LECTURE #10

DIVERSITY – (continued)

– Combining techniques:

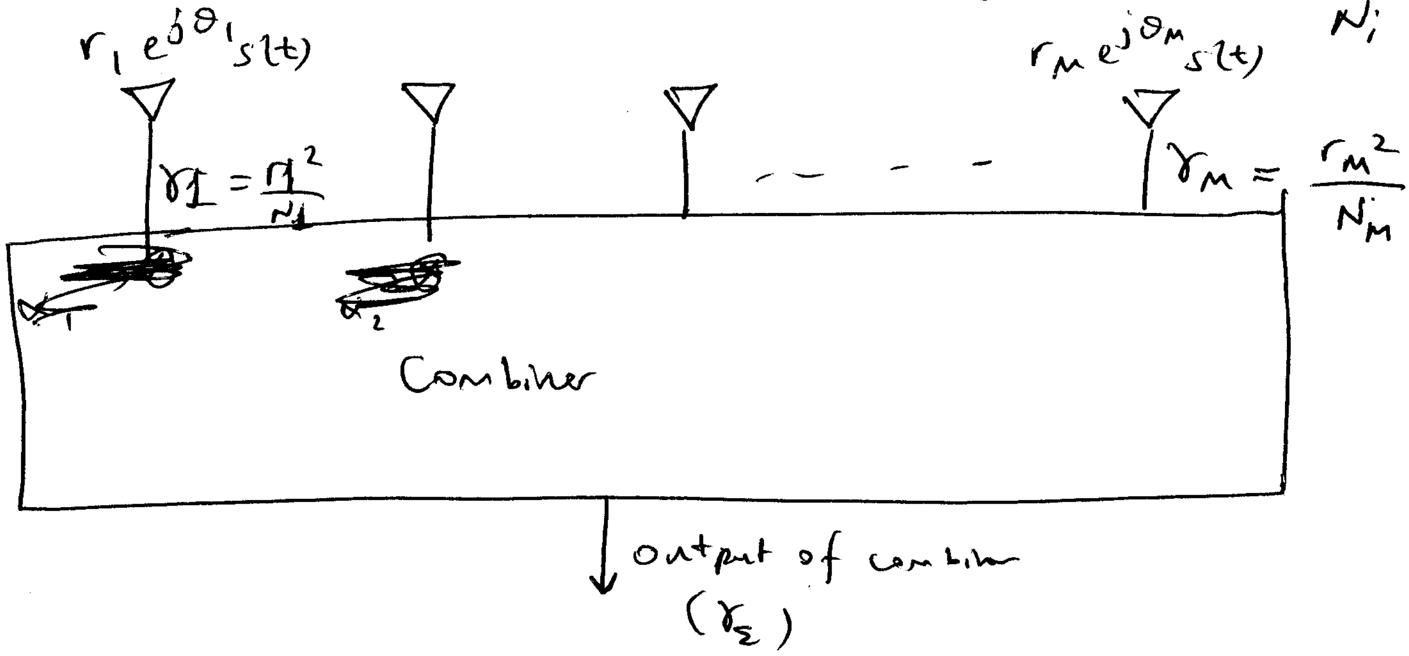
A. selection ~~com~~

B. MRC

C. Equal-gain combining

A. Selection combining.

Idea: choose the branch with the highest $SNR_i = \gamma_i = \frac{r_i^2}{N_i}$



SNR at output of combiner: $\gamma_\Sigma = \max_{1 \le i \le M} (\gamma_i)$

- Assume uncorrelated Rayleigh fading amplitudes r_i.

$\gamma_i = \frac{r_i^2}{N}$. (assume $N_i = N$)

Last time we show that $r_i \sim$ Rayleigh

$\Rightarrow \frac{r_i^2}{N_i} \sim$ exponential. $\quad \gamma_b \sim$ exponential $\left(\frac{1}{\bar{\gamma}_b}\right)$

$$\gamma_b \equiv \frac{r^2}{N} = \frac{r^2}{2\sigma_n^2}$$

(where σ_n = standard deviation of noise in each of the complex components).

$\left(\bar{\gamma}_b = \frac{\sigma^2}{\sigma_n^2}\right.$: σ^2: parameter of Rayleigh var
σ_n^2: variance of each dimension of thermal noise. $\left.\right]$

Then: $p(\gamma_i) = \frac{1}{\bar{\gamma}_i} e^{-\gamma_i/\bar{\gamma}_i}$, $\gamma_i \geq 0$ (pdf)

- Probability of outage :

$$P_{out}(i) = Pr\{\gamma_i < \gamma_0\}$$

$$= \int_0^{\gamma_0} \frac{1}{\bar{\gamma}_i} e^{-\gamma_i/\bar{\gamma}_i} d\gamma_i = 1 - e^{-\gamma_0/\bar{\gamma}_i}.$$

$$P_{out}\text{ (combiner)} = Pr\{\gamma_\Sigma < \gamma_0\}$$

$$= P\{\max_{1 \leq i \leq M}(\gamma_i) \leq \gamma_0\}$$

$$= P\{\gamma_1 \leq \gamma_0, \ldots, \gamma_M \leq \gamma_0\}$$

$$= P\{\gamma_1 \leq \gamma_0\} \cdot \ldots \cdot P\{\gamma_M \leq \gamma_0\}$$ (ind. of branch

$$= \prod_{i=1}^{M} P\{\gamma_i \le \gamma_0\}$$

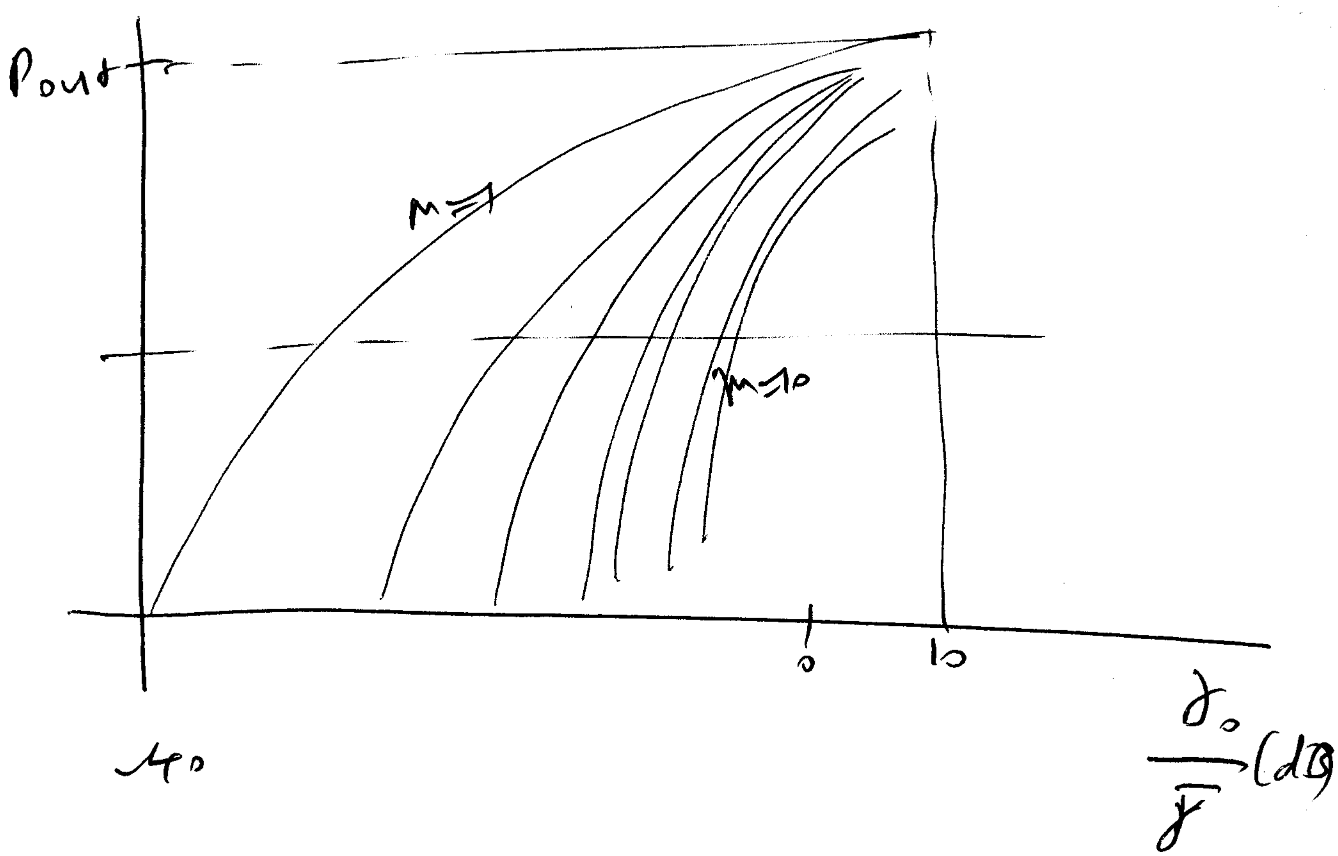
$$= \prod_{i=1}^{M} (1 - e^{-\gamma_0/\bar{\gamma}_i}).$$

$$= (1 - e^{-\gamma_0/\bar{\gamma}})^M.$$

if $\bar{\gamma}_i = \bar{\gamma}$.

$$p_{\gamma_\Sigma}(\gamma) = \frac{d}{d\gamma} P\{\gamma_\Sigma < \gamma\} = \frac{M}{\bar{\gamma}} (1 - e^{-\gamma/\bar{\gamma}})^{M-1} e^{-\gamma/\bar{\gamma}}$$

$$\boxed{\bar{\gamma}_\Sigma = \bar{\gamma} \cdot \sum_{i=1}^{M} \frac{1}{i}}$$



∴ can determine # of branches required to satisfy a fixed P_{out}, at a threshold γ_0 and avg. SNR per branch $\bar{\gamma}$.

- Note that $M \to \infty \Rightarrow \bar{\gamma}_\Sigma \to +\infty$.

  But not physically possible: the antennas in a given area will become correlated.

<u>Calculation of $\bar{P_b}$ under diversity:</u>

$$\bar{P}_b = \int_0^\infty \underbrace{P_b(\gamma)}_{\text{prob of bit error of modulation scheme}} \cdot P_{\gamma_\Sigma}(\gamma)\, d\gamma$$

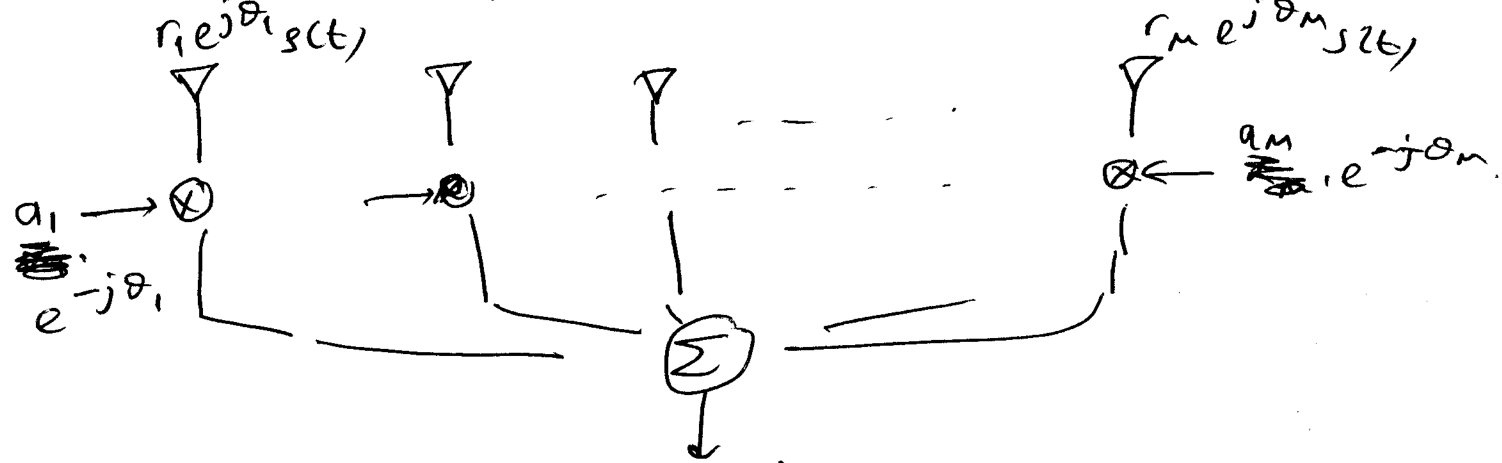
pdf after combining (pdf of SNR at output of combiner)

prob of bit error of modulation scheme.

Ⓑ <u>MRC</u>: Maximum ratio combining.

<u>Idea</u>: weight each branch based on its SNR.

— Remove the phase & then combine.



$$r = \sum_{i=1}^{M} a_i r_i$$

Assume $N_i = N$.

$$N_{total} = \sum_{i=1}^{M} a_i^2 \cdot N.$$

$$\gamma_\Sigma = \frac{r^2}{N_{total}} = \frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{\left(\sum_{i=1}^{M} a_i^2\right) \cdot N}$$

maximize this by adjusting $\vec{a}$.

— using the Schwartz inequality, can show that:

$$a_i^{*2} = \frac{r_i^2}{N}$$ — are the optimal weights

$$\boxed{\gamma_\Sigma^* = \sum_{i=1}^{M} \frac{r_i^2}{N} = \sum_{i=1}^{M} \gamma_i}$$

- SNR increases linearly with the # of diversity branches.

— For analysis, let $\bar{\gamma}_i = \bar{\gamma} \ \forall i$. $(\Rightarrow \gamma_i \sim \exp(1/\bar{\gamma}))$

Then: $\gamma_\Sigma \sim \text{Gamma}\left(\frac{1}{\bar{\gamma}}, M\right)$

$$\boxed{\overline{\gamma_\Sigma} = M\bar{\gamma}}$$

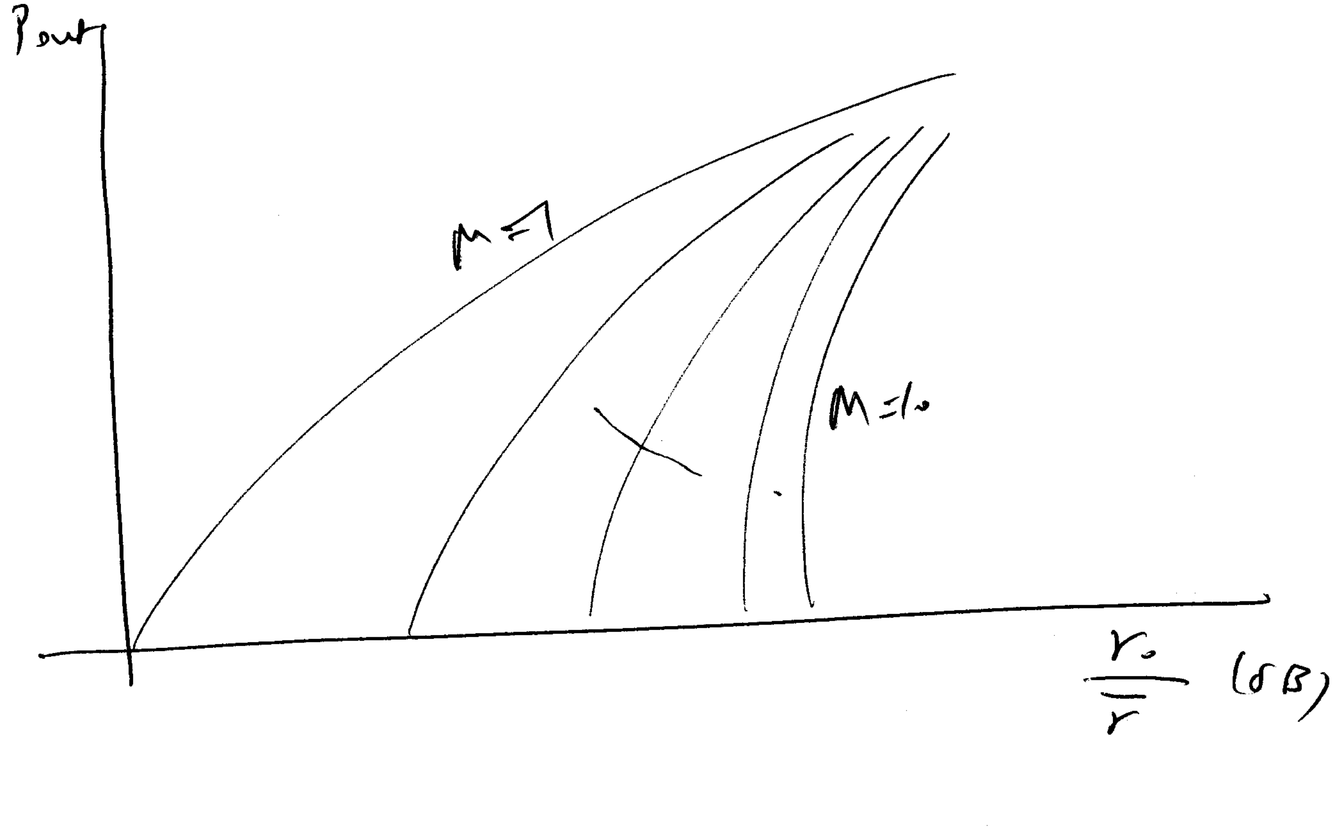
$$\mathrm{Var}(\gamma_\Sigma) = \mathrm{Var}\left(\sum_{i=1}^{M} \gamma_i\right)$$

$$= \sum_{i=1}^{M} \mathrm{Var}(\gamma_i) \quad \text{(independence)}$$

$$\underset{\bar{\gamma}_i = \bar{\gamma}}{=} \sum_{i=1}^{M} \bar{\gamma}^2 = M\bar{\gamma}^2.$$

$$P_{out} = \Pr(\gamma_\Sigma < \gamma_0) = \int_0^{\gamma_0} p_{\gamma_\Sigma}(\gamma)\, d\gamma$$

$$= 1 - e^{-\gamma/\bar{\gamma}} \sum_{k=1}^{M} \frac{(\gamma/\bar{\gamma})^{k-1}}{(k-1)!}$$



(c) <u>Equal-gain combining</u>:

<u>Idea</u>: MRC requires that $\frac{r_i^2}{N}$ be known to adjust the weights (to optimal settings.)

EGC: sets all $a_i = 1$, $\forall i \in \{1, .., M\}$.

$$\gamma_\Sigma = \frac{(\sum r_i)^2}{\underbrace{N_{total}}_{N \cdot M}} = \frac{1}{NM} \left(\sum_{i=1}^{M} r_i \right)^2$$

$$\bar{\gamma}_\Sigma = E\left[\frac{1}{NM} \left(\sum_{i=1}^{M} r_i \right)^2 \right]$$

$$= \frac{1}{NM} \left[\sum_{i=1}^{M} \sum_{j=1}^{M} \underbrace{E[r_i r_j]}_{\substack{r_i \perp\!\!\!\perp r_j \;\; i \neq j \\ \downarrow \\ E[r_i]E[r_j]}} \right]$$

$$= \frac{1}{NM} \left[(E[r_1])^2 + (E[r_2])^2 + \dots + (E[r_M])^2 + \sum_{i=1}^{M} \sum_{\substack{j=1 \\ i \neq j}}^{M} E[r_i]\, E[r_j] \right] \quad \text{and } \bar{r_i} = \pi$$

assume $\bar{r_i} = \bar{r}$ $\forall i$

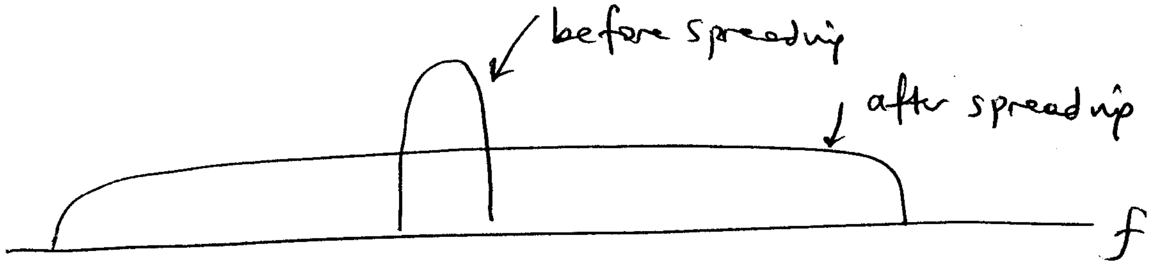
$$\frac{r_i^2}{N} = \gamma_i$$

$$\stackrel{\downarrow}{=} \frac{1}{M} \left[M \cdot \underbrace{\sum_{i=1}^{M} \frac{\overline{r_i^2}}{N}}_{\bar{\gamma}} + M(M-1) \underbrace{\frac{\sqrt{\pi}}{2}}_{\substack{\uparrow \\ \text{(avg. of a Rayleigh r.v.)}}} \cdot \frac{\sqrt{\pi}}{2} \cdot \bar{\gamma} \right]$$

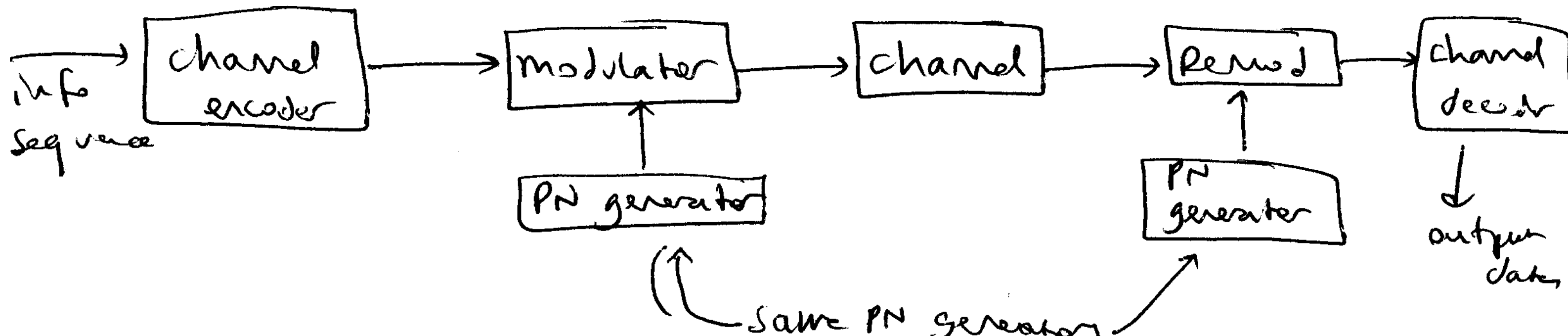
$$\boxed{\bar{\gamma}_\Sigma = \bar{\gamma} \left[1 + (M-1) \cdot \frac{\pi}{4} \right]}$$

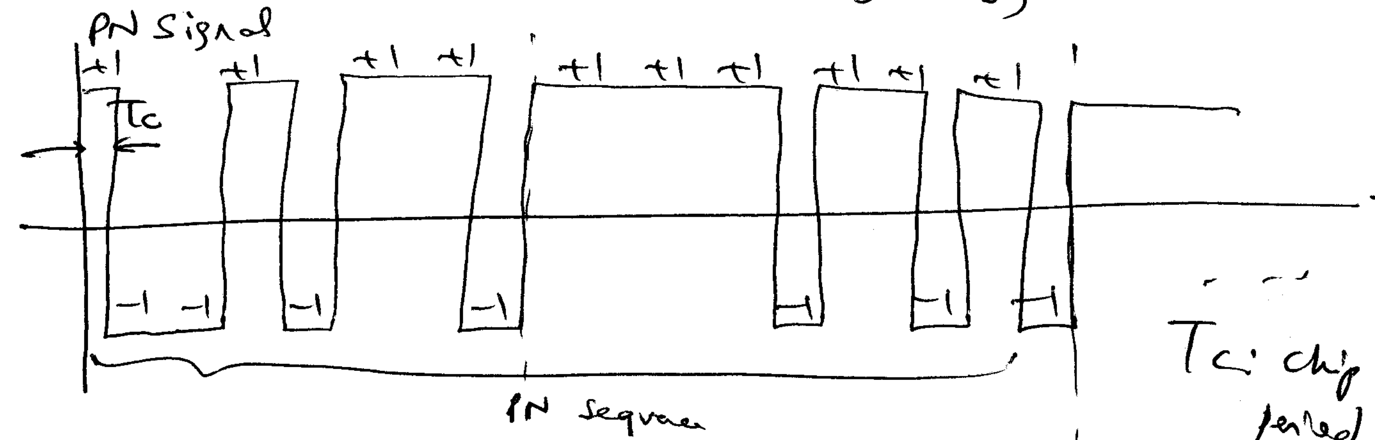
As $M \to \infty$, the "gap" between EGC and MRC is <u>1.05 dB</u> (small).

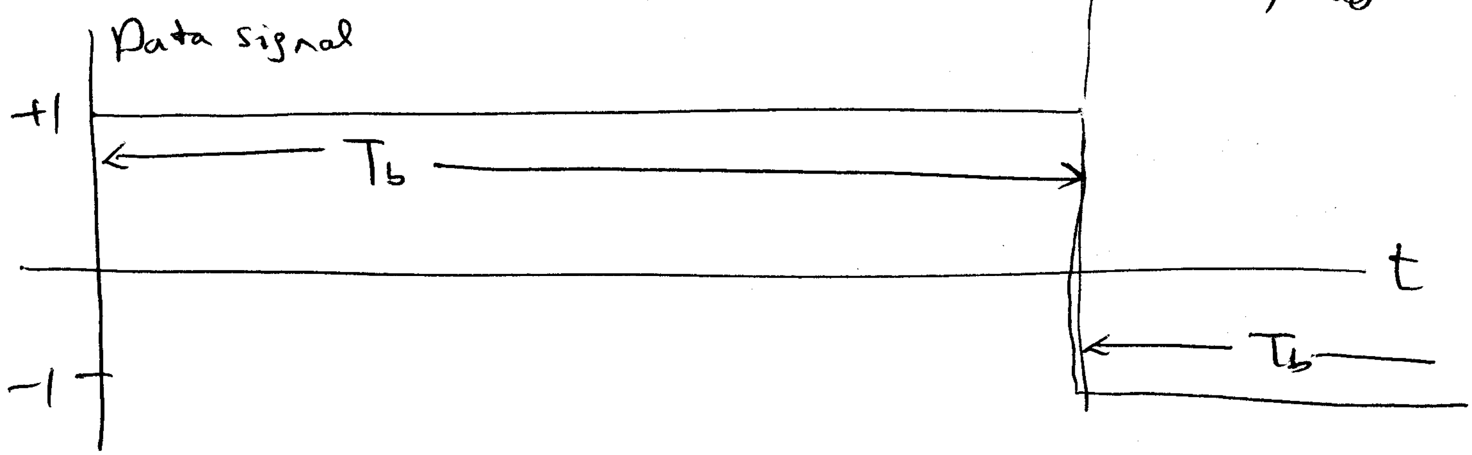
At $M = 3$ antennas, in 99.9% of <u>no</u> outage, "gap" $\simeq$ <u>1–2 dB</u>.



Model of DS-SS system:

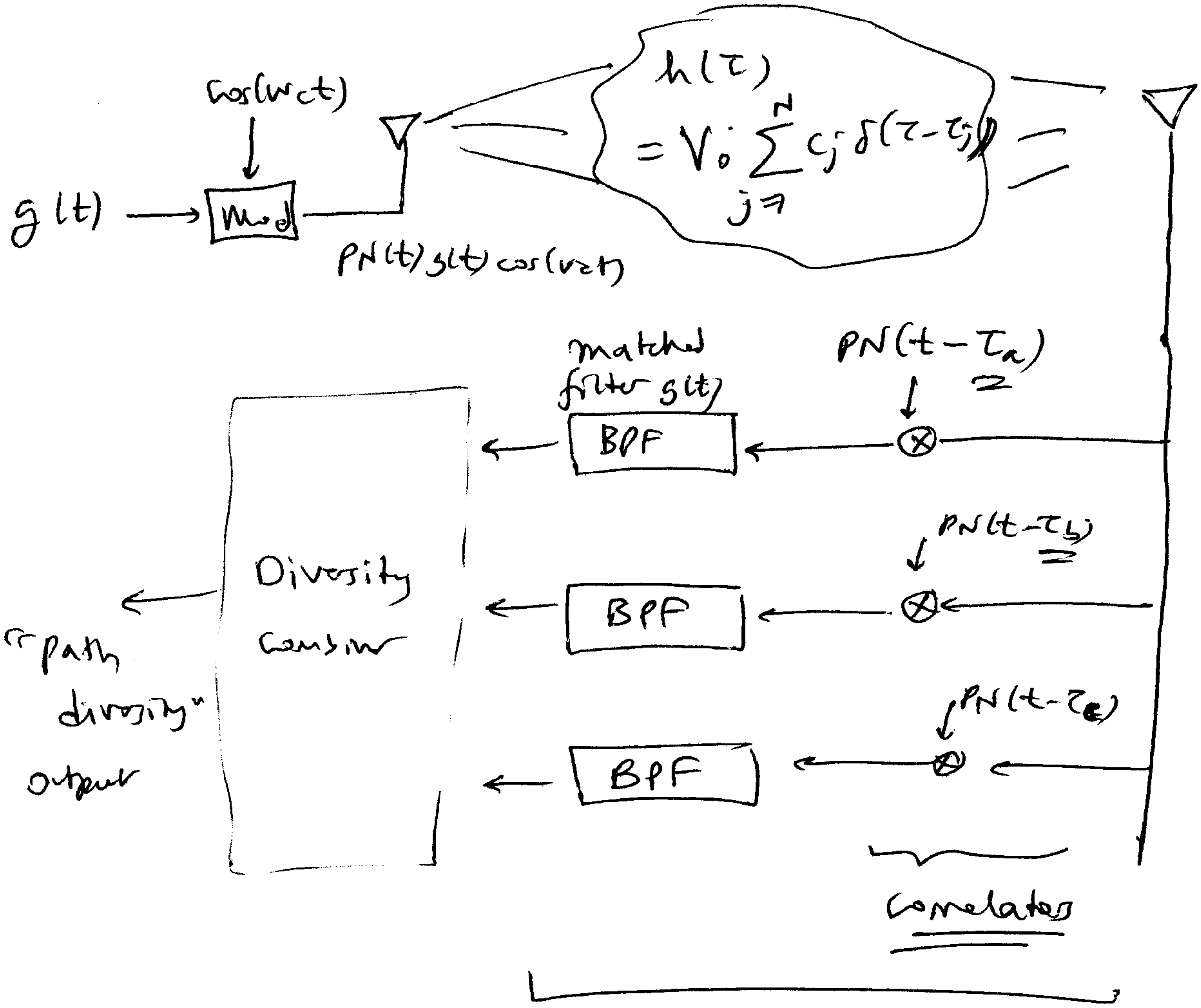






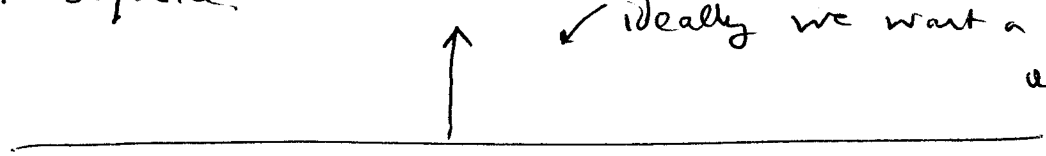
- It is clear that if you multiply the data signal with PN signal twice, you will get the data signal back.

RAKE receiver + diversity combining.

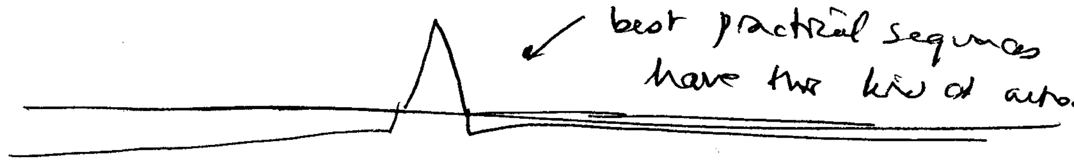


RAKE receiver.

- We see that the quality / error rate will depend directly on the autocorrelation properties of the PN sequence:



✓ ideally we want a δ fn as autocorrelation

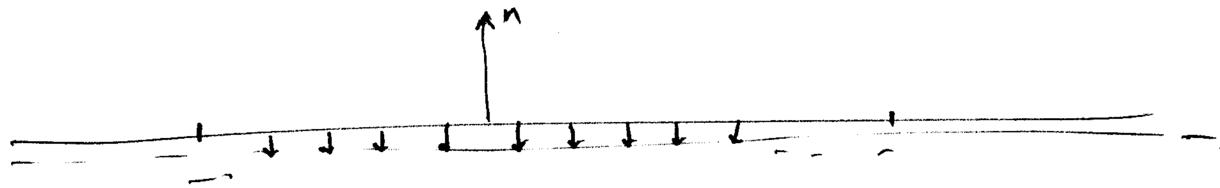


✓ best practical sequences (~~[illegible]~~ have this kind of autocorrelation

- Hence we should clearly focus on the design of PN sequences.

- Further, their periodic autocorrelation fn

$$\phi(j) = \begin{cases} n & j=0 \\ -1 & 1 \le j \le n-1 \end{cases}.$$



For large n ; $\dfrac{\phi(j)\ (j\neq 0)}{\phi(0)} = -\dfrac{1}{n}$ is small.

→ This is definitely good for a multipath channel single-user + RAKE receivers

~~But, in CDMA ; multiple users,~~

~~∃ multiple~~

<u>generation of m-sequences</u>

Ex: $m=3$: