{h\nu}{q}(I_0 - I_{th})$$

$L_{tot} = L_c + L_a$

Assume a current injection width of $3\mu m$.

$I_0 = J_{max} w L_a = (8 \text{ kA}/cm^2)(3\mu m)(300\mu m) = 72 \text{ mA} / \eta_i = 102.8 \text{ mA}$

$\eta_{d1} = \eta_i \frac{\alpha_m}{\alpha_m + <\alpha_i>} F_1$

$F_1 = \left(\frac{\alpha_{m1}}{\cancel{\alpha_m}}\right) = \frac{|t_g|^2}{(1-|r_g|^2) + \frac{|r_g|}{r}(1-r_1{}^2)}$

$\eta_{d1} = \eta_i \frac{\alpha_{m1}}{\alpha_m + <\alpha_i>}$

For a 100% coupler, the energy in the coupled optical mode is in the top guide for 1/2 of the propagation distance and is in the bottom guide for the other 1/2 of the propagation distance. Therefore, the modal loss of the optical mode in the passive section (coupling section) can be written as $< \alpha_i >_c = (< \alpha_i >_{top} + < \alpha_i >_{bot})/2 = 10$ cm^{-1}. For the active region, the mode is primarily in the top guide; so $< \alpha_i >_a = < \alpha_i >_{top} = 15$ cm^{-1}

$$\alpha_{m_{1,2}} = \frac{1}{L_a}\ln\left(\frac{1}{r_{1,2}}\right)$$

$$r_1 = \sqrt{0.32}$$

$$r_2 = \sqrt{e^{L_c<\alpha_i>}0.32\, e^{L_c<\alpha_i>}}$$

$$\alpha_{m1} = \frac{1}{0.03\text{ cm}}\ln\left(\frac{1}{\sqrt{0.32}}\right) = 19.0\text{ cm}^{-1}$$

$$\begin{aligned}
\alpha_m &= \alpha_{m_1} + \alpha_{m_2} \\
&= \frac{1}{L_a}\ln\left(\frac{1}{0.32}\right) + \frac{L_c}{L_a} < \alpha_i >_c \\
&= \frac{1}{0.03\text{ cm}}\ln\left(\frac{1}{0.32}\right) + \frac{L_c}{300\mu\text{m}}(10\text{ cm}^{-1}) \\
&= (38.0\text{ cm}^{-1}) + \frac{L_c}{30\mu\text{m}}\text{ cm}^{-1}
\end{aligned}$$

$$\begin{aligned}
\eta_{d1} &= \eta_i \frac{\alpha_{m1}}{\alpha_m + < \alpha_i >} \\
&= (0.70)\frac{(19.0\text{ cm}^{-1})}{(38.0\text{ cm}^{-1}) + \frac{L_c}{30\mu\text{m}}\text{ cm}^{-1} + (15\text{ cm}^{-1})} \\
&= \frac{(19.0\text{ cm}^{-1})}{(53.0\text{ cm}^{-1}) + \frac{L_c}{30\mu\text{m}}\text{ cm}^{-1}}
\end{aligned}$$

Calculate the lateral confinement factor:

$n_1 = n_3 = 3.17$ and $n_2 = 3.40$

$$\begin{aligned}
V^2 &= k_0^2 d^2 (n_2^2 - n_1^2) \\
&= \frac{2\pi}{1.55\mu\text{m}}(3\mu\text{m})(3.40^2 - 3.17^2) \\
&= 223
\end{aligned}$$

$$\rightarrow \qquad \Gamma_y \approx \frac{V^2}{2+V^2} = 0.99 \approx 1.$$

Using the gain-current relation from Fig. 4.25, we can calculate the threshold current:

$$\Gamma_{xy} g_{th} = < \alpha_i >_a + < \alpha_i >_c \frac{L_c}{L_a} + \frac{1}{L_a}\ln\left(\frac{1}{R}\right)$$

$$\begin{aligned}
g_{th} &= \frac{1}{0.06}\left[15 + 10\frac{L_c}{300\mu\text{m}} + \frac{1}{0.03}\ln\left(\frac{1}{0.32}\right)\right] \\
&= (16.67\text{ cm}^{-1})\left[53 + \frac{L_c}{30\mu\text{m}}\right]
\end{aligned}$$

$$\begin{aligned}
I_{th} &= J_{th} w L_a \\
&= J_{tr}\, e^{g_{th}/g_0} w L_a \\
&= (81\text{ A/cm}^2)\, e^{(16.67\text{ cm}^{-1})[53+\frac{L_c}{30\mu\text{m}}]/(583\text{ cm}^{-1})}(3\times10^{-4}\text{ cm})(0.03\text{ cm}) \times 4QW/\eta_i \\
&= \cancel{(3.317\text{ mA})\, e^{\frac{L_c}{1080\mu\text{m}}}}
\end{aligned}$$

$$\begin{aligned}
P_{out} = 5\text{ mW} &= \eta_d \frac{h\nu}{q}(I_0 - I_{th}) \\
&= \frac{(19.0\text{ cm}^{-1})}{(53.0\text{ cm}^{-1}) + \frac{L_c}{30\mu\text{m}}\text{ cm}^{-1}}(0.8\text{ mW/ mA})\left[\overset{102.8}{\cancel{(72\text{ mA})}} - \overset{I_{th}}{\cancel{(3.317\text{ mA})\, e^{\frac{L_c}{1080\mu\text{m}}}}}\right]
\end{aligned}$$

Solve numerically for L_c:

$$L_c = \overset{1176}{\cancel{2292}}\,\mu\text{m}$$

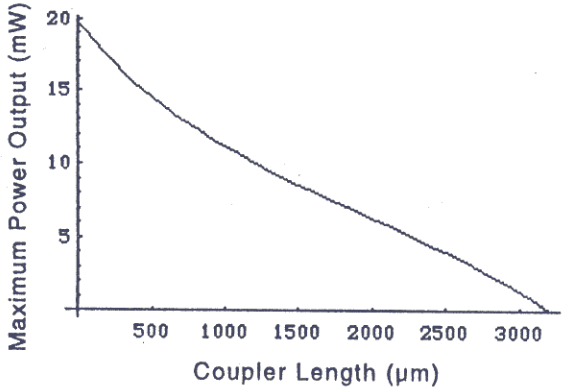

Figure 8.4c. Maximum output power for $J = 8$ kA/cm^2 vs. coupler length.

---

b)

For 100% coupling, κ can be found using Eq. 6.71

$$\begin{aligned}\kappa &= \frac{\pi}{2L_c} \\ &= \frac{\pi}{2(\cancel{0.2292\ \text{cm}}\ 0.1176)} \qquad (6.71) \\ &\cancel{= 0.85\ \text{cm}^{-1}} \\ &= 13.36\ \text{cm}^{-1}\end{aligned}$$

---

d) Calculate the LI curve using η_{d1} and I_{th} from part (c).

$$\begin{aligned}I_{th} &= J_{th} w L_a \times 4/\eta_i \qquad = 49.2\,\text{mA} \\ &\cancel{= (3.317\ \text{mA})\, e^{\frac{L_c}{1050\mu m}}} \\ &\cancel{= (3.317\ \text{mA})\, e^{\frac{2292}{1050}}}\ 1176 \\ &\cancel{= 29.4\ \text{mA}}\end{aligned}$$

$$\begin{aligned}\eta_{d1} &= \eta_i \frac{\alpha_m F_1}{\alpha_m + \langle \alpha_i \rangle} \\ &= \cancel{\frac{(19.0\ \text{cm}^{-1})}{(53.0\ \text{cm}^{-1}) + \frac{L_c}{30\mu m}\ \text{cm}^{-1}}} \\ &= \cancel{\frac{(19.0\ \text{cm}^{-1})}{(53.0\ \text{cm}^{-1}) + \frac{2292}{30\mu m}\ \text{cm}^{-1}}} \\ &\cancel{= 0.1468} \\ &= 0.12\end{aligned}$$

$$\begin{aligned}P_{out} &= \eta_{d1} \frac{h\nu}{q}(I - I_{th}) \\ &= \cancel{0.1468}\ 0.12\,(0.8\ \text{mW}/ma)(I - 49.2\ \text{mA}) \\ &\cancel{= (0.1175\ \text{mW}/ma)(I - 29.4\ \text{mA})} \\ &= 0.096\,(I - 49.2\,\text{mA})\ \text{mW}/\text{mA}\end{aligned}$$