ECE 236
PK 2007

Homework #1

Due 17 Jan

1.1 Solve one or both of these problems.

a) **1.5** The nonlinear dynamic equations for a single-link manipulator with flexible joints [185], damping ignored, is given by

$$\begin{aligned} I\ddot{q}_1 + MgL\sin q_1 + k(q_1 - q_2) &= 0 \\ J\ddot{q}_2 - k(q_1 - q_2) &= u \end{aligned}$$

where q_1 and q_2 are angular positions, I and J are moments of inertia, k is a spring constant, M is the total mass, L is a distance, and u is a torque input. Choose state variables for this system and write down the state equation.

b) **1.8** A synchronous generator connected to an infinite bus can be represented [148] by

$$\begin{aligned} M\ddot{\delta} &= P - D\dot{\delta} - \eta_1 E_q \sin\delta \\ \tau\dot{E}_q &= -\eta_2 E_q + \eta_3\cos\delta + E_{FD} \end{aligned}$$

where δ is an angle in radians, E_q is voltage, P is mechanical input power, E_{FD} is field voltage (input), D is damping coefficient, M is inertial coefficient, τ is time constant, and η_1, η_2, and η_3 are constant parameters.

(a) Using δ, $\dot{\delta}$, and E_q as state variables, find the state equation.

(b) Let $P = 0.815$, $E_{FD} = 1.22$, $\eta_1 = 2.0$, $\eta_2 = 2.7$, $\eta_3 = 1.7$, $\tau = 6.6$, $M = 0.0147$, and $D/M = 4$. Find all equilibrium points.

(c) Suppose that τ is relatively large so that $\dot{E}_q \approx 0$. Show that assuming E_q to be constant reduces the model to a pendulum equation.

1.2

a) In the Furuta Pendulum equations assume that the angular speed is kept at a constant value $\dot{\varphi} = c$ and compare the first equation of FP with the Bead Handout. Discuss the equilibria and explain their positions for the pendulum.

b) Briefly re-do the Brake problem and explain its phase plot.

1.3

2.2 For each of the following systems, find all equilibrium points and determine the type of each isolated equilibrium:

both! (1) $\dot{x}_1 = x_2, \qquad \dot{x}_2 = -x_1 + \frac{1}{16}x_1^5 - x_2$

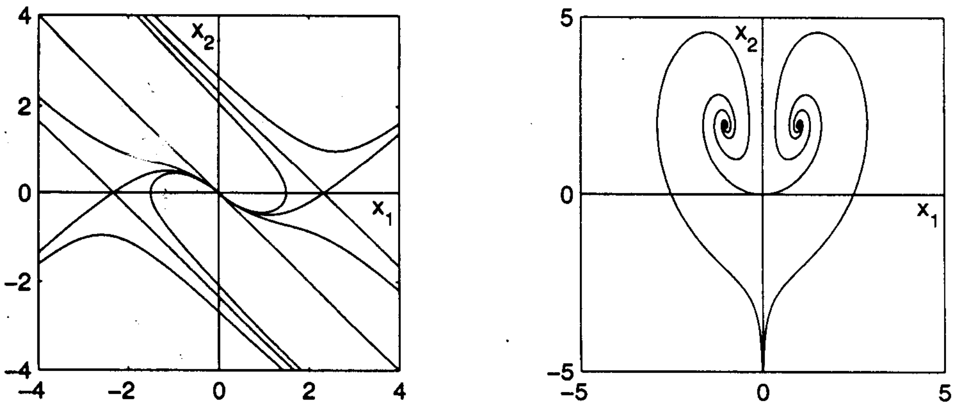
(2) $\dot{x}_1 = 2x_1 - x_1 x_2, \qquad \dot{x}_2 = 2x_1^2 - x_2$

1.4

2.4 The phase portraits of the following two systems are shown in Figure

both! (1) $\dot{x}_1 = x_2, \qquad \dot{x}_2 = x_1 - 2\tan^{-1}(x_1 + x_2)$

(2) $\dot{x}_1 = 2x_1 - x_1x_2, \qquad \dot{x}_2 = 2x_1^2 - x_2$

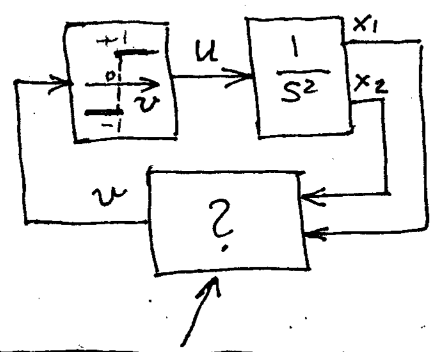


(1) (2)

1.5

2.15 Consider the system

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = u$$



where the control input u can take the values ± 1.

(a) Sketch the phase portrait when $u = 1$.

(b) Sketch the phase portrait when $u = -1$.

(c) By superimposing the two phase portraits, develop a strategy for switching the control between ± 1 so that any point in the state plane can be moved to the origin in finite time.

1.6

2.17 For each of the following systems, use the Poincaré–Bendixson's criterion to show that the system has a periodic orbit:

(2) $\dot{x}_1 = x_2, \qquad \dot{x}_2 = -x_1 + x_2(2 - 3x_1^2 - 2x_2^2)$

one or both

(3) $\dot{x}_1 = x_2, \qquad \dot{x}_2 = -x_1 + x_2 - 2(x_1 + 2x_2)x_2^2$

2.20 For each of the following systems, show that the system has no limit cycles:

(2) $\dot{x}_1 = -x_1 + x_1^3 + x_1x_2^2, \qquad \dot{x}_2 = -x_2 + x_2^3 + x_1^2x_2$

one or both

(3) $\dot{x}_1 = 1 - x_1x_2^2, \qquad \dot{x}_2 = x_1$