**Homework #5** **Due 16 May**

5.1 Problem 3 in the "Parametric Resonance" Handout.

Hint: Compose the period transition map as the product of the half-period maps.

In the "plus" half-period use cosh $(k\tau)$, sinh $(k\tau)$, $k^2 = \alpha^2 + \omega^2$

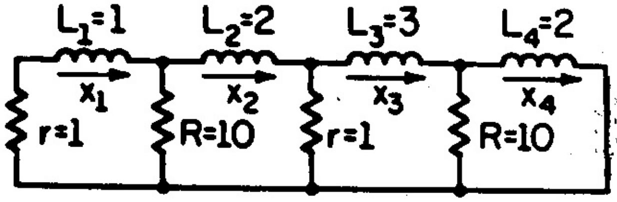
In the "minus" half-period use cos $(\Omega\tau)$, sin $(\Omega\tau)$, $\Omega^2 = \alpha^2 - \omega^2$

In the final calculation approximate these functions with a couple of terms each.

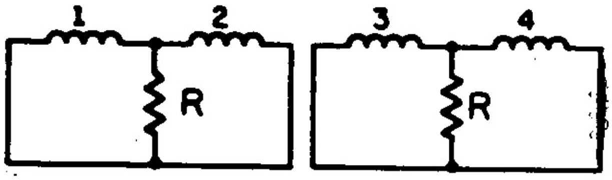
5.2 Simulate the system in the handout for $\varepsilon = 1, 0.8, 0.6$ etc.

5.3 Complete the Mechanical Network example.

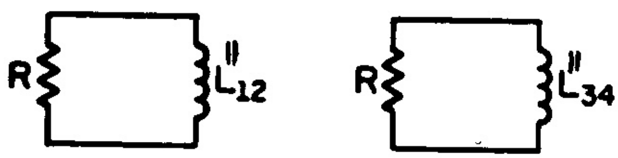
# A CIRCUIT EXAMPLE



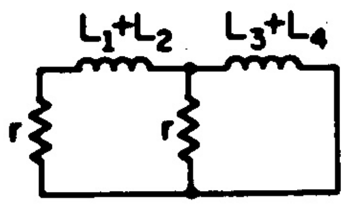

$$\dot{x}=A_\varepsilon x=\begin{bmatrix} -\frac{r+R}{L_1} & \frac{R}{L_1} & 0 & 0 \\ \frac{R}{L_2} & -\frac{r+R}{L_2} & \frac{r}{L_2} & 0 \\ 0 & \frac{r}{L_4} & -\frac{r+R}{L_3} & \frac{R}{L_3} \\ 0 & 0 & \frac{R}{L_4} & -\frac{R}{L_4} \end{bmatrix}$$




$$A_\varepsilon=\begin{bmatrix} -\frac{R}{L_1} & \frac{R}{L_1} & 0 & 0 \\ \frac{R}{L_2} & -\frac{R}{L_2} & 0 & 0 \\ 0 & 0 & -\frac{R}{L_3} & \frac{R}{L_3} \\ 0 & 0 & \frac{R}{L_4} & -\frac{R}{L_4} \end{bmatrix}+\varepsilon\begin{bmatrix} -\frac{R}{L_1} & 0 & 0 & 0 \\ 0 & -\frac{R}{L_2} & \frac{R}{L_2} & 0 \\ 0 & \frac{R}{L_3} & -\frac{R}{L_3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$




$$(\dot{x}_1-\dot{x}_2)_o=-\frac{R}{L_{12}}(x_1-x_2)_o \qquad (\dot{x}_3-\dot{x}_4)_o=-\frac{R}{L_{34}}(x_3-x_4)_o$$




FAST SUBNETWORKS OBTAINED BY CONSIDERING THE SMALL RESISTORS **r** AS SHORT CIRCUITS, SLOW SUBNETWORK BY CONSIDERING THE LARGE RESISTORS **R** AS OPEN CIRCUITS.

| | | | | |
|---|---|---|---|---|
| $-\lambda_i$ (NETWORK) | .08 | .77 | 8.47 | 15.8 |
| $-\lambda_i$ (SUBNETWORKS) | .09 | .78 | 8.33 | 15.0 |

SLOW STATES

$$Y_1 = \frac{L_1}{L_1+L_2} X_1 + \frac{L_2}{L_1+L_2} X_2$$

$$Y_2 = \frac{L_3}{L_3+L_4} X_3 + \frac{L_4}{L_3+L_4} X_4$$

AND FAST STATES

$$Z_1 = X_1 - X_2$$
$$Z_2 = X_3 - X_4$$

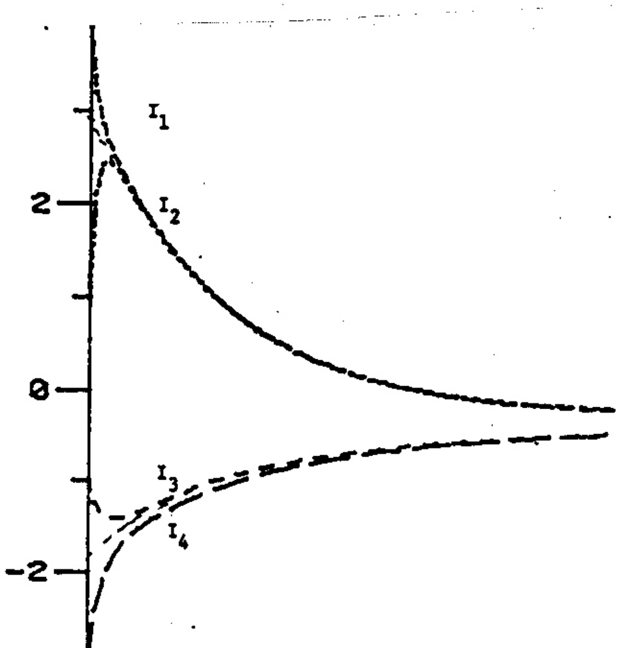
AFTER $R = \frac{r}{\varepsilon}$ GIVE

$$\dot{Y}_1 = -\frac{2r}{L_1+L_2} Y_1 + \frac{r}{L_1+L_2} Y_2 + A_1 Z_1 + A_2 Z_2$$

$$\dot{Y}_2 = \frac{r}{L_3+L_4} Y_1 - \frac{r}{L_3+L_4} Y_2 + A_3 Z_1 + A_4 Z_2$$

$$\varepsilon \dot{Z}_1 = \varepsilon A_5 Y_1 + \varepsilon A_6 Y_2 - r\left(\frac{1}{L_1} + \frac{1}{L_2}\right) Z_1 + \varepsilon A_7 Z_2$$

$$\varepsilon \dot{Z}_2 = \varepsilon A_8 Y_1 + \varepsilon A_9 Y_2 + \varepsilon A_{10} Z_1 - r\left(\frac{1}{L_3} + \frac{1}{L_4}\right) Z_2$$



# A MECHANICAL NETWORK

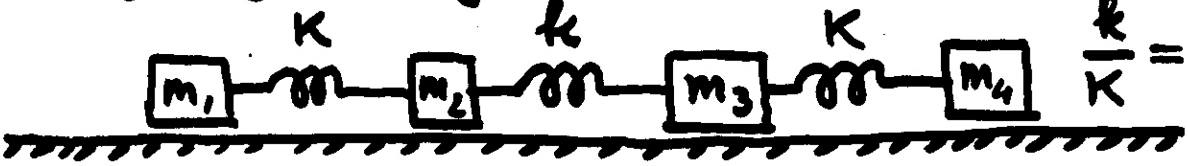
what is fast/slow?

fast time scale:

$$\tau = \frac{t}{\varepsilon}: \quad \frac{d^2y}{d\tau^2} = \mathcal{F}(y,\varepsilon), \ y \in \mathbb{R}^{n+m}, \ \left\{ \begin{matrix} x \in \mathbb{R}^n \\ z \in \mathbb{R}^m \end{matrix} \right\} ?$$

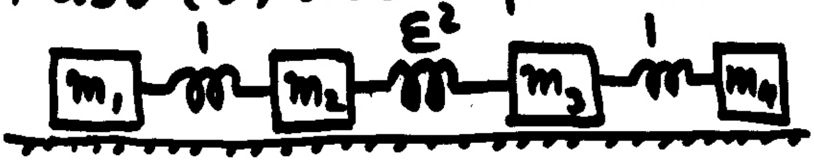
<u>CRUCIAL:</u> $\frac{d^2y}{d\tau^2} = \mathcal{F}(y,0)$ has an equlibrium manifold $\mathcal{M}_0$, so that the slow manifold is $\mathcal{M}_\varepsilon$

For aggregability $\mathcal{M}_0$ has a "simple" structure

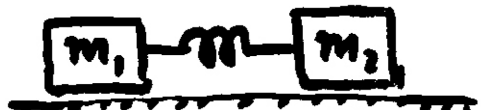



$$\frac{k}{K} = \varepsilon^2$$

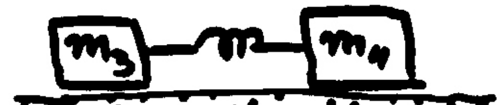
Fast ($\tau$) decomposition




Resut: Two fast subsystems

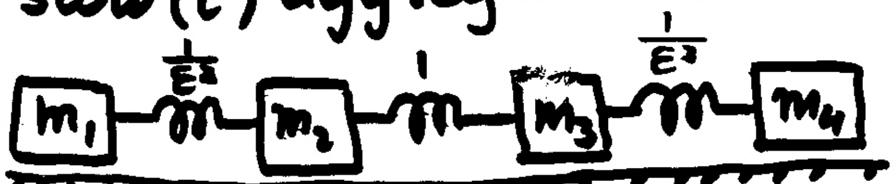


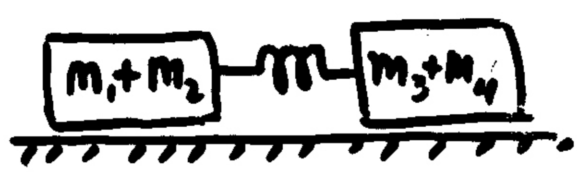

$\eta_{12} = y_1 - y_2$




$\eta_{34} = y_3 - y_4$

slow ($t$) aggregation




Result: One slow subsystem which is an approximate model on the manifold $\boxed{S}$

$$x_{12} = \frac{m_1}{m_1+m_2} y_1 + \frac{m_2}{m_1+m_2} y_2$$

$$x_{34} = \frac{m_3}{m_3+m_4} y_3 + \frac{m_4}{m_3+m_4} y_4$$

<u>HW 5.3</u> Continue and write it in a standard form. (in analogy with the RL circuit)