LECTURE #7

Topics: A Review of AWGN channel capacity; definitions)

B Review of modulation

C Capacity of wireless fading & ISI channels

D Performance of digital modulation over wireless fading & ISI channels

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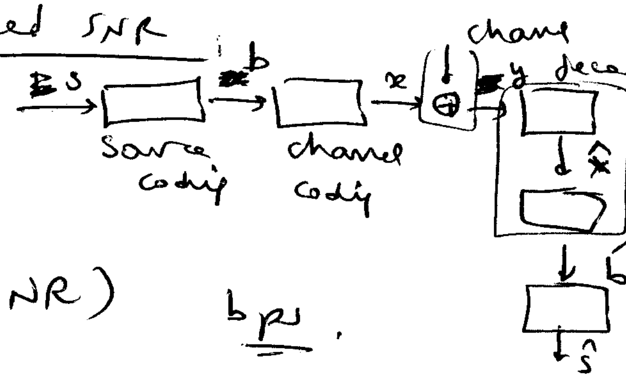
(A): Channel capacity and normalized SNR



Bandlimited AWGN channel:

$$C_{[b/s]} = W \cdot \log_2 (1 + SNR) \quad \text{bps}.$$

- If $R < C_{[b/s]}$, then $\exists$ a channel coding scheme s.t. a data rate of R can be transmitted with arbitrary low P_e.
- If $R > C_{[b/s]}$, no coding scheme exists that can achieve arbitrary low P_e's.

Note that: 1) C depends only on W and SNR.

2) does not matter whether the channel is baseband or passband.

Spectral efficiency of a coding scheme

$$\rho = \frac{R}{W} \quad \text{bps/Hz} \qquad \left[\frac{\text{bits}}{\cancel{\text{second}} \cdot \frac{1}{\cancel{\text{second}}}} = \underline{\underline{\text{bits}}} \right]$$

Then Shannon formula

$$\Leftrightarrow \quad \rho < \log_2(1+SNR) \quad \text{b/s/Hz}.$$

$$\Leftrightarrow \quad SNR > 2^{\rho} - 1.$$

(a) Normalized SNR: SNR_{norm}:

$$\boxed{SNR_{norm} \equiv \frac{SNR}{2^{\rho}-1}}$$

$$\Leftrightarrow \quad SNR_{norm} > 1 \quad (0\ \text{dB})$$

↓
measures how far a given coding scheme operates ~~away~~ from the Shannon limit
"gap to capacity" (dB)

(b) $$\boxed{\frac{E_b}{N_0}} \equiv \frac{\text{average information energy per bit}}{\text{noise spectral density}}$$

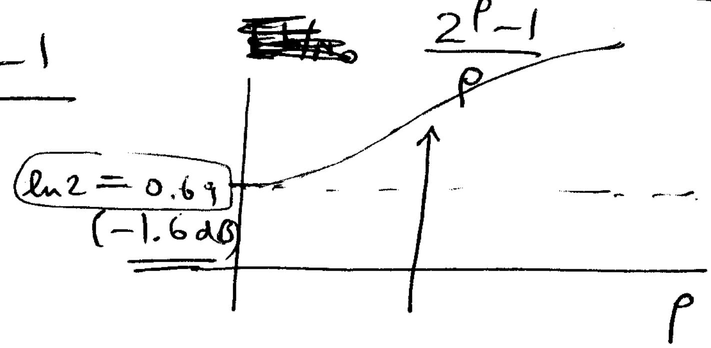
$$= \frac{P}{RN_0} = \boxed{\frac{SNR}{\rho}}$$

$$\boxed{\frac{E_b}{N_0} = \frac{SNR}{\rho}}$$

SNR: dimensionless

$$\frac{E_b}{N_0} := \frac{1}{\text{bits}}.$$

Shannon formula: $\dfrac{E_b}{N_0} > \dfrac{2^{\rho}-1}{\rho}$



Monotonically increasing function of ρ.

$\rho = 2$: $\dfrac{2^{\rho}-1}{\rho} = 1.76\,\text{dB}$

$$\lim_{\rho\to 0} \frac{2^{\rho}-1}{\rho} = 0.69 (= \ln 2) = \underline{-1.6\,\text{dB}}$$ "Shannon limit".

- Power-limited vs. Bandwidth limited channels;

no sharp dividing line, but

$\boxed{\rho = 2 \ \ b/s/Hz}$ usually taken as boundary

(highest spectral efficiency by binary transmission

Power-limited regime: $\equiv$ modulation scheme has $\boxed{\rho < 2}$

Behavior:

$$C_{[b/s]} \triangleq W \log_2 (1+\underset{\text{SNR small}}{SNR})$$

$$\simeq W \underbrace{(SNR \cdot \log_2 e)}_{\text{(linear in SNR)}} \quad \text{(by Taylor expansion)}$$

$$\Leftrightarrow \quad \rho < SNR \log_2 e$$

$$SNR_{norm} \triangleq \frac{SNR}{\rho \ln 2} = \left[\frac{E_b}{N_0}\right] \log_2 e.$$

* In power-limited regime $C_{[b/s]}$ and ρ (achievables) grow linearly with SNR

Bandwidth-limited regime:

$$C_{[b/s]} \triangleq W \log_2 (1+\underset{\text{SNR large}}{SNR})$$

$$\simeq W \log_2(SNR)$$

$$\rho < \log_2 (SNR)$$

$$SNR_{norm} = \frac{SNR}{2^\rho}$$

* $C_{[b/s]}$ and ρ (achievables) go logarithmically in SNR. [need 3 dB increase in SNR to get additional 1 b/s/Hz.]

Examples:

1. wired voice channel:

$$W = 3.5 \text{ kHz}$$

$$SNR = 37 \text{ dB}$$

Shannon formula: $\rho < \frac{37}{3} = 12.3$ b/s/Hz

$$R < 43 \text{ kbps}$$

Increase SNR by 3 dB $\rightarrow$ ρ goes up to $\boxed{13.3}$

, R $\uparrow$ 43 + 3.5 kbps

= 46.5 kbps

(small increase)

(use multilevel modulation)

(E_b/N_0 used).

2. Deep-space comm. link (no bandwidth restriction)

+ (power limitations).

$$R < \frac{P}{N_0} \log_2 e.$$

Increase SNR by 3 dB doubles the data rate.

[use binary coding & modulation]

(SNR_{norm} used)

Ch#1 Signals & Detection: Discrete Data Transmission:

$$\boxed{\text{message Source}} \xrightarrow{\{m_i\}} \boxed{\text{encoder}} \xrightarrow[\text{Symbol}]{\{x_i\}} \boxed{\text{mod}} \xrightarrow{\{x_i(t)\}} \boxed{\text{Channel}} \longrightarrow$$

$$\longrightarrow \boxed{\text{Demod}} \xrightarrow{\{y\}} \boxed{\text{Detector (Decoder)}} \xrightarrow{\{\hat{m}\}} \boxed{\text{message sink}}$$

$$\boxed{x(t) \triangleq \sum_{n=1}^{N} x_n \varphi_n(t)}$$

modulation by orthonormal basis fns.

$\boxed{\text{modulated waveform}}$

$$\langle \varphi_m, \varphi_n^* \rangle = \int_{-\infty}^{\infty} \varphi_m(t)\, \varphi_n^*(t)\, dt = \delta_{mn}$$

<u>Signal constellation</u>: set of M vectors $\{\underline{x}_i\}$

$\{x_i(t)\}$ is called the <u>signal set</u>

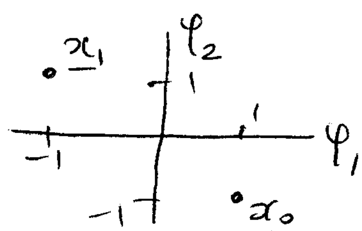
<u>Average energy</u>:

$$\boxed{\mathcal{E}_x \triangleq E[\|x\|^2] = \sum_{i=0}^{M-1} \|x_i\|^2 p_x(i)}$$

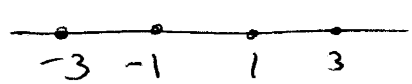
BPSK basis fns:

$$\varphi_1(t) = \sqrt{\frac{2}{T}} \cos\left(\frac{2\pi t}{T} + \frac{\pi}{4}\right)$$

$$\varphi_2(t) = \sqrt{\frac{2}{T}} \cos\left(\frac{2\pi t}{T} - \frac{\pi}{4}\right)$$



<u>2B1Q</u>:



$(d = 2)$

<u>Invariance of inner product</u>: $\langle \underline{v}, \underline{v} \rangle = \langle v(t), v(t) \rangle$.

<u>DEMODULATION</u>: Recovering vectors from modulated waveforms

$$x(t) \longrightarrow \underset{\uparrow \varphi_1(t)}{\otimes} \longrightarrow \boxed{\int_{-\infty}^{\infty} (\cdot)\, dt} \longrightarrow \underline{x}_1$$

Recall $x_i(t) = \sum_{n=1}^{N} x_{in} \varphi_n(t)$

$$\boxed{x_{in} = \int_{-\infty}^{\infty} x_i(t)\, \varphi_n^*(t)\, dt}$$ correlative demodulator.

OR

$$\boxed{x(t) * \varphi_n(T-t) \Big|_{t=T} = x_{in}}$$ matched filter demodulator.

MAP Detector: (p.15)

min $P_e \Longleftrightarrow$ max $P_{X|Y}(i|v)$ given a received vector $\underline{y} = \underline{v}$.

MAP Detection Rule:

$\hat{m} \Rightarrow m_i$ if $P_{y|x}(\underline{v}|i)\, p_x(i) \geq P_{y|x}(\underline{v}|j)\, p_x(j)$

ML detection rule:

$\hat{m} \Rightarrow m_i$ if $P_{\underline{y}|\underline{x}}(\underline{v}|i) \geq P_{\underline{y}|\underline{x}}(\underline{v}|j)$

p.22 AWGN Channel (Detection Rules)

MAP: $\|\underline{v} - x_i\|^2 - \cdots\cdots -$

ML: $\|\underline{v} - \underline{x}_i\|^2 \leq \|\underline{v} - \underline{x}_j\|^2$

<u>Bits per Dimension</u>: # of bits per dimension in a constellation

$$\boxed{\bar{b} \triangleq \frac{b}{N} = \frac{\log_2(M)}{N}}$$

$$\boxed{\text{\# of bits per Hz} = 2\bar{b}}$$

<u>CFM</u>:

$$\boxed{\zeta_{\underline{x}} = \frac{\left(\frac{d_{min}}{2}\right)^2}{\bar{\mathcal{E}}_x}}$$

(unitless)

- must use for systems w/ same $\bar{b}$ (N may be different)

<u>Energy per bit</u>: (in a constellation $\{\underline{x}\}$):

$$\boxed{\mathcal{E}_b = \frac{\mathcal{E}_x}{b} = \frac{\bar{\mathcal{E}}_x}{\bar{b}}}$$

<u>CONSTELLATIONS</u>: p. 41–62

<u>ADDITIVE SELF-CORRELATED NOISE</u>:

I. <u>FILTERED ONE-SHOT AWGN CHANNEL</u>:

$$x(t) \rightarrow \boxed{h(t)} \xrightarrow{\tilde{x}(t)} \oplus \longrightarrow \tilde{y}(t)$$

$\downarrow n(t) \longrightarrow$ <u>white</u> Gsn noise

$$\tilde{x}_i(t) = h(t) * x(t)$$

$$= h(t) * \sum_{n=1}^{N} x_n \varphi_n(t)$$

$$= \sum_{n=1}^{N} x_n \underbrace{[h(t) * \varphi_n(t)]}_{\triangleq \phi_n(t)}$$

$\{\phi_n(t)\}$ are <u>not</u> necessarily orthogonal. For no loss of information (i.e. for channel to convey all M messages) $\{\phi_n(t)\}$ have to be linearly independent.

Using Gram-Schmidt, generate $\{\Psi_n(t)\}$ ~~=~~, new orthonormal basis fns from $\{\Phi_n(t)\}$.

filtered message signals

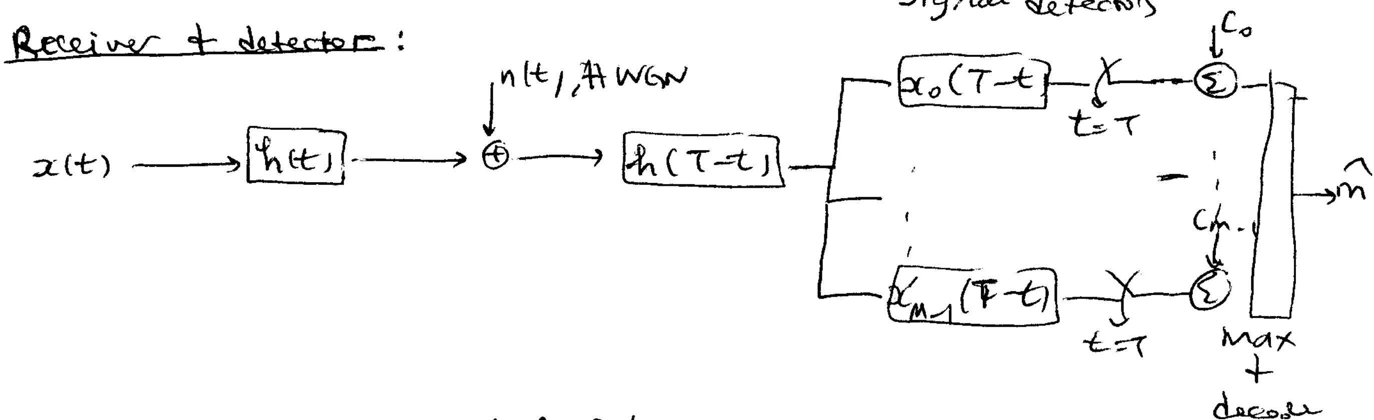
so: $\tilde{x}_{in} = \langle \tilde{x}_i(t), \Psi_n(t) \rangle$.

new signal constellation

in new signal constellation

Then: $P_e \leq N_e Q\left(\dfrac{d_{min}}{2\sigma}\right)$

<u>Receiver + detector</u>:



II. OPTIMUM DETECTION IN AWGN

BOUNDS (on AWGN channels..)

① **UNION BOUND**:

$$\boxed{P_e \le (M-1)\cdot Q\left(\frac{d_{min}}{2\sigma}\right)}$$

② **NNUB**:

$$N_e \triangleq \sum_{i=0}^{M-1} N_i \, p_{\underline{x}}(i)$$

Average # of nearest neighbors.

N_i = # of constellation points with which i shares a decision boundary.

$$\boxed{P_e \le N_e \, Q\left(\frac{d_{min}}{2\sigma}\right)}$$

ALTERNATIVE PERFORMANCE MEASURES

① **BER**:

$$P_b \triangleq \sum_{i=0}^{M-1} \sum_{\substack{j \\ j\ne i}} p_{\underline{x}}(i)\, p\{\epsilon_{ij}\}\, n_b(i,j).$$

For AWGN channel:

use this.

$$P_b \lesssim N_b \cdot Q\left[\frac{d_{min}}{2\sigma}\right]$$

where $n_b(i) \triangleq \sum_{j=1}^{N_i} n_b(i,j)$

$$N_b \triangleq \sum_{i=0}^{M-1} p_{\underline{x}}(i)\, n_b(i)$$

$n_b(i,j)$ = # of bit errors for the particular choice of encoder when i is detected as j.

② PROB. OF BIT ERROR :

$$\boxed{\bar{P}_b \triangleq \frac{P_b}{b}} \qquad 0 \le \bar{P}_b \le 1 \quad \text{always}$$

③ Normalized Prob. of ERROR

$$\boxed{\bar{P}_e \triangleq \frac{P_e}{N}}$$

④ Normalized N_e : $\quad \bar{N}_e = \frac{N_e}{N}$.

$$\therefore \boxed{\bar{P}_e \le \bar{N}_e \, Q\left(\frac{d_{min}}{2\sigma}\right)}$$

CONSTELLATIONS AND MODULATION

$$\boxed{\bar{\mathcal{E}}_x = \frac{\mathcal{E}_x}{N}}$$

- Average power : $\boxed{P_x = \frac{\mathcal{E}_x}{T}}$

- Noise energy per dimension:

$$\boxed{\bar{\sigma}^2 = \frac{\sum_{l=1}^{N} \sigma^2}{N} = \sigma^2 = \frac{N_0}{2}}$$

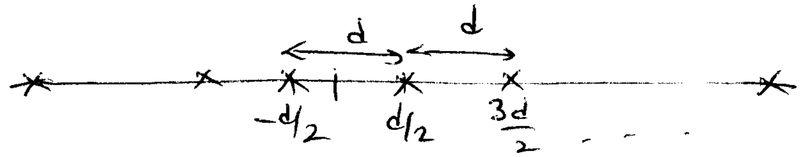
- SNR : $\boxed{SNR = \frac{\bar{\mathcal{E}}_x}{\sigma^2}}$ $\leftarrow$ per dimension

$M = \#$ of data symbols
$b = \log_2(M)$
$R = \frac{b}{T}$

RECTANGULAR CONSTELLATIONS :

I. PAM :

$d_{min} = d$



$x = f(d,M)$ $\quad \mathcal{E}_x = \bar{\mathcal{E}}_x = \frac{d^2}{2M}\left[\frac{M^3}{6} - \frac{M}{6}\right].$ $\quad$ (as a fn of (d, M))

$$\boxed{\mathcal{E}_x = \bar{\mathcal{E}}_x = \frac{d^2}{12}\left[M^2 - 1\right].}$$

$l = f(\mathcal{E}_x, M)$ $\quad \boxed{d = \sqrt{\frac{12\,\mathcal{E}_x}{M^2-1}}}$

$, = f(\mathcal{E}_x, d)$ $\quad \boxed{\bar{b} = \log_2(M) = \frac{1}{2}\log\left(12\,\frac{\bar{\mathcal{E}}_x}{d^2} + 1\right)}$

$l = f(b, \mathcal{E}_x)$ $\quad \boxed{d = \sqrt{\frac{12\,\mathcal{E}_x}{4^b - 1}}}$

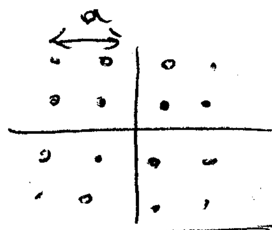
- Increasing b while keeping d_{min} constant:

$$\boxed{\bar{\mathcal{E}}_x(b+1) = 4\,\bar{\mathcal{E}}_x(b) + \frac{d^2}{4}}$$

$P_e = f(d, \bar{b})$ $\quad \boxed{P_e = \bar{P}_e = 2\left(1 - \frac{1}{2^{\bar{b}}}\right)\cdot Q\left(\frac{d_{min}}{2\sigma}\right)}$ $\quad$ Exact. (also NNUB)

$P_e = f(SNR, M)$ $\quad \boxed{P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{3\,SNR}{M^2-1}}\right)} = 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{3\,SNR}{2^{2\bar{b}} - 1}}\right)$

II. QAM



(a) QAM-SQ : $\boxed{b \text{ even}}$

$$\boxed{\mathcal{E}_x = d^2\left(\frac{M-1}{6}\right)}$$

$$\boxed{\bar{\mathcal{E}}_x = \frac{\mathcal{E}_x}{2} = \frac{d^2}{12}(M-1)}$$

$\boxed{d_{min} = d}$

$d_{min} = f(\mathcal{E}_x, M)$:

$$\boxed{d = \sqrt{\frac{6\,\mathcal{E}_x}{M-1}} = \sqrt{\frac{12\,\bar{\mathcal{E}}_x}{M-1}}}$$

$d = f(\bar{b}, \bar{\mathcal{E}}_x)$

$$\boxed{d = \sqrt{\frac{12\,\bar{\mathcal{E}}_x}{4^{\bar{b}}-1}}}$$

- Increasing b while maintaining d :

$$\boxed{\mathcal{E}_x(b+1) = 2\,\mathcal{E}_x(b) + \frac{d^2}{6}}$$

$$\boxed{P_e = 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\frac{d}{2\sigma}\right) - 4\left(1-\frac{1}{\sqrt{M}}\right)^2 \left[Q\left(\frac{d}{2\sigma}\right)\right]^2 \quad \underline{\text{exact}}}$$

NNUB :

$$\boxed{P_e < 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\frac{d}{2\sigma}\right) \qquad \text{NNUB}}$$

$$\boxed{\bar{P}_e \le 2\left(1-\frac{1}{2^{\bar{b}}}\right) Q\left(\frac{d}{2\sigma}\right)} = 2\left(1-\frac{1}{2^{\bar{b}}}\right) Q\left(\sqrt{\frac{3\,SNR}{M-1}}\right)$$

$$= 2\left(1-1/2^{\bar{b}}\right) Q\left(\sqrt{\frac{3\,SNR}{2^{2\bar{b}}-1}}\right)$$

CFM for QAM-SQ:

$$\boxed{\mathcal{Z}_x = \frac{3}{M-1} = \frac{3}{4^{\bar{b}}-1} = \frac{3}{2^{b}-1}}$$

II. QAM (continued):

(b) QAM−CR : $\boxed{b \text{ odd}}$

$\mathcal{E}x = f(M,d)$ $$\boxed{\mathcal{E}_x = \frac{d^2}{6}\left[\frac{31}{32}M - 1\right]}$$ $d_{min} = d$.

$$\boxed{d = \sqrt{\frac{6\mathcal{E}_x}{\frac{31}{32}M - 1}} = \sqrt{\frac{12\bar{\mathcal{E}}_x}{\frac{31}{32}M - 1}}}$$

- Inc b while keep d same:

$$\boxed{\mathcal{E}_x(b+1) = 2\mathcal{E}_x(b) + \frac{d^2}{6}}$$

$P_e = f(d,M)$ $$\boxed{P_e < 4\left(1 - \frac{1}{\sqrt{2M}}\right) Q\left(\frac{d_{min}}{2\sigma}\right) \quad \text{NNUB}}$$

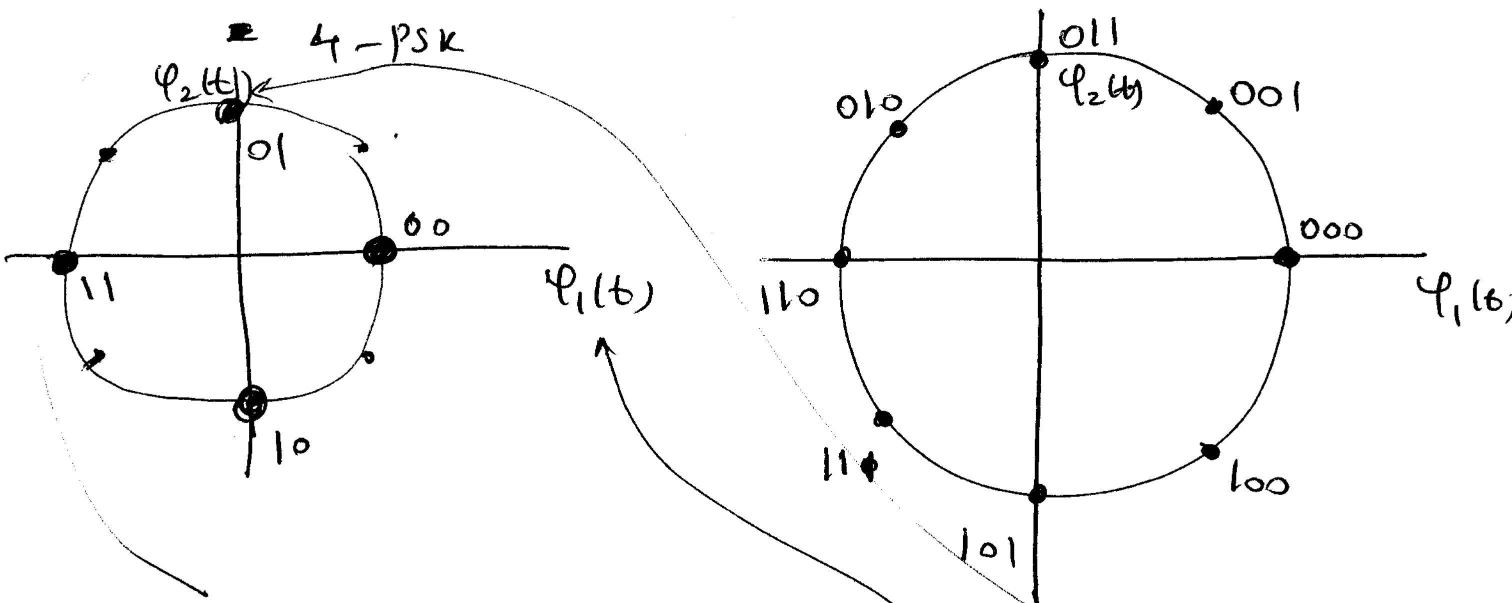
$P_e = f(SNR, M)$ $$\frac{d_{min}}{2\sigma} = \sqrt{\frac{3\,SNR}{\frac{31}{32}M - 1}}$$

(B1) PHASE SHIFT KEYING.

$$S_m(t) = Re\{g(t)\, e^{j2\pi(m-1)/M}\, e^{j2\pi f_c t}\} \quad 0 \le t \le T_s$$

$$= \boxed{g(t)\cos\left(2\pi f_c t + \frac{2\pi}{M}(m-1)\right)}$$

4-PSK



$$= g(t)\cos\left[\frac{2\pi}{M}(m-1)\right]\cos(2\pi f_c t)$$

$$-g(t)\sin\left[\frac{2\pi}{M}(m-1)\right]\sin(2\pi f_c t).$$

$$\boxed{E_{S_m} = \frac{1}{2}E_g.}$$

$$\boxed{d_{min} = \sqrt{E_g \cdot \left(1-\cos\left(\frac{2\pi}{M}\right)\right)}.}$$

4/21/99
Thursday

Announcements

HW2 due today

Last time:

— Capacity of flat-fading channels.
— " " frequency-fading ".

Today:

— Digital modulation
— Performance in AWGN
— " " fading.

Linear modulation:

PAM
PSK
QAM

for QPSK
$-T_s/4$
or
$-T_s/2$

$$s(t) = Re\left\{ u(t)\, e^{j2\pi f_c t} \right\}$$

$$= \left[\sum_n a_n g(t-T_s)\right]\cos(2\pi f_c t) - \left[\sum_n b_n g(t-T_s)\right] \sin(2\pi f_c t)$$

T_s = symbol time, M bits/symbol

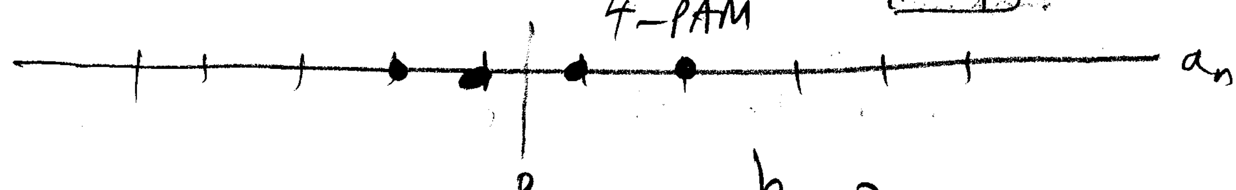
$T_s = M T_b$.

a_n, b_n – info bits.

$g(t)$ – pulse shape.

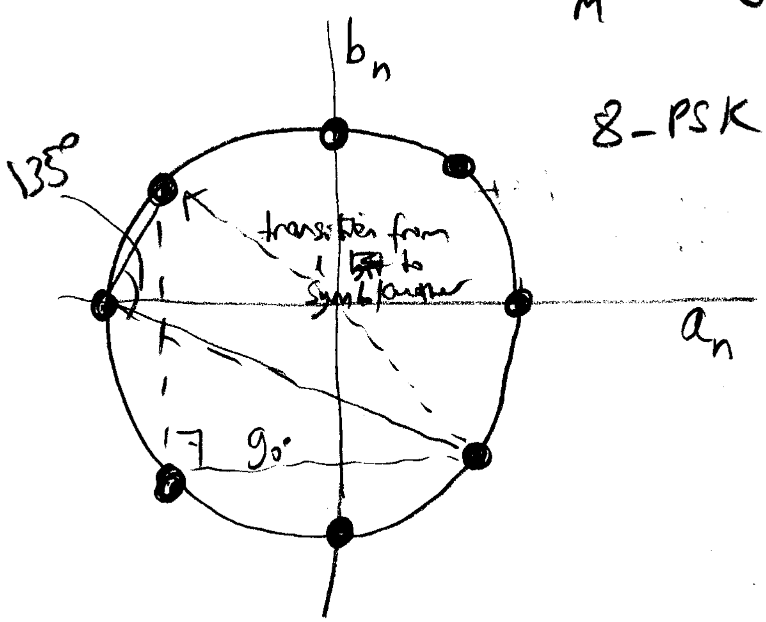
- For each symbol time T_s, map M bits into (a_n, b_n).

<u>MPAM</u> – $b_n = 0$.



<u>M-PSK</u>: Each symbol selected from points equally spaced in angle.

M bits/symbol $\Rightarrow \frac{2\pi}{M}$ = angular spacing



8-PSK. $(M=8)$

SK ?

$\frac{T_s}{2}$ $90°$ phase transition

$\frac{T_s}{4}$ $135°$ " "

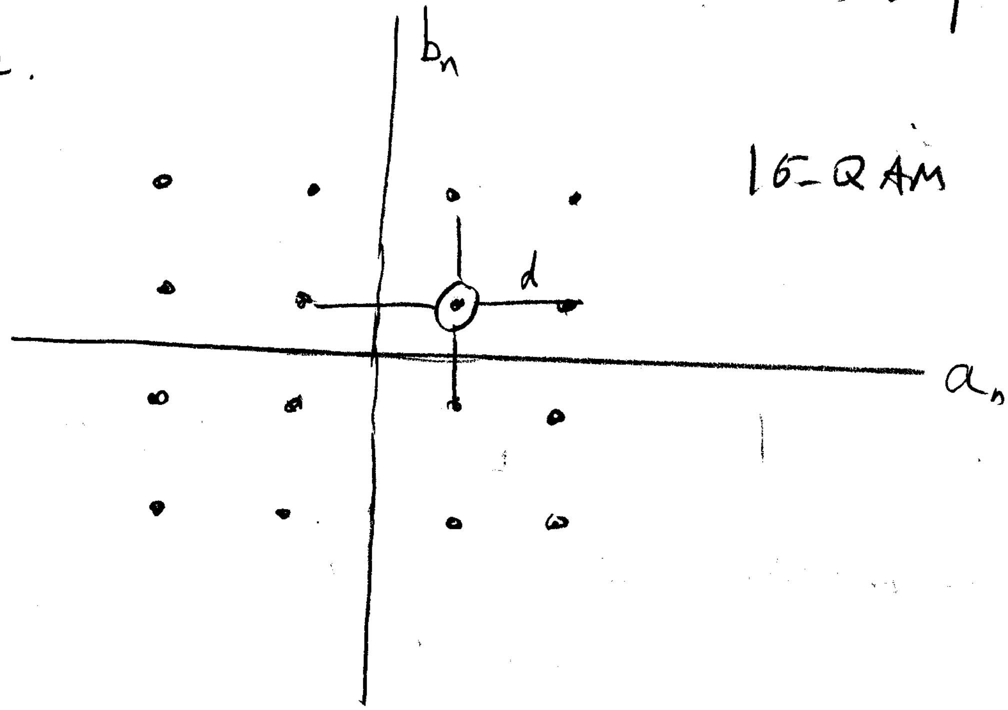
<u>Quadrature offset</u>:

- If the offset is $\frac{T_s}{2}$, then $90°$ phase transitions; called OMPSK <u>or</u> OPSK
- Offset $\frac{T_s}{4} \longrightarrow$ get $135°$ phase transition called $\frac{\pi}{4}$-MPSK.

better than $\frac{T_s}{4}$.

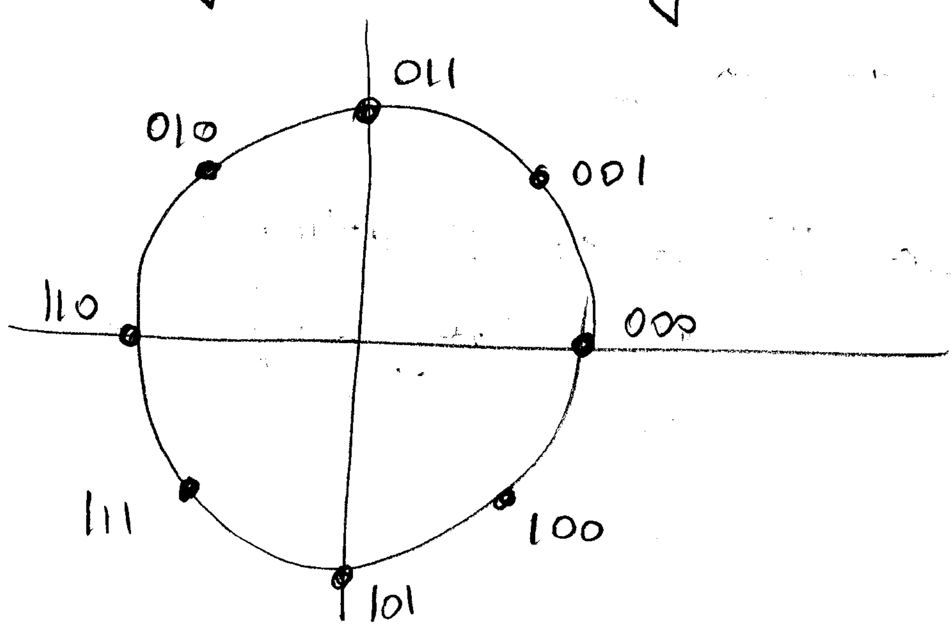
But $\frac{T_s}{4} \rightarrow$ can do <u>differential</u> phase encoding. ($\frac{\pi}{4}$-DPSK). used in IS-54

MQAM: Encode information in both amplitude and phase.



Bit Mapping: M bits $\rightarrow$ 1 symbol (a_n, b_n)

- typically use Gray coding.



∴ at most 1 bit error if you mistake as neighbor.

- minimizes # of bit errors for adjacent symbol errors.

<u>Distance between points</u> : (a_m, b_m) and (a_n, b_n).

$$d_{mn} = \sqrt{(a_n - a_m)^2 + (b_n - b_m)^2}.$$

d_{mn} is a fn of average energy per symbol E_s.

$P_s = Pr\{$mistaking a symbol for another$\}$

at <u>high SNR's</u>:

$\approx Pr\{$mistaking a symbol for its nearest neighbor$\}$

P_s at high SNRs determined by the # of nearest neighbors and the distance between nearest neighbors, $\underline{d_{min}}$,

$$d_{min} = \min_{n,m} d_{mn}.$$

<u>Pulse shaping</u>:

- to control BW of transmitted signal.
- Controls spectral properties of tx signal.
- may also introduce self-interference (ISI).
  (ISI introduced by <u>designer</u> to get smaller BW) This ISI is not channel induced.

• Family of Nyquist pulses : have <u>zero</u> ISI (introduced by designer) if <u>timing is perfect</u>.

$\Rightarrow g(t) = 0$ at any nT_s for $n \neq 0$.

- Rectangular pulse
  ✓

- Raised cosine pulse.
  parametrized by β.
  (spectral eff vs. timing jitter)

See $\boxed{\text{Figure 5.6}}$.

<u>GSM pulse shape</u>, Non-Nyquist
(self-ISI present).
trade-off between spectral properties and ISI you introduce.

<u>Differential Modulation</u> - used to get around a coherent phase reference.

<u>Motivation</u>: It's difficult to obtain a coherent phase reference in a fading environment.

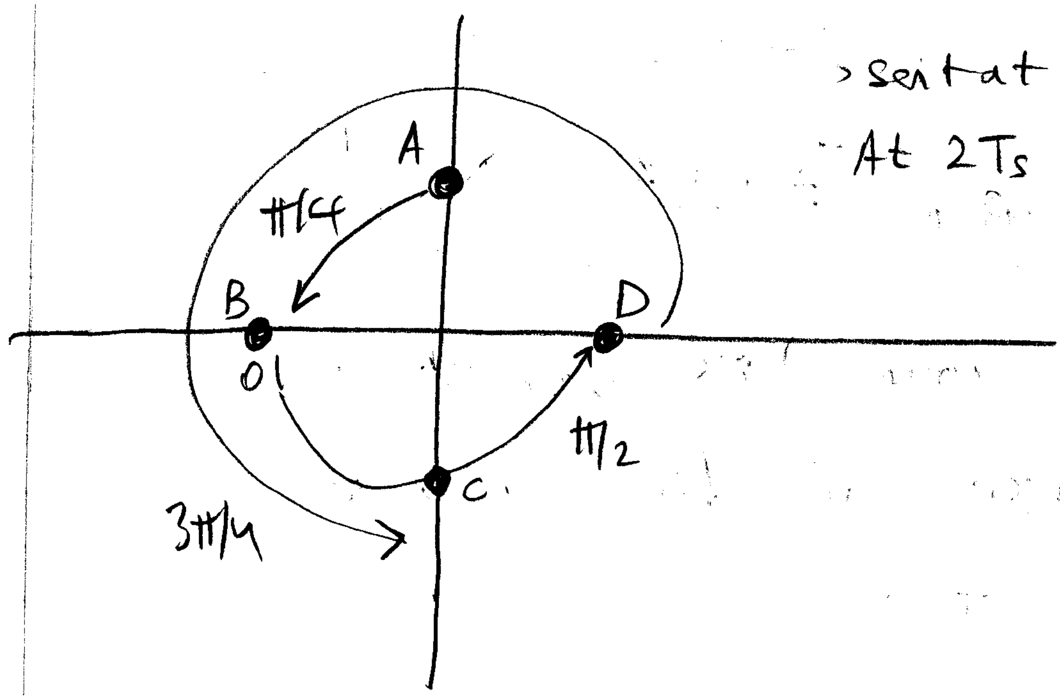
Also, phase drift impacts P_b of linear modulation.

<u>Basic Premise</u>: Use previous symbol as a phase reference for current symbol.

$\Rightarrow$ information bits are encoded as the phase difference between symbols.

<u>Example</u>: 4-DPSK.

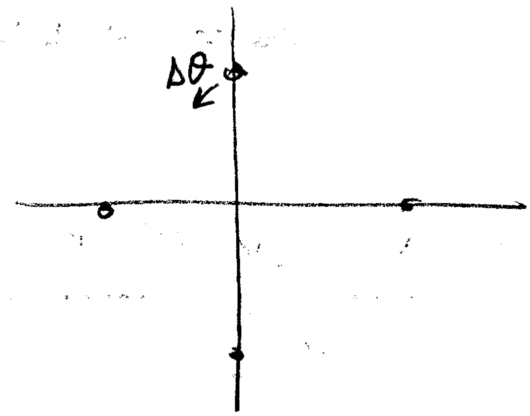
Information bits	$\Delta\phi$
00	0
01	$\pi/4$
10	$\pi/2$
11	$3\pi/4$



t	Send	info
sent at T_s	A	
At $2T_s$	A	00
$3T_s$	B	01
$4T_s$	D	10
$5T_s$	C	11

- Removes most of impact of phase drift.

Drift: $\Delta\theta$



$\boxed{\text{If } \Delta\theta \ll \frac{2\pi}{M} \text{ over } [0, T_s]}$

then, we never make a symbol error.

difference over only 1 symbol time matters.

Then phase drift won't hurt you

Power penalty: Double the power

(But get receiver simplicity)

- If channel Doppler is high;

coherence time Δt_c of channel.

If $T_s > \Delta t_c$ $\Rightarrow$ Channel completely

decorrelated $\Rightarrow$ phase reference is independent of current symbol phase!

$\Rightarrow$ You will make an error no matter what.

$\Rightarrow$ Irreducible Error Floor at high Dopplers.

II. Nonlinear Modulation:

$$s(t) = \underbrace{A}_{\text{constant envelope.}} \cos\left(2\pi f_c t + 2\pi \alpha_M \Delta f_c t\right)$$

α_M = varied w.r.t info bits

$\alpha_M = \mp 1$ binary FSK.

To maintain continuous phase, use FM (FSK is a special case of FM)

$$s(t) = A \cos\left[2\pi f_c t + 2\pi K \int_{-\infty}^{t} m(\tau)\,d\tau\right]$$

$$m(t) = \sum_n a_n g(t - nT_s)$$ is a PAM signal

By Carson's Rule,

$$BW_{FM} \approx 2\Delta f_c + 2B_u$$

B_u is BW of $m(t)$

$$= \text{BW of } g(t)$$

$\therefore$ can use pulse-shaping to control spectrum.

Note that

$$BW_{FM} > B_u = \text{BW for linear modulation}$$

$$= \text{BW of } g(t).$$

<u>Minimum frequency spacing in FSK</u> is

$$\Delta f_c = \frac{1}{2T_b}$$

$\Rightarrow$ MSK is FSK with this spacing

If $g(t) \to$ Gsn pulse,

then GMSK.

$+$ • constant envelope ($\Rightarrow$ nonlinear amplifiers)

$-$ not spectrally efficient

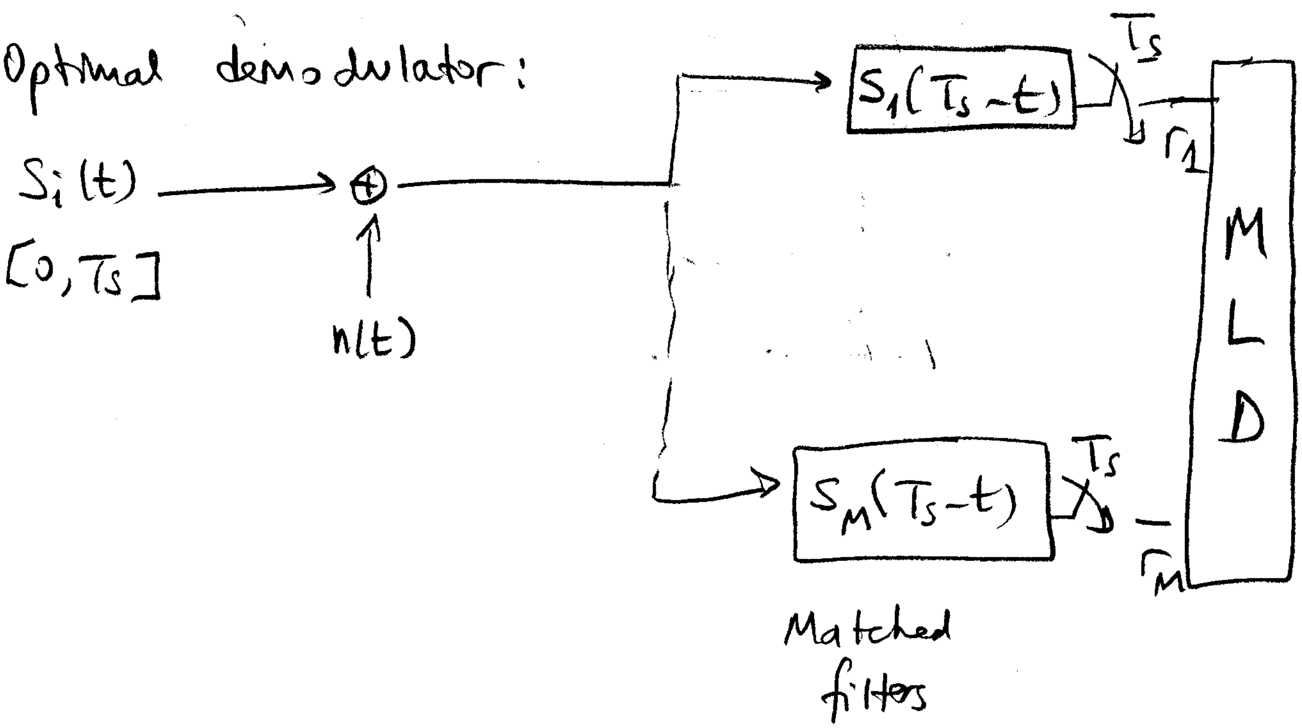
Performance in AWGN:

$$SNR_{\text{received}} = \frac{\text{received signal power}}{\text{noise power}} = \frac{P_r}{N_0 \cdot B}$$

$$\left(P_r = \frac{E_s}{T_s}\right) \qquad = \boxed{\frac{E_s}{N_0 B \cdot T_s}} \triangleq \gamma_s$$

$$= \boxed{\frac{E_b}{N_0 \cdot B \cdot T_b}}$$

Optimal demodulator:



$$\Pr\{r_i | s_i(t)\} > \Pr\{r_j | s_j(t)\} \quad j \neq i$$

MLD

For coherent detection of linear modulation at high SNRs,

$$P_s \approx \underbrace{\alpha}_{\text{\# of nearest neighbors at distance } d_{min}} \cdot Q\left(\sqrt{\frac{d_{min}}{2N_0}}\right).$$

$$\boxed{P_s \approx \underbrace{\alpha_M}_{\text{\# of nearest neighbors for particular modulation } M} Q\left(\sqrt{\beta_M \gamma_s}\right)}$$

a unified formula for different mods