



Lecture 2- Photon Statistics and Basics of Propagation in Dielectric Media



Photodetection

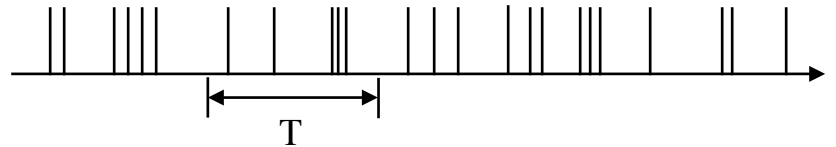
Detection of Optical Signals



- ⇒ Thermal: Temperature change with photon absorption
 - ⇒ Thermoelectric
 - ⇒ Pyromagnetic
 - ⇒ Pyroelectric
 - ⇒ Liquid crystals
 - ⇒ Bolometers
- ⇒ Wave Interaction: Exchange energy between waves at different frequencies
 - ⇒ Parametric down-conversion
 - ⇒ Parametric up-conversion
 - ⇒ Parametric amplification
- ⇒ Photon Effects: Generation of photocarriers from photon absorption
 - ⇒ Photoconductors
 - ⇒ Photoemissive
 - ⇒ Photovoltaics

Photon Statistics

- ⇒ Photon sources can in general be characterized as coherent or incoherent[†]
 - ⇒ Coherent: Probability that a photon is generated at time t_0 is mutually independent of probability of photons generated at other times (Markov Process)
 - ⇒ Poisson Process: Probability of finding n photons in time interval T
 - ⇒ Bunching is a trait of the Poisson process
 - ⇒ Interarrival time is decaying exponentially distributed



$$P(n | T) = \frac{(rT)^n e^{-rT}}{n!}$$

[†] Can also be a combination of these two types -> partially coherent

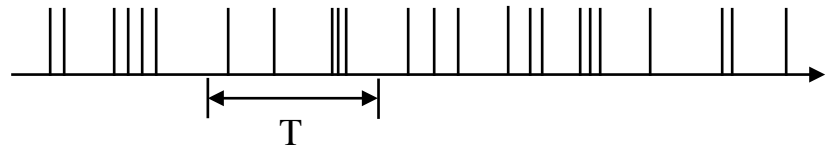
Where :

$P(n|T)$ is probability of finding n photons in time interval T
 R is mean photon arrival rate (photons/second)

Photon Statistics (II)

⇒ Narrowband Thermal (Gaussian):

⇒ Bose-Einstein Process: Probability of finding n photons in time interval T



$$P(n) = \left(\frac{1}{1 + n_b} \right) \left(\frac{n_b}{1 + n_b} \right)^n$$

Where :

$P(n)$ = probability of finding n photons given

n_b = mean number photons from incoherent source = $N_0/h\nu_0$

N_0 = spectral density of source = P_{opt}/B_0

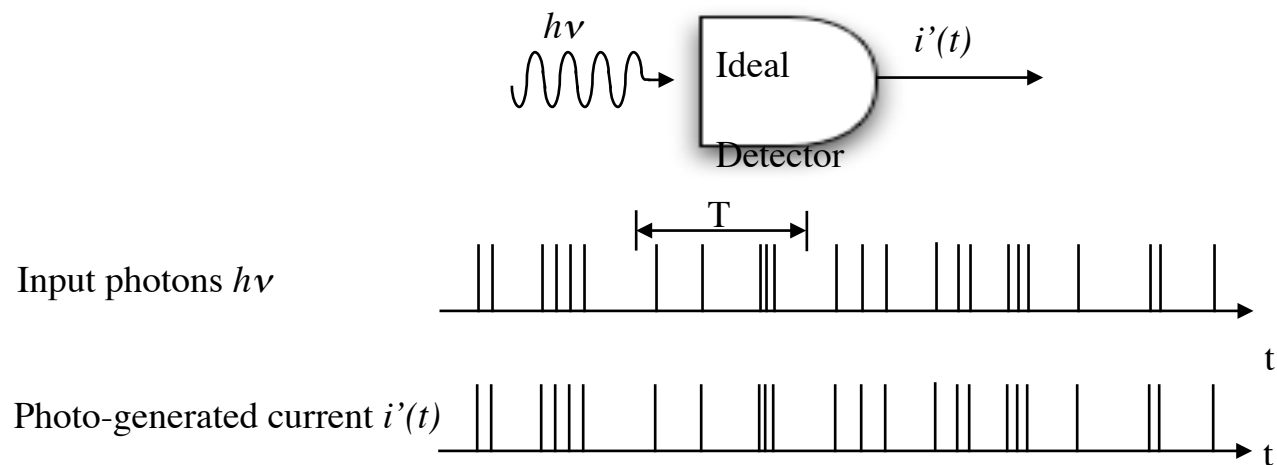
P_{opt} = total optical power from source

B_0 = source optical bandwidth

T = observation time $\leq 1/B_0$

Detecting Photons (1)

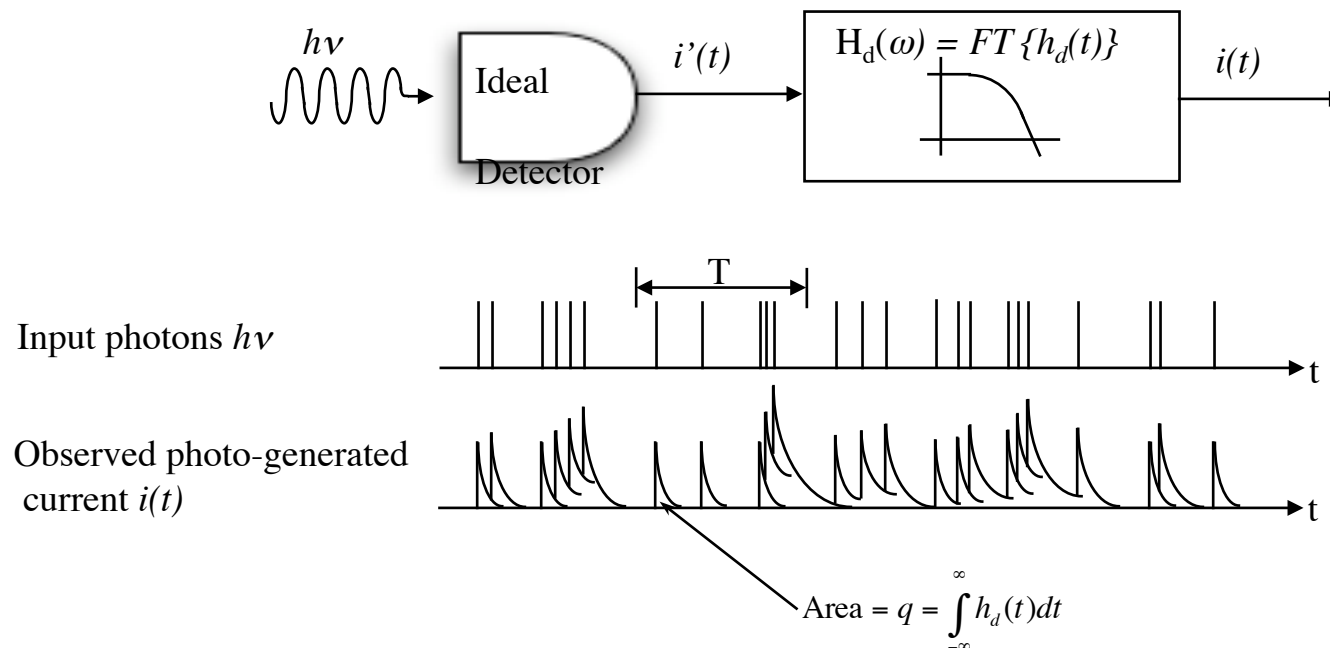
- ⇒ Any material that can respond to single photons can be used to count photons
- ⇒ Ideal Detector
 - ⇒ Generation of a electron-hole pair per absorbed photon results in an instantaneous current pulse



Detecting Photons (2)

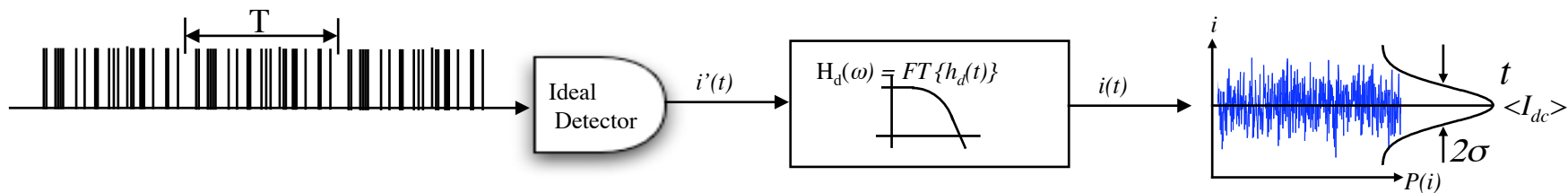
⇒ Real Detector

- ⇒ Has an inherent “impulse response,” $h_d(t)$, due to built in resistance and/or capacitance.
- ⇒ Can be modeled as an RC filter with low pass response



Detecting Photons (3)

- ⇒ As the average photon rate increases, the observed photo-current starts smoothing out, with a variance around the mean (average) count that is based on the statistics (which tends to Gaussian for large photon arrival rate)
- ⇒ $P(i)$ is the probability function of measuring the current at a certain value at time t .



Detecting Photons (4)

⇒ The detector output current $i(t)$ can be modeled as a discrete “filtered Poisson” process

$$i(t) = \sum_{j=1}^N h_d(t - \tau_j)$$

⇒ Where $h_d(t)$ is PD impulse response, N is total number e-h pairs generated, τ_j is the random time the j^{th} photocarrier is generated.

⇒ Define: Quantum Efficiency (QE), unitless, as

$$\eta = \frac{\text{number of photocarriers produced}}{\text{number of incident photons}}, 0 \leq \eta \leq 1$$

⇒ Define: Time varying photon rate parameter ($\lambda(t)$) in units of photocarriers/second as

$$\lambda(t) = \frac{\eta}{h\nu} P_{\text{recvd}}(t)$$

Detecting Photons (5)

⇒ The power incident on a photodetector of area A , in units of Watts, is

$$P(t) = \int_A I(\vec{p}, t) dA$$

⇒ where the instantaneous optical intensity at an observation point $\vec{p}$ is given by

$$I(\vec{p}, t) = \frac{1}{Z_0} |E(\vec{p}, t)|^2$$

⇒ The time varying photon rate parameter $\lambda(t)$ can then be written in terms of $P(t)$

$$\lambda(t) = \frac{\eta}{h\nu} \frac{|E(t)|^2}{Z_0}$$

Detecting Photons (6)

- ⇒ If we consider an observation interval, over which we are going to average our photon count over
 - ⇒ This can be due either to the inherent bandwidth of the detector or (as we will see later) on purpose to match the receiver bandwidth to the data bit rate
- ⇒ Then the number of photocarriers generated over the interval T counted at the j^{th} observation interval

$$N_j = \int_0^T \lambda_j(\tau) d\tau$$

- ⇒ Assuming a coherent source, the *conditional inhomogeneous Poisson process* describes this photon count during the j^{th} observation interval

$$P(N_j = N) = \frac{\left(\int_0^T \lambda_j(\tau) d\tau \right)^N}{N!} e^{-\int_0^T \lambda_j(\tau) d\tau}$$

Detecting Photons (7)

- ⇒ If we assume a constant rate parameter over the time interval T (independent of j), then the photo-generated current can be written as

$$i(t) = \lambda(t)q$$

$$\lambda(t) = \frac{N}{T}$$

- ⇒ Then the photocurrent produced by the photodetector can be written in Amperes, assuming the observation time is normalized to one second

$$\begin{aligned} i(t) &= \lambda(t)q = \frac{\eta q}{h\nu} P_{rcvd}(t) \\ &= \Re P_{rcvd}(t) \end{aligned}$$

- ⇒ Where we have defined the detector responsivity as

$$\Re = \frac{\eta q}{h\nu}$$



Review of The Wave Equation in Dielectric Media

Notation



- ⇒ MKS units
 - ⇒ Lower case for time varying quantities
 - ⇒ Capitals for the amplitudes of time varying quantities
 - ⇒ Complex quantities used to represent amplitude and phase:
- ⇒ (at least in Chapter 1. In Chapter 2, $E(x,y,z,t) = \text{Re} [E(x,y,z) e^{i\omega t}]$)

$$a(t) = \text{Re}[Ae^{i\omega t}]$$

Maxwell's Equations



$$\nabla \times \bar{h} = i + \frac{\partial d}{\partial t}$$

$$\nabla \times \bar{e} = -\frac{\partial b}{\partial t}$$

$$\nabla \cdot \bar{d} = 0$$

$$\nabla \cdot \bar{b} = 0$$

where $\bar{e}$ and $\bar{h}$ are the electric and magnetic field vectors
 $\bar{d}$ and $\bar{b}$ are the electric and magnetic displacement vectors
No free charge.

Constitutive Relations



$$\vec{d} = \epsilon_0 \vec{e} + \vec{p}$$

$$\vec{b} = \mu_0 (\vec{h} + \vec{m})$$

$\vec{p}$ and $\vec{m}$ are the electric and magnetic polarizations of the medium
 ϵ_0 and μ_0 are the electric and magnetic permeabilities of vacuum
 $\vec{e}$ and $\vec{h}$ are the electric and magnetic field vectors
 $\vec{d}$ and $\vec{b}$ are the electric and magnetic displacement vectors

Electric Susceptibility χ (Isotropic)



Isotropic Media: χ is a complex number

$$\bar{P} = \epsilon_0 \chi_{ij} \bar{E}$$

The real part determines the index (velocity) and the imaginary part determines the gain or absorption.

Isotropic media: Vacuum, gasses, glasses (optical fibers)

Anisotropic media: Semiconductors, crystalline materials.