Output Regulation for Nonlinear Systems: an Overview *

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Abstract

This paper reviews recent advances in the theory of output regulation for nonlinear systems, in the presence of exosystem-generated commands and disturbances.

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1 Introduction

The problem of controlling the output of a system so as to achieve asymptotic tracking of prescribed trajectories and/or asymptotic rejection of undesired disturbances is a central problem in control theory. A classical setup in which the problem was posed and successfully addressed – in the context of linear, time-invariant and finite dimensional systems – is the one in which the exogenous inputs, namely commands and disturbances, may range over the set of all possible trajectories of a given autonomous linear system, commonly known as the *exosystem*. The cases when the exogenous command is a constant (i.e. set point control) or the exogenous disturbance is a sinusoidal signal of know frequency (i.e. the exosystem is a harmonic oscillator) are, of course, classical. In these special cases, it is well known that a controller which incorporates an *internal model* of the exosystem is able to secure asymptotic decay to zero of the tracking error for *every possible exogenous input* in the class of signals generated by the exosystem (i.e for every desired set point or every amplitude and phase of the disturbing sinusoid), and does it *robustly* with respect to parameter uncertainties. This is in sharp contrast with alternative approaches, such as the one aiming at "perfectly" tracking a fixed trajectory, where in lieu of the assumption that a signal is within a class of signals generated by an exogenous system, one instead needs to assume complete knowledge of the past, present and future time history of the trajectory to be tracked, as well as perfect knowledge of all system parameters.

An exhaustive presentation of the theory of output regulation for multivariable linear, time-invariant and finite dimensional systems can be found in the works of Davison, Francis and Wonham. The latter authors had also initiated the study of the corresponding design problem for nonlinear systems. Their contribution were followed by our earlier work on solving the problem of output regulation for nonlinear systems in the case of neutrally stable exosystems, which led to the formulation of the so-called nonlinear regulator equations, and their existence theory based on zero dynamics. This work has stimulated more extensive research on special classes of exosystems, and on the development of computational approaches to solving the regulator equations by Huang, Rugh, and Krener, as well as the recent development of methods for robust output regulation by Huang, Khalil and ourselves. The purpose of this paper is to review some relevant features of the theory of output regulation as well as to outline the major problems on which the research is presently progressing.

2 Linear Systems

For the sake of completeness, we review here some of the most relevant facts about output regulation of linear systems. Consider a linear system modeled by equations of the form

$$\begin{aligned} \dot{x} &= Ax + Bu + Pw \\ e &= Cx + Qw\,, \end{aligned} \tag{1}$$

with state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}^m$, regulated output $e \in \mathbb{R}^m$ and exogenous disturbance input $w \in \mathbb{R}^r$ generated by an exosystem

$$\dot{w} = Sw \tag{2}$$

which is assumed to be *neutrally stable*.

The problem of output regulation is the problem of finding a feedback law (the *regulator*)

$$\begin{aligned} \dot{\xi} &= F\xi + Ge \\ u &= H\xi \end{aligned} \tag{3}$$

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such that, in the "forced" closed loop system

$$\begin{aligned}\dot{x} &= Ax + BH\xi + Pw \\ \dot{\xi} &= F\xi + GCx + GQw \\ \dot{w} &= Sw\,,\end{aligned} \tag{4}$$

for every initial condition $(x(0), \xi(0), w(0))$, the response $(x(t), \xi(t))$

(a) is bounded, and

(b) such that

$$\lim_{t\to\infty} e(t) = 0\,.$$

Conditions for the existence of a solution of the problem of output regulation have been established in the works of Davison [2], Francis and Wonham [7], Francis [6], Wonham [25]. In particular, the work of Francis has shown that the solvability of this problem corresponds to the solvability of a system of two linear matrix equations.

Theorem 1 *There exists a regulator if and only if the pair (A, B) is stabilizable, the pair (C, A) is detectable, and the linear matrix equations*

$$\begin{aligned}\Pi S &= A\Pi + B\Gamma + P \\ 0 &= C\Pi + Q\end{aligned} \tag{5}$$

have a solution Π, Γ

As observed in the introduction, it is of paramount importance that the property of asymptotic tracking achieved by solving a problem of output regulation be insensitive to parameter variations. This requirement leads to the notion of *structurally stable regulator*, defined as follows. Viewing the set of matrices $\{A, B, C, P, Q\}$ which characterize (1) as an element of a *space of parameters*

$$\mathcal{P} = \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m} \times \mathbb{R}^{m\times n} \times \mathbb{R}^{n\times r} \times \mathbb{R}^{m\times r},$$

a regulator of the form (3) is said to be structurally stable at $\{A_0, B_0, C_0, P_0, Q_0\}$ if it solves the problem of output regulation for any set $\{A, B, C, P, Q\}$ of parameters in some open neighborhood $\mathcal{P}_0$ of the nominal *nominal* set $\{A_0, B_0, C_0, P_0, Q_0\}$ in the parameter space $\mathcal{P}$. Conditions for the existence of a structurally stable regulator are the following ones (see again [6]).

Theorem 2 *There exists a structurally stable regulator at $\{A_0, B_0, C_0, P_0, Q_0\}$ if and only if the pair (A_0, B_0) is stabilizable, the pair (C_0, A_0) is detectable, and the linear matrix equations*

$$\begin{aligned}\Pi S &= A_0\Pi + B_0\Gamma + P \\ 0 &= C_0\Pi + Q\end{aligned} \tag{6}$$

have a solution Π, Γ *for all* P, Q.

Remark. The last condition in this Theorem is equivalent to the condition that matrix

$$\begin{pmatrix} A_0 - \lambda I & B_0 \\ C_0 & 0 \end{pmatrix} \tag{7}$$

is nonsingular for each λ which is an eigenvalue of S, or – what is the same – to the condition that none of the trasmission zeros of the controlled plant coincides with an eigenvalue of the exosystem. ◁

As far as actual construction of a structurally stable regulator is concerned, suppose, Without loss of generality, that the matrix S which characterizes the exosystem has been transformed into a block-diagonal matrix of the form

$$S = \begin{pmatrix} * & 0 \\ 0 & S_{\min} \end{pmatrix}$$

in which $S_{\min}$ is a matrix whose characteristic polynomial coincides with the minimal polynomial of S. Let q denote the dimension of $S_{\min}$ and let Φ be a $qm \times qm$ matrix defined as

$$\Phi = \begin{pmatrix} S_{\min} & 0 & \cdots & 0 \\ 0 & S_{\min} & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & S_{\min} \end{pmatrix}.$$

(the block $S_{\min}$ is repeated m times in Φ, where m is the number of input and output components of (1)).

Let Ψ and Γ be matrices, of dimensions $qm \times m$ and $m \times qm$ respectively, such that the pair (Φ, Ψ) is controllable and the pair (Γ, Φ) is observable (standard controllability/observability tests for systems in Jordan form show that matrices of this kind always exist). As a consequence of the hypothesis that the matrix (7) is nonsingular for each λ which is an eigenvalue of S (or, what is the same, an eigenvalue of Φ), the triplet

$$\begin{pmatrix} A_0 & B_0\Gamma \\ \Psi C_0 & \Phi \end{pmatrix}, \quad \begin{pmatrix} B_0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} C_0 & 0 \end{pmatrix} \tag{8}$$

is necessarily stabilizable and detectable and there exist matrices K, L, M such that

$$\begin{pmatrix} \begin{pmatrix} A_0 & B_0\Gamma \\ \Psi C_0 & \Phi \end{pmatrix} & \begin{pmatrix} B_0 \\ 0 \end{pmatrix} M \\ L\begin{pmatrix} C_0 & 0 \end{pmatrix} & K \end{pmatrix}$$

has all eigenvalues with negative real part.

Using the matrices Φ, Ψ, Γ and K, L, M thus defined, it is possible to construct a structurally stable regulator, in the form of the parallel connection of a subsystem

$$\begin{aligned}\dot{\xi}_1 &= \Phi\xi_1 + \Psi e \\ u_1 &= \Gamma\xi_1\,,\end{aligned} \tag{9}$$

known as *internal model* (in the terminology of Francis and Wonham [7]) or *servocompensator* (in the terminology of Davison [2]), and of a *stabilizer*

$$\begin{aligned}\dot{\xi}_2 &= K\xi_2 + Le \\ u_2 &= M\xi_2\,.\end{aligned} \tag{10}$$

3 Perfect Versus Asymptotic Tracking

As mentioned in the introduction, one could in principle pose the problem of "perfect" reproduction of a given trajectory. If the system to be controlled is "invertible", to address such a problem consists in computing a precise initial state and a precise control input (or equivalently a reference trajectory of the state), such that, if the system is accordingly initialized and driven, its output exactly reproduces the reference signal. This process, which is sometimes referred to as "dynamic inversion", does not pose any particular difficulty in the case of invertible "minimum-phase" systems. It becomes however pretty delicate in the case of non-minimum-phase systems, where, in order to avoid unbounded responses, special strategies have to be implemented, which may require (see [5]) a control action, on the system, prior to the instant of time from which perfect tracking is supposed to take place (thus, in a certain sense, a "non-causal" control action). However, regardless of the fact that the plant to be controlled is minimum-phase or not, it must be stressed that the basic assumptions needed to successfully solve a problem of "perfect tracking" are, indeed, the "perfect knowledge" of the entire reference trajectory as well as the "perfect knowledge" the model of the plant to be controlled. Since the values of the parameters which characterize the model of the plant and/or the trajectory to be tracked *uniquely* determine the input required to enforce an identically zero tracking error, uncertainties on these parameters – which are inevitable in any realistic practical setup – render the problem of perfect tracking an ill-posed design problem.

To be more specific, consider the following simple example (see [24]) in which the output x_1 of a two-dimensional linear system

$$\begin{array}{rclr} \dot{x}_1 & = & x_2 & x_1(0)=0 \\ \dot{x}_2 & = & -\mu x_2 + u & x_2(0)=1 \end{array} \qquad (11)$$

is required to track a sinusoidal signal $x_{1d}(t) = \sin t$. Set $\mu = \mu_{\text{nom}} + \Delta\mu$, where μ_{nom} represents the nominal value of μ.

Let $x_d(t)$ be the (unique) state trajectory which complies with the constraint $e(t) = 0$ for all $t \geq 0$ and let $u_d(t)$ be the (unique) input able to enforce this particular trajectory. Perfect tracking can be achieved, if and only if $x(0) = x_d(0)$, setting $u(t) = u_d(t)$. The plant being unstable, this control mode should be supplemented with some stabilizing action. In this respect, a convenient choice is to add a correction term proportional to the difference between the desired $x_d(t)$ and the actual $x(t)$, namely to use the control (see e.g. [5])

$$u(t) = u_d(t) + K(x_d(t) - x(t)) \,. \qquad (12)$$

Due to the presence of the correction term $K(x_d(t) - x(t))$, which is trivially zero in the case of perfect tracking, this law is able of securing asymptotic decay to zero of the tracking error in case the initial state of the plant is different from $x_d(0)$. However, it is not able to compensate, even asymptotically, for any imperfect knowledge of the value of the (possibly uncertain) parameter μ.

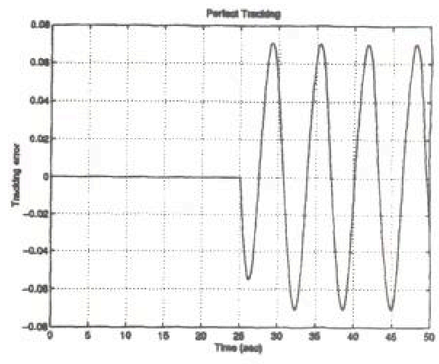

Figure 1: Error due to parameter change at time $t = 25$.

The control law (12), computed for $\mu = \mu_{\text{nom}}$, specializes as

$$u = \mu_{\text{nom}} \cos t - \sin t + k_1(\sin t - x_1) + k_2(\cos t - x_2) \qquad (13)$$

where k_1, k_2 are chosen so as to asymptotically stabilize the system. This control law secures perfect tracking in case $\Delta\mu = 0$ but, when $\mu \neq \mu_{\text{nom}}$, it is no longer able to secure $e(t) = 0$, even asymptotically. In fact, in case $\Delta\mu \neq 0$, the control input needed to impose $e(t) = 0$ for all $t \geq 0$ is equal $u(t) = u_{\text{nom}}(t) + \Delta\mu \cos t$. As a consequence, if the control law (13) is used, a sinusoidal steady state error occurs, whose maximum amplitude is given by

$$e_{\max} = \frac{\Delta\mu}{\sqrt{(k_1-1)^2 + (k_2 + \mu_{\text{nom}} + \Delta\mu)^2}}. \qquad (14)$$

Fig.1 highlights the non robustness of this approach, by showing how an unpredicted change in the value of the parameter μ induces a persistent tracking error. In particular, it describes the case in which, at time $t = 25\,sec$, the parameter μ switches from the nominal value $\mu = \mu_{\text{nom}} = 2$ to a new value $\mu = 1$.

Conversely, if a control law of the form (9)-(10) is used, asymptotic tracking can be achieved, with internal model

$$\begin{aligned} \dot{\zeta}_1 &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e \\ u_1 &= \begin{bmatrix} c_1 & c_2 \end{bmatrix} \xi \end{aligned}$$

and the stabilizer $u_2 = Ke$, where the constants c_1, c_2 and K are such that the overall closed loop system is

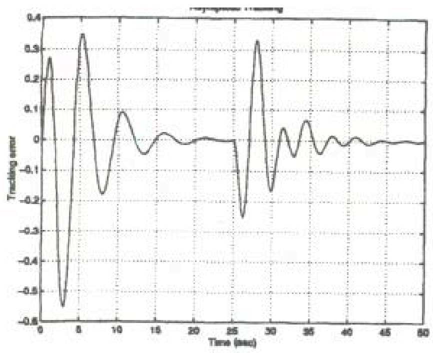

Figure 2: Error due to state mismatch at $t = 0$ and parameter change at $t = 25$.

asymptotically stable for all possible values of μ. In this case, by fixing $c_1 = 1.70$, $c_2 = 0.17$ and $K = 3.25$, and by assuming the same behavior as above for μ, the simulation results shown in Fig.2 are obtained (assuming, in this case, $x(0) = 0$). Note that the control law (9)-(10) is able to (asymptotically) overcome both the the lack of knowledge of $x(0)$ and $w(0)$, which is responsible for the transient at $t = 0$, as well as the unpredicted switch of the value of μ at $t = 25$ *sec.*

4 Structurally Stable and Robust Regulation

Consider now a nonlinear system modeled by equations of the form

$$\begin{aligned} \dot{x} &= f(x, w, u, \mu) \\ e &= h(x, w, \mu) \end{aligned} \qquad (15)$$

with state $x \in X \subset \mathbb{R}^n$, control input $u \in \mathbb{R}^m$, exogenous input $w \in W \subset \mathbb{R}^r$ and tracking error $e \in \mathbb{R}^m$. In these equations, $\mu \in \mathbb{R}^p$ represents a vector of parameters whose value may not be accurately known. It is assumed that the exogenous input is generated by an autonomous dynamical system of the form

$$\dot{w} = s(w)\,, \qquad (16)$$

which is supposed to be *neutrally stable* (i.e. Lyapunov stable in forward and backward time). It also is assumed that $f(x, w, u, \mu)$, $h(x, w, \mu)$, $s(w)$ are sufficiently smooth functions of their arguments and $f(0, 0, 0, \mu) = 0$, $h(0, 0, \mu) = 0$, $s(0) = 0$.

In (15) the roles of the exogenous input w and that of the (possibly uncertain) parameter μ have been kept separate. Note, however, that such a separation is superfluous if the parameter μ is constant. In this case, in fact, μ can be viewed as generated by the trivial dynamical system $\dot{\mu} = 0$ which is indeed neutrally stable. Thus, w and μ can be viewed as components of an "augmented" exogenous input w^a modeled by

$$\dot{w}^a = \begin{pmatrix} \dot{w} \\ \dot{\mu} \end{pmatrix} = \begin{pmatrix} s(w) \\ 0 \end{pmatrix} = s^a(w^a). \qquad (17)$$

The problem of ***structurally stable output regulation*** is the problem of finding a controller modeled by equations of the form

$$\begin{aligned} \dot{\xi} &= \eta(\xi, e) \\ u &= \theta(\xi) \end{aligned} \qquad (18)$$

with state $\xi \in \Xi^\circ \subset \mathbb{R}^\nu$, in which $\eta(\xi, e)$ and $\theta(\xi)$ are sufficiently smooth functions of their arguments, $\eta(0, 0) = 0$ and $\theta(0) = 0$, such that, in the forced closed loop system

$$\begin{aligned} \dot{x} &= f(x, w, \theta(\xi), \mu) \\ \dot{\xi} &= \eta(\xi, h(x, w, \mu)) \\ \dot{w} &= s(w) \\ \dot{\mu} &= 0\,, \end{aligned} \qquad (19)$$

for every initial condition $(x(0), \xi(0), w(0), \mu(0))$ in *some* neighborhood S of $(0, 0, 0, 0)$, the response $(x(t), \xi(t))$

(a) is bounded, and

(b) such that

$$\lim_{t\to\infty} e(t) = 0\,.$$

The problem of ***robust output regulation*** is to find a controller of the form (18) such that the properties indicated in (a) and (b) hold for every initial condition $(x(0), \xi(0), w(0), \mu(0))$ in an ***a priori fixed*** neighborhood S of $(0, 0, 0, 0)$.

5 Nonlinear Regulator Equations and Nonlinear Internal Model

Beginning with the work [19], the problem of nonlinear output regulation has been the subject of intensive investigation. One of the highlights of this work was the proof of a (partial) nonlinear version of Theorem 1, showing that the role of the linear equations (5) is played, for a nonlinear system, by equations of which take the form

$$\begin{aligned} \frac{\partial \pi(w, \mu)}{\partial w} s(w) &= f(\pi(w, \mu), c(w, \mu), w, \mu) \\ 0 &= h(\pi(w, \mu), w, \mu) \end{aligned} \qquad (20)$$

in the unknowns $\pi(w, \mu)$ and $c(w, \mu)$, which are C^k functions (for some large k) defined in a neighborhood of $(0, 0)$ and such that $\pi(0, \mu) = 0$ and $c(0, \mu) = 0$.

As shown in [19], solvability of these equations is a basic necessary condition for the solution of a problem of output regulation. Paper [19] also showed that, under appropriate additional hypotheses expressing certain properties of stabilizability and detectability of the linear approximation of the plant, solvability of these equations suffices to guarantee the existence of a solution of a problem of output regulation. However, the regulator constructed in [19], explicitly dependent on the functions $\pi(\cdot,\mu)$ and $c(\cdot,\mu)$, was not guaranteed to be structurally stable.

The approach of [19] to the problem of output regulation was further pursued by Huang and Rugh in [14], [15]. In particular, the first paper improved the stabilization results of [19] and addressed the issue of finding a power series expansion of the solution $\pi(w,\mu)$ of the regulator equation (20). The second paper elaborated on the property that if the solution in question is determined only up to a certain degree of accuracy, then output regulation can be secured up to a steady-state error of the same degree. Issues related to polynomial approximation and/or power series expansions for the determination of the solution of (20) were also considered by Krener [22].

The problem of output regulation for nonlinear systems in the presence of parameter uncertainties was originally addressed in [7] and [9], where it was shown that, when the exogenous input is *constant* (i.e. set point control under constant disturbances), the incorporation of an internal model into the compensator (i.e. integral control) suffices to guarantee output regulation in the presence of small parameter variations (i.e. structurally stable regulation).

The solution of the problem of output regulation, in the presence of small parameter uncertainties, when the exogenous input is *time-varying* (e.g. when the exosystem is a harmonic oscillator) requires that the internal model be not only able to generate inputs corresponding to the trajectories of the exosystem, but also a number of their "higher order" nonlinear "deformations". For example, in the case of a cubic nonlinearity with unknown coefficient and sinusoidal reference output, the internal model must generate the sinusoid in question *and* its third harmonic. This key idea was simultaneously elaborated by Huang and Lin [10, 11, 12], by Khalil [20, 21] and by Delli Priscoli [3, 4]. In particular Huang and Lin, appealing to concept of "regulation of order k" (namely, regulation up to a steady-state error which is infinitesimal of order k with respect to the amplitude of the disturbance input) introduced earlier in [15], provided in [10] a methodology for the design of a controller which, regardless of small parameter perturbations, achieves regulation of order k. This methodology was proven in [11] to yield exact regulation, regardless of small parameter variations, for some relevant classes of nonlinear systems [11]. In fact, [11] shows that if the function $c(w,\mu)$, which renders the regulator equations (20) satisfied for some $\pi(w,\mu)$, is a *polynomial* of (some) degree k in w and the exosystem is a *linear system* then it is possible to obtain *structurally stable* regulation by designing an internal model which generates all the exogenous inputs as well all higher harmonics, up to order k. Delli Priscoli, in [3] and [4], arrived at the similar (though somewhat more general) conclusion that structurally stable regulation is possible if the family of all functions of the form $u = c(w(t),\mu)$, where $w(t)$ is any exogenous input generated by the exosystem, can be seen as a subset of the set of all possible solutions of a fixed ordinary differential equation.

More precisely, suppose there exists a nonlinear differential equation

$$\zeta^{(\nu)} + \phi(\zeta^{(\nu-1)},\ldots,\zeta^{(1)},\zeta) = 0$$

with the property that, for every $w(0),\mu(0)$ there is $\zeta(0)$ such that

$$\zeta(t) = c(w(t),\mu)$$

(this is always the case, for instance, whenever the exosystem is linear and $c(w,\mu)$ is a polynomial – with μ-dependent coefficients – in w). Then, a structurally stable regulator can be constructed, in a form which is identical to the one described before in the case of linear systems, where the role of *internal model* is now played by a subsystem of the form

$$\begin{aligned} &\xi_1^{(\nu)} + \phi(\xi_1^{(\nu-1)},\ldots,\xi_1^{(1)},\xi_1) = e \\ &u_1 = \xi_1 \,. \end{aligned} \qquad (21)$$

The other component of the regulator, namely subsystem (10), has to be designed so that the resulting interconnection

$$\begin{aligned} \dot{x} &= f(x,0,\theta(\xi),0) \\ \dot{\xi} &= \eta(\xi,h(x,0,0)) \end{aligned}$$

is asymptotically stable (in the first approximation) at the equilibrium point $(x,\xi)=(0,0)$ and this is indeed possible if the linear approximation of the plant

$$\begin{aligned} \dot{x} &= f(x,0,u,0) \\ e &= h(x,0,0) \end{aligned}$$

at the equilibrium point $(x,\xi)=(0,0)$ is stabilizable and detectable, and none of its trasmission zeros coincide with an eigenvalue of the linear approximation of the internal model (21) at the equilibrium $\xi_1 = 0$.

The problem of designing a *semiglobal robust regulator* can be approached in a similar way, i.e. as the parallel interconnection of the internal model (21) and of a stabilizer, but now the challenge is to achieve the required convergence to zero of the tracking error (with bounded internal trajectories) not just in *some* neighborhood of the given equilibrium but for any initial condition in a given *a priori fixed* (compact) set. The issue of designing a unit, which replaces the role of subsystem (10), capable of securing the required convergence properties on a possibly large domain is indeed rendered more difficult by the fact that the controlled plant is interconnected with an internal model which is only neutrally stable. The first important contribution in this area can be found in the work [21] of Khalil. This paper studies a class of system having relative degree equal to the dimension of the state space (i.e. having a trivial zero dynamics) and utilizes a technique developed earlier by Esfandjari and Khalil to design an error-feedback controller in which the components of the internal state are estimated by means of a "saturated" high-speed observer. Extensions of this result to classes of systems having asymptotically stable zero dynamics have been presented in [23] and in [18]. However, the solution of the problem under the weaker hypothesis that the controlled plant is just semiglobally stabilizable by output feedback is, to the best of our knowledge, not known yet.

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