

PROGRAMMING ASSIGNMENT

Due Thursday, December 3, 2009 (5:00 p.m.)

A second-order autoregressive (AR) process $\{u(n)\}$ is described by the following difference equation:

$$u(n) + a_1 u(n-1) + a_2 u(n-2) = v(n)$$

where $v(n)$ is a white-noise process with zero mean and variance σ_v^2 . The parameters of this process are assigned each of the following three sets of values:

- (i) $a_1 = -0.5359$, $a_2 = 0.0718$, $\sigma_v^2 = 0.7461$
- (ii) $a_1 = -1.0390$, $a_2 = 0.2699$, $\sigma_v^2 = 0.3065$
- (iii) $a_1 = -1.6364$, $a_2 = 0.6694$, $\sigma_v^2 = 0.0216$.

The AR process is applied to a two-weight *linear predictor*. Repeat Problems 1 – 3 below for each set of parameters. Note that only one computer program should be written; use a separate subroutine for each algorithm.

1. Steepest-Descent (SD) Algorithm:

- (a) Calculate the eigenvalues of the 2×2 input autocorrelation matrix R . Determine the corresponding stable range for the step-size parameter α .
- (b) Using two elements of the weight-error vector (i.e., $v_1(n)$ and $v_2(n)$) as the variables in the SD algorithm, construct loci for constant values of the mean-square error (MSE). Superimpose the trajectory that describes the change in the coordinates $v_1(n)$ and $v_2(n)$ with time n , assuming that $\alpha = \alpha_{\max}/10$ and the initial values of the predictor weights are zero.
- (c) Repeat the computations in part (b) using the original weights $w_1(n)$ and $w_2(n)$ as the variables in the SD algorithm.
- (d) Plot the learning curves for the following three step-size parameter values: $\alpha = \alpha_{\max}/100$, $\alpha = \alpha_{\max}/50$, and $\alpha = \alpha_{\max}/10$.

2. Least-Mean-Square (LMS) Algorithm:

- (a) What is the stable range of values for the step-size parameter α for convergence in the mean?
- (b) What is the stable range of values for the step-size parameter α for convergence in the mean square?
- (c) Generate a 2000-sample sequence representing the AR process $\{u(n)\}$. By averaging over 100 independent trials, plot the MSE learning curves of the LMS algorithm for the following three step-size parameter values: $\alpha = \alpha_{\max}/1000$, $\alpha = \alpha_{\max}/100$, and $\alpha = \alpha_{\max}/10$.
- (d) Estimate the mean values of $w_1(\infty)$ and $w_2(\infty)$ for the three step-size parameter values in part (c). You may do this by averaging the steady-state values of the weights (obtained from the last iteration) over 100 independent trials of the experiment. Compare your results with the theoretical values.
- (e) Repeat parts (a) – (d) using the sign-data LMS algorithm:

$$W(n+1) = W(n) + \alpha e(n) \text{sgn}[U(n)]$$

where the signum function retains the sign of each component of $U(n)$.

- (f) Repeat parts (c) and (d) using the sign-error LMS algorithm:

$$W(n+1) = W(n) + \alpha \text{sgn}[e(n)]U(n).$$

- (g) Repeat parts (c) and (d) using the normalized LMS algorithm:

$$W(n+1) = W(n) + \alpha \frac{e(n)U(n)}{U^T(n)U(n)}.$$

3. Recursive-Least-Squares (RLS) Algorithm:

- (a) Repeat parts (c) and (d) of Problem 2 using the RLS algorithm.
 - (b) Suppose that the AR parameter a_1 is made positive; otherwise, its magnitude is left as specified for each of the three cases. How would you assess the effect of this change on the performance of the RLS algorithm?
4. **Discussion:** Compare and discuss your results. What can you say about the computational complexity and speed of convergence for each algorithm?