237/2007 PK Homework # 1 Due 11 April 20

READING: $\boxed{8.3}$ L8.2 ("Barbalat"), T.8 ("convergence")
T8.5 ("Window"), Example 8.11 (connect with 230!)

$\boxed{9.1}$ L9.1 ("Exp"+Lin 11.11), Quadratic-Like p345-46

$\boxed{9.2}$ L9.2 ("Exp" to $\|V\| \le \delta$), L9.3 ("UAS" to $\|V\| \le \rho(\delta)$)

$\boxed{9.5}$ T9.2 (M-matrix connections!)

Handouts:
- Signal convergence (connect with L8.2) useful.
- Optimality and Passivity ~ Important! Paralel to 14.4
- Lyapunov Redesign ~ Paralel to 14.2, 14.3

1.1

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\sin x_1 - g(t)x_2$$

$g(t)$ is continuously differentiable and $0 < k_1 \le g(t) \le k_2$.

(i) What are the equilibrium points?

(ii) To analyze the stability properties of (0,0) try $V = 1 - \cos x_1 + \frac{1}{2}x_2^2$ and see if you can establish a convergence property.

(iii) Use a linearization theorem from Chpt 4 to see if you can prove Exp. S.

1.2 Problem 9.2 ~ Very useful in applications!

1.3 Problem 9.7 ~ How much do you remember from 236?

1.4 Find an optimal stabilizing control for

$\dot{x} = xu$, $\quad J = \int_0^\infty (x^{2p} + u^2)\,dt$ first for $p=2$

and then try $p=1$. In the case $p=1$ the value $V(x)$ is not differentiable, but you can propose a "solution"! In the case $p=2$, show the passivity property