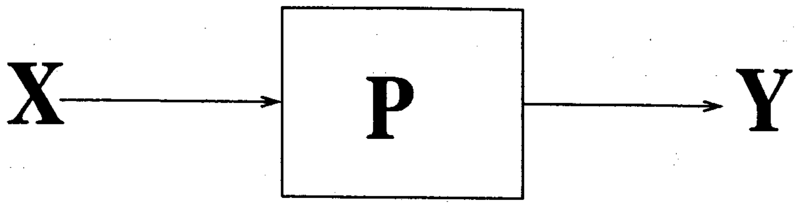
ECE 250 HW #5

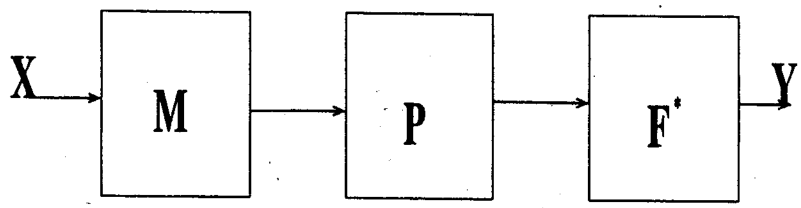
1. *Vector Coding*

This is the first problem on Channel Partitioning and Vector Coding.

(a) (2 pts) Consider the following vector channel :



We would like to add a matrix transmit filter, M, and a matrix receiver filter, F^*, such that $FF^* = MM^* = I$ i.e. the matrix filter doesn't change the energy. These filters are merely rotations in $R^{N+\nu}$ and R^N, respectively.



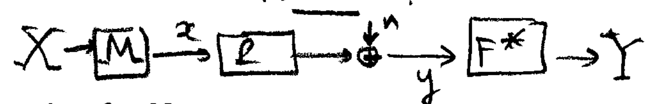
Moreover, we want that $Y_n = \Lambda_n X_n + N_n$

Given that the channel is $1+0.5D$, calculate P, M, F^* and $\Lambda's$. Assume $SNR_{mfb} = 10dB$, $\bar{Ex} = 1$, and $N = 8$.

(b) (3 pts) Using the value of Λ's, calculate the optimal energy and bit distribution using a loading algorithms with $\Gamma = 3dB$.

(c) (1 pt) What is the SNR_{vc} for the vector-coded system? Noise

2. *Theoretical questions on Vector Coding*



(a) (2 pts) Verify (10.149) and give an expression for N

(b) (1 pt) Show that N in (10.149) is indeed white and Gaussian with variance $N_0/2$.

(c) (3 pts) Calculate R_{yy} and show that $det(R_{yy})$ is as assumed in (10.155).

(d) (2 pts) Prove (10.167). Notice that now we have $N+\nu$ instead of N as the nth-root. Under what conditions can we get similar results to the ideal case (10.16) described in section 10.2.2?

(e) (Optional) Prove that $F^* = \begin{bmatrix} 1+\frac{1}{SNR_1} & & 0 & 0 \\ & 1+\frac{1}{SNR_2} & & 0 \\ & & \ddots & \vdots \\ 0 & & & \\ 0 & 0 & \cdots & 1+\frac{1}{SNR_N} \end{bmatrix} \Lambda R_{Xy} R_{yy}^{-1}$

3. *DMT for the* $1 + 0.5D^{-1}$

   As always, $N = 8$, $\bar{E} = 1$, $SNR_{MFB} = 10\text{dB}$, and $\Gamma = 3\text{dB}$

   (a) (2 pts) Solve for the DMT gains, P_n, and the optimal bit distribution. What is the bit rate? Use Water-fill for optimal bit distribution.

   (b) (2 pts) Compare the gains P_n to the gains λ_n of vector coding for the same channel. Which one is better in terms of the ratio of largest to smallest bit assigned? Why is that?

   (c) (3 pts) Plot the Fourier spectra (i.e. the fft with appropriate zero-padding) of the columns of M for VC and for DMT in case of N=8,16,32. What is the change of pattern in spectra as N increases?

4. *Grasping Vector Coding*

   Assume the channel is $1 + aD^{-1}$. For this problem, we will vary 'a' and 'N' and notice their effects on F, Λ and M.

   (a) (1 pt) For N=4 and a=0.9, compute F, Λ and M. Plot the modulus of the spectra of the eigenvectors of M, that is, take the columns of M as time sequences and calculate their abs(fft). Use at least 32 points for the fft's. Turn in this plot. What shape do these filters have?

   (b) (2 pts) Recompute for N=4 and a=0.1 Do you notice any differences in the resulting filter's ffts compared to (a)? What explains this behavior?
   (Hint: Part (c) might be helpful)

   (c) (2 pts) Set a=0.9 and N=8. Calling λ_i the diagonal elements of the Λ matrix, calculate $\lambda_1, (\lambda_2 + \lambda_3)/2$, $(\lambda_4 + \lambda_5)/2$, $(\lambda_6 + \lambda_7)/2$ and compare to the values of H_n in Example 10.1.1. Explain the similarity.

   (d) (Optional) For all the examples tested, matrix F seems to have some special structure. Describe it. How can you use this to reduce the number of operations in the receiver?