

HOMework #5

Due Friday, November 6, 2009 (5:00 p.m.)

Reading: Chapter 8 (8.1–8.8)

Problems:

1. Chapter 6: Problem 1
2. Chapter 6: Problem 7
3. Verify equations (42) and (46) in the paper by Horowitz and Senne.
4. Consider the modified LMS algorithm

$$W(n+1) = W(n) + 2\alpha R^{-1}X(n)e(n)$$

where R is the autocorrelation matrix of the input signal $X(n)$.

- (a) Find the range of values for α such that $W(n)$ converges in the mean.
 - (b) Specify the value of α which results in one-step convergence in the mean.
 - (c) Derive an equation for this algorithm similar to (37) in the paper mentioned above, and indicate how the new equation is different from (37).
5. Consider a system-identification application where the desired response $d(n)$ is obtained as follows (using mixed time-domain and z -domain notation):

$$d(n) = G(z)x(n) + v(n)$$

where the input signal $x(n)$ is a zero-mean random sequence with autocorrelation matrix R , and $G(z)$ is the system to be identified. The additive white-noise sequence $v(n)$ has zero mean and variance σ_v^2 , and is uncorrelated with $x(n)$. Assume that the LMS algorithm with M coefficients is used to identify

$$G(z) = 1 - 0.2z^{-1} + 0.4z^{-2}$$

where

$$R = \begin{bmatrix} 1 & 0.2 & 0 \\ 0.2 & 1 & 0.2 \\ 0 & 0.2 & 1 \end{bmatrix},$$

$\sigma_v^2 = 0.01$, and $M = 3$.

- (a) Determine $\xi_{\min}$ for the adaptive filter, and find the Wiener weight vector.
 - (b) For convergence in the mean, find the stable range of α .
 - (c) Repeat part (b) for convergence in the mean square (using the results of Horowitz and Senne).
 - (d) Plot $D(z)$ for real z (from the paper mentioned above) showing all M roots for a value of α that satisfies (b) but not (c).
6. (a) Verify that the following two LMS stability conditions are equivalent:

$$\begin{aligned} 0 < \mu < 1/3\lambda_{\max} & \quad \text{and} \quad \eta(\mu) < 1 \\ 0 < \mu < 1/2\lambda_{\max} & \quad \text{and} \quad \eta(\mu) < 1 \end{aligned}$$

where

$$\eta(\mu) = \sum_{k=0}^{M-1} \frac{\mu\lambda_k}{1 - 2\mu\lambda_k}.$$

- (b) Find examples for $M = 3$ where (i) $\eta(\mu) > 1$, $\mu < 1/2\lambda_{\max}$, and (ii) $\eta(\mu) < 1$, $\mu > 1/2\lambda_{\max}$.