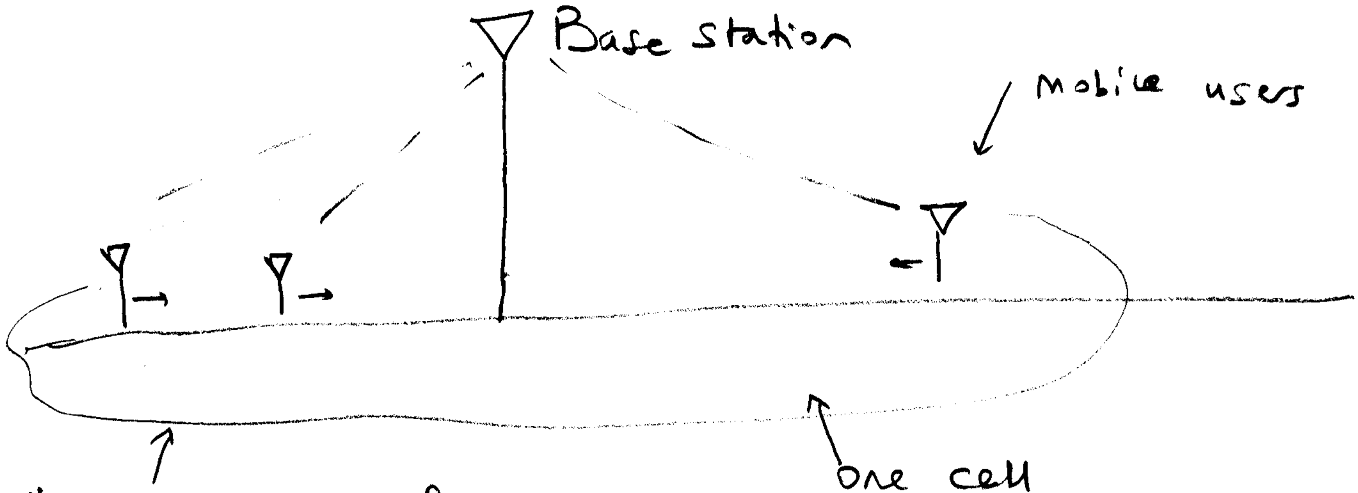
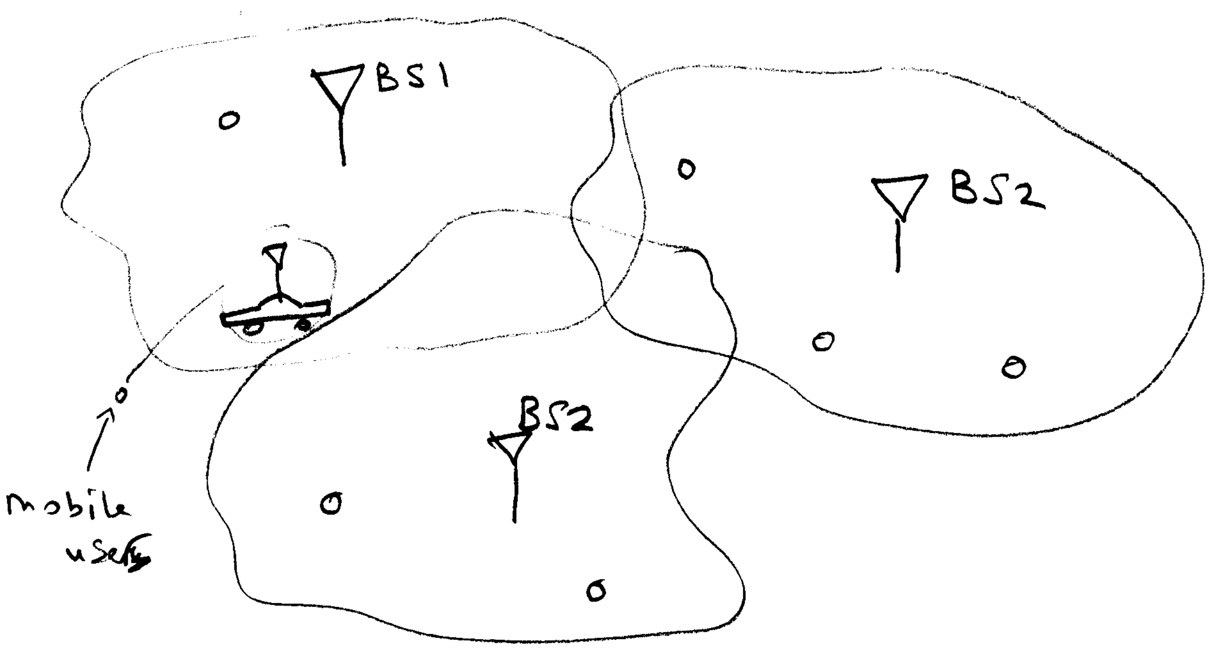
ECE 250 : LECTURE #2

CELLULAR SYSTEMS



"coverage area" of a base station:
the area in which the transmissions from a BS can be detected correctly.



"cellular concept"

Path Loss Model.

- predicts the average signal strength at a given distance from the base station, and from mobile to BS.

Empirical model:

$$P_r = P_t \cdot \alpha \cdot \left(\frac{d}{d_0}\right)^{-n}$$

d_0: "reference distance".
[must be in the far field of the transmit antenna.]

$d \geq d_0$ required (i.e. model holds only for $d \geq d_0$.

α: depends on n, and d_0. (Do not assume same d_0 for different n.)

P_t: transmit power (at antenna of transmitter)

P_r: receive power (of reception measured at receiver.)

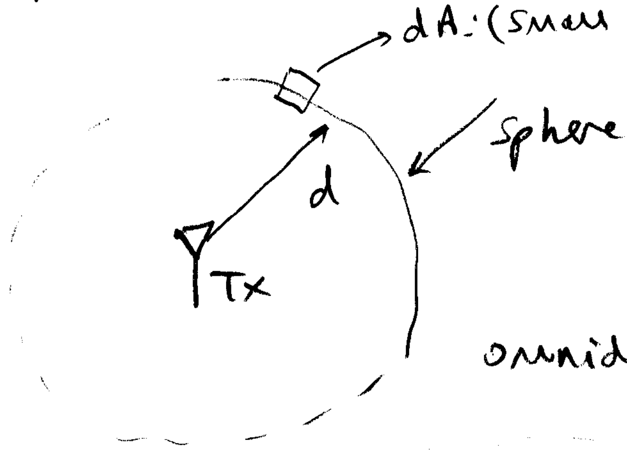
Requirements:

- Distance $d \gg h$: height of the antennas. (transmit and receive antennas.) and $d \gg \lambda$

$$n: \text{usually: } \begin{cases} n \geq 2 & \text{for outdoors.} \\ n \geq 1 & \text{indoor.} \end{cases}$$

Simple path loss derivations (to gain intuition)

(A) Free space propagation:



omnidirectional antenna.

$$P_r(d) = P_t \cdot \frac{G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2 L}$$

$\longrightarrow \underbrace{(G_t \cdot \lambda)}_{A_e^{(t)}} \cdot \underbrace{(G_r \cdot \lambda)}_{A_e^{(r)}}$

G_t : transmit antenna gain

G_r : receive antenna gain

λ ; wavelength. $(\lambda \cdot \nu = c.)$

[L : system loss factor: not related to propagation
$L \geq 1$. $L = 1$ is ideal model.]

$P_r \propto \frac{1}{d^2}$ because power gets divided over an area of $4\pi d^2$ (area of sphere) in space.

- good model for inter-satellite communication.

Antenna gain:

$$G = \frac{4\pi A_e}{\lambda^2}$$

A_e → effective area of antenna aperture

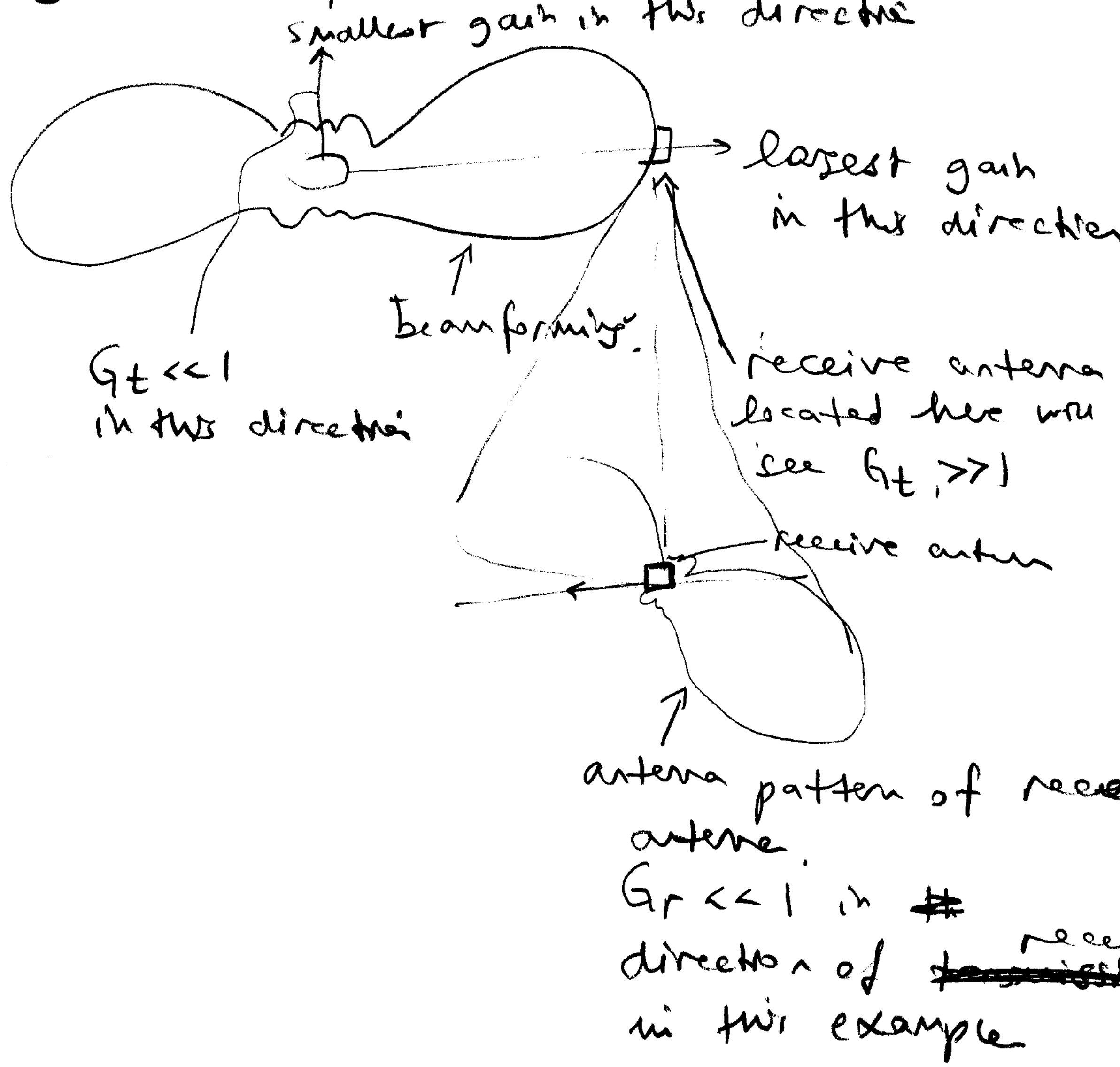
- note that G is dimensionless.

- omnidirectional antenna (isotropic radiator)

$$G = 1.$$

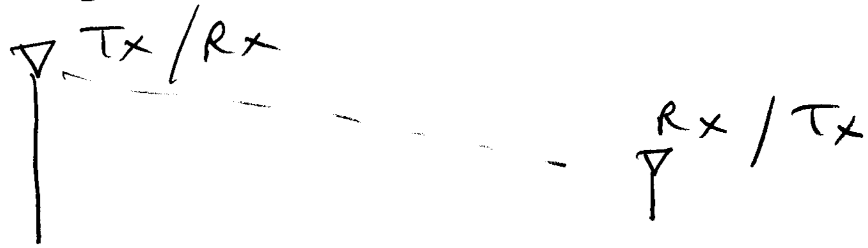
In reality: antenna pattern

Ex)



Antenna theory:

- <u>Reciprocity</u>: the ~~tx~~ and receive antennas are reciprocal. This means that it does not matter (as far as channel gain is concerned) which one is receiving & which one is transmitting.



[Do <u>not</u> confuse with uplink / downlink differences: those occur due to the fact that UL and DL are located ~~at~~ different frequencies in practical systems.]

<u>dB</u>: decibel.

<u>Definitions</u>:

- <u>Power</u>: $P_{dB} \equiv 10 \log_{10}(P)$

- <u>Amplitude</u>: $P_{dB} \equiv 20 \log_{10}(V)$
  (e.g. voltage)

- Reason for this definition:

$$P = \frac{V^2}{R} = I^2 R \;.\quad P_1 = \frac{V_1^2}{R}\;,\quad P_2 = \frac{V_2^2}{R}$$

(• Assume that R constant.)

We want the difference between the two systems to be the <u>same</u> in dB (when we compare them) no matter whether we are using voltage or power.

$$\boxed{10 \log_{10} \frac{P_1}{P_2}} = 10 \log_{10} \frac{V_1^2}{V_2^2} = \boxed{20 \log_{10} \frac{V_1}{V_2}}$$

we want these #'s to be the same

(dB) (dB)

• Free space propagation in dB:

$$10 \log_{10} P_r = 10 \log_{10} \left[P_t \cdot \frac{G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2 L} \right]$$

$$= P_t\,(dB) + G_t\,(dB) + G_r\,(dB)$$

$$+ 10 \log_{10} (\lambda^2) - 10 \log_{10} (4\pi)^2$$

$$- 20 \log_{10} (d) - 10 \log_{10} (L)$$

Note that $G(dB) = 0$ for an omnidirectional antenna.

- Empirical path loss model:

$$P_r = P_t \cdot \left(\frac{d}{d_0}\right)^{-n}$$

$$\boxed{P_r(dB)} = 10 \log_{10}\left[P_t \cdot \left(\frac{d}{d_0}\right)^{-n}\right]$$

$$= \boxed{P_t(dB) - 10n \cdot \log_{10}\left(\frac{d}{d_0}\right)}$$

<u>dBm</u>:

, P: <u>watts</u>, $\longrightarrow$ dBW. (or dB)

$$10 \log_{10} P_{(watts)}$$

, dBm : mW $\longrightarrow$ dBmW. (or dBm)

$$10 \log_{10} \left(P_{(mWatts)}\right)$$

$$10 \log_{10} \overset{x}{P}_{(watts)} = 10 \log_{10}\left[1000 \cdot P_{(mWatts)}\right]$$

$$\boxed{x(dBW) = x(dBmW) - 30\, dB}$$

<u>Note</u>.

1 mW $\rightarrow$ 0 dBm,

1 W $\rightarrow$ 0 dB.

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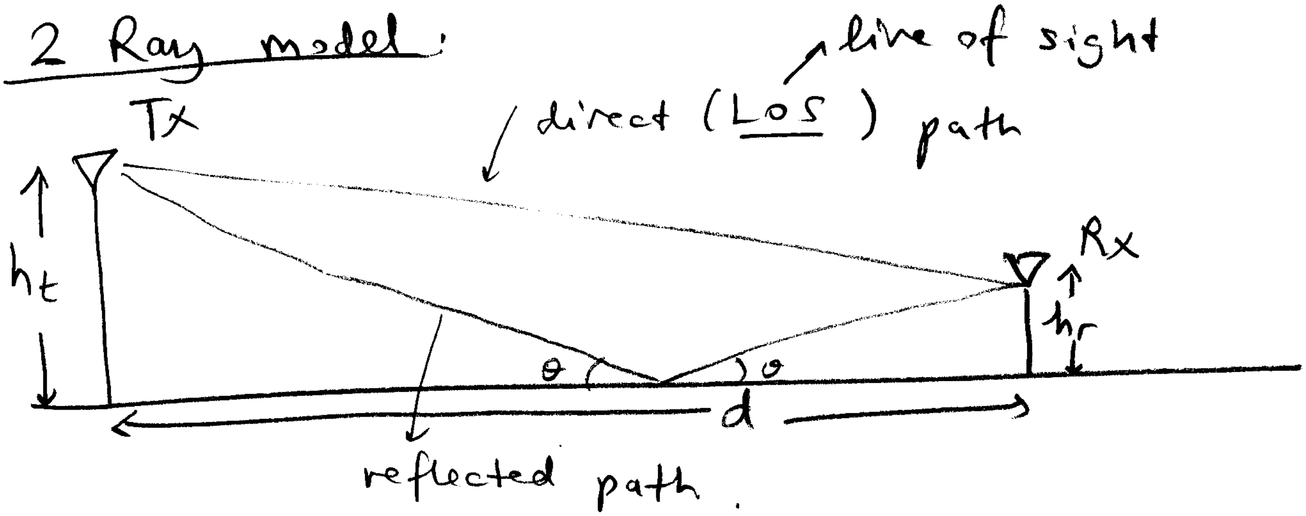
Ⓑ 2 Ray model:



[Derivation given in Rappaport using the method of images.]

Result:

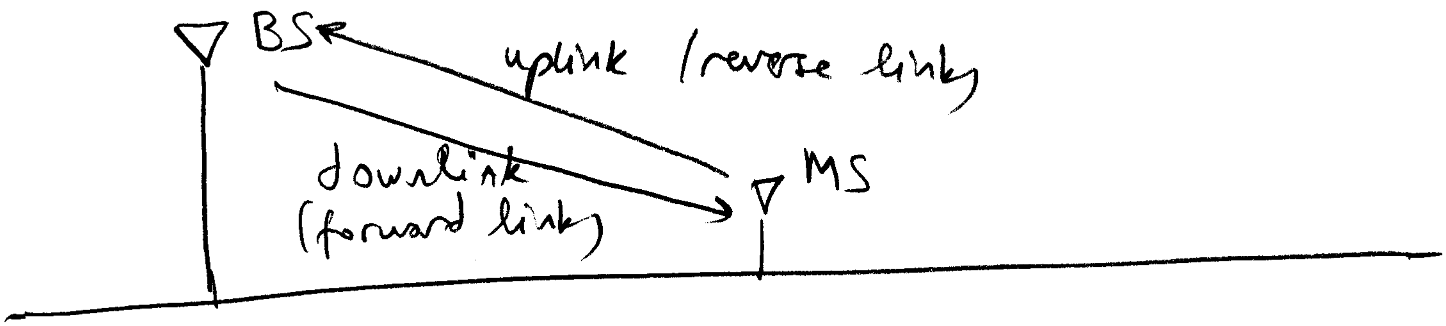
If $d > \frac{20\, h_t \cdot h_r}{\lambda}$ and $d \gg h_t + h_r$, then

$$\boxed{P_r = P_t \cdot G_t \cdot G_r \cdot \frac{h_t^2\, h_r^2}{d^4}}$$

Hence, $P_r \propto 1/d^4$.

- fall-off faster than free space
- Intuition: Reflected path causes destructive interference.
- this derivation gives intuition for the observed (empirical) $1/d^4$ fall-off in macrocells.

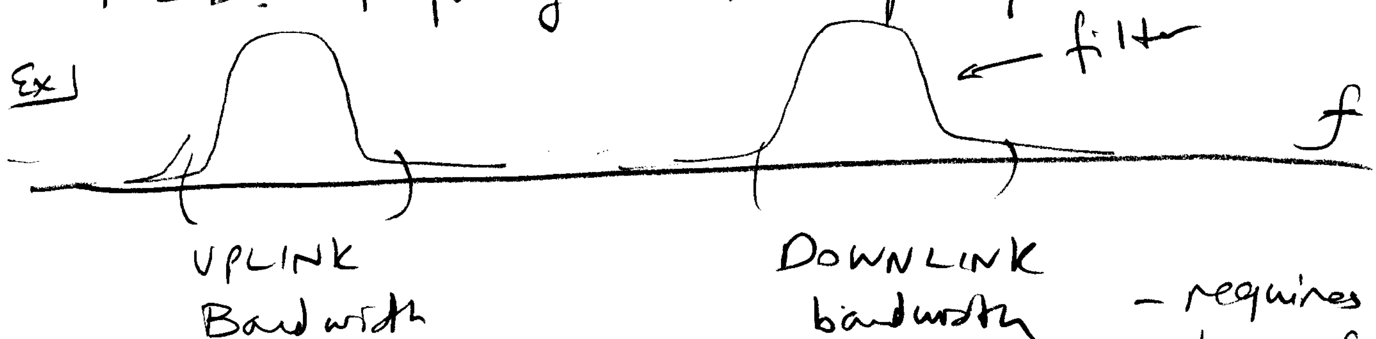
Cellular fundamentals:



- Duplexing: separation of uplink & downlink transmissions (between a Tx and Rx)

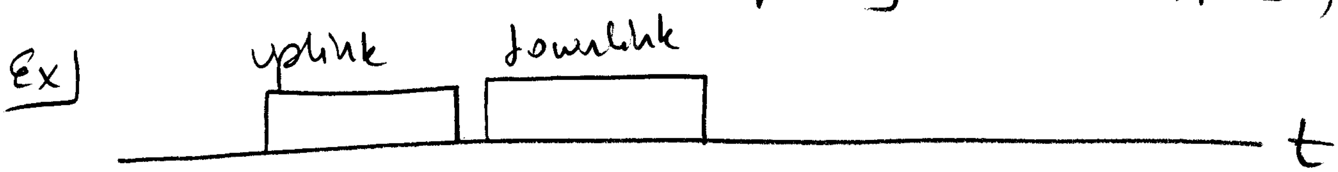
Main duplexing techniques:

- FDD: frequency division duplexing



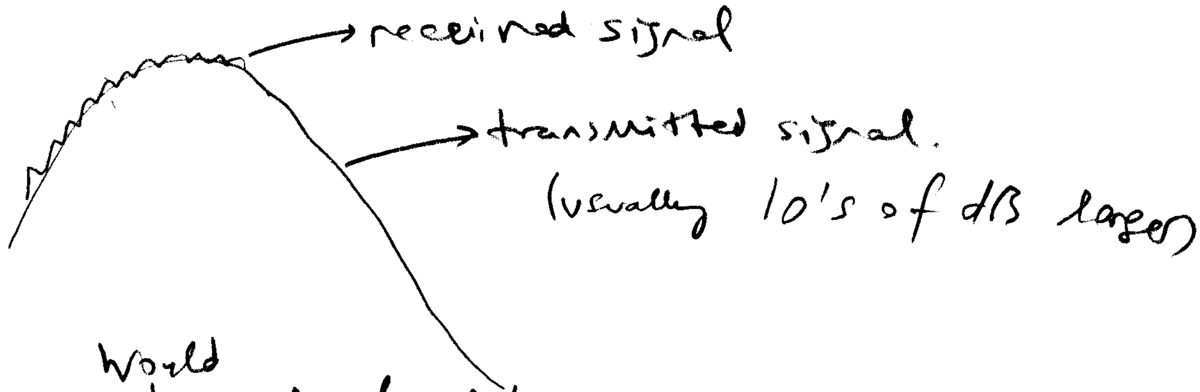
– requires sharp filters (to eliminate ACI.)

- TDD: time division duplexing



requires synchronization between Tx and rx.

* At a terminal, the Tx and receive signals must be separated either in time or in frequency domain — a fact that has to do with circuit technology.



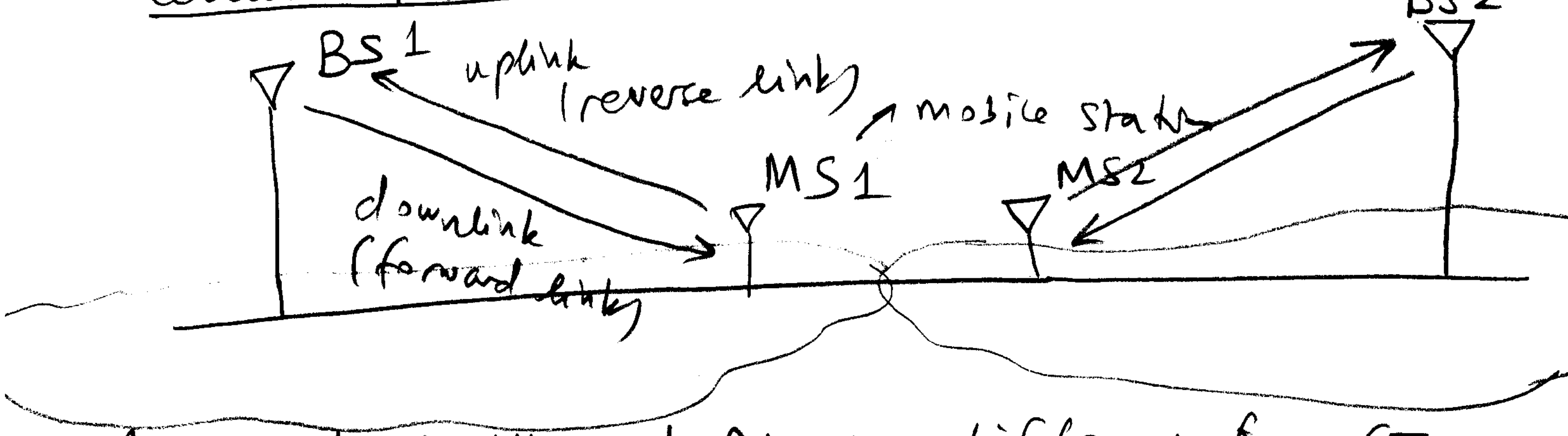
- need v. high A/D resolution to actually ~~cat~~ detect the received signal.

- usually never worth the circuit cost to transmit and receive at same time or same W.

- If the signals are separated by sharp duplexing filters, simultaneous tx and rx possible.
  [These filters are expensive.]

- TDD usually preferred in practical systems.

Cellular Fundamentals: Interference



- Assume that UL and DL use different freq. (FDD)

① When MS 2 $\longrightarrow$ BS 2, this uplink transmission causes interference at BS1's receiver (if same bandwidth W is used.)

"out-of-cell uplink interference".

② When BS 2 $\longrightarrow$ MS 2, this downlink transmission interferes with BS1 $\rightarrow$ MS1 (at MS1's receiver)

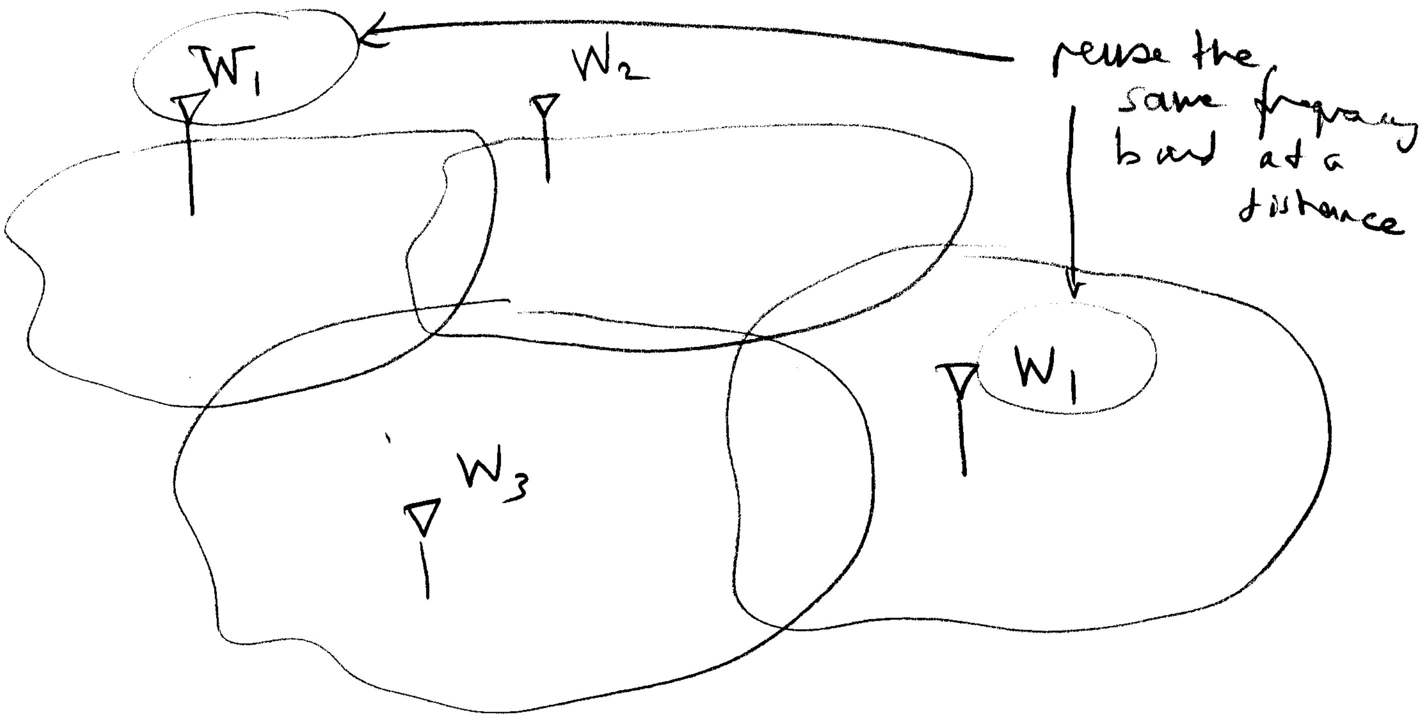
"out of cell downlink interference".

- Key challenge in cellular systems: use total system bandwidth W efficiently over the area.

Frequency Reuse:

— System Bandwidth limited.

— Should use the bandwidth cleverly over the area.



- Main idea:

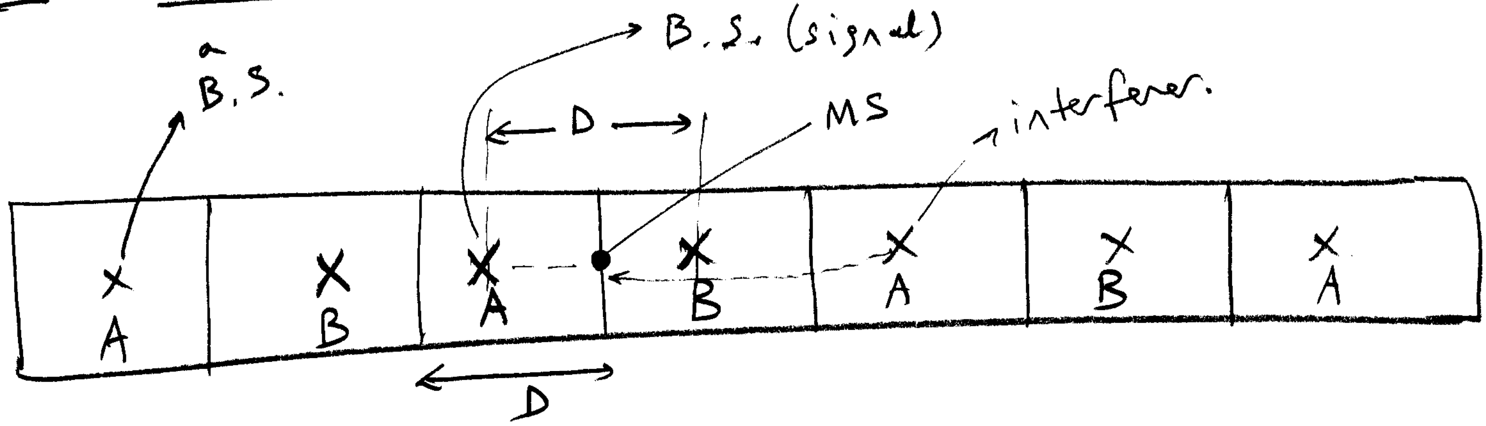
UL ⟵ FDD example ⟶ DL

$[\,W_1^{UL} \mid W_2^{UL} \mid W_3^{UL}\,]$ $\quad$ $[\,W_1^{DL} \mid W_2^{DL} \mid W_3^{DL}\,]$

— Due to the path loss (tx power falloff), the interference at some large enough distance will become negligible. Hence can reuse frequency band W_1.

We will develop intuition starting with simple models.

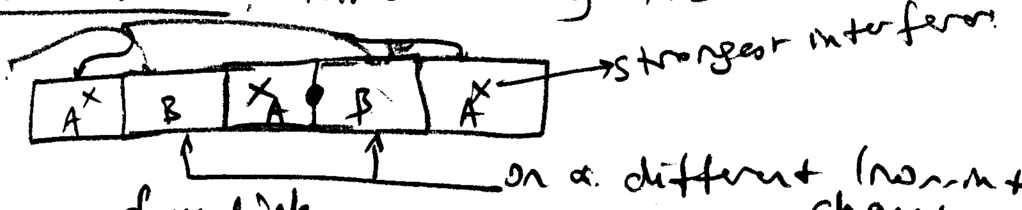
$\boxed{A}$: <u>One-dimensional models</u>:



- Two channels are used A and B.
- frequency reuse factor $M = 2$.
- Find the worst-case SIR (signal-to-interference ratio) at a MS (downlink) and at a BS (uplink).

<u>Downlink</u>:

- Largest (worst-case) downlink interference occurs when the MS is located at the cell boundary.
  - <u>First-order calculation</u>: involves only the neighboring cells.



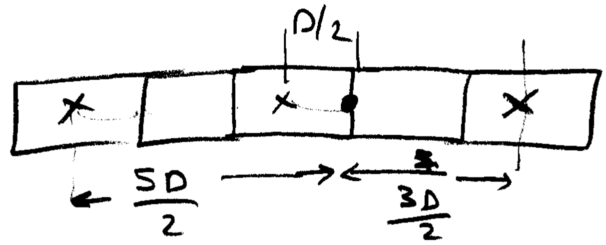
- Assume that the downlink transmit power same at each B.S.

Signal power at MS's receiver:

assumes same antenna gains for all antennas.

$$S = P_A \cdot \beta \cdot \left(\frac{D}{2}\right)^{-n}$$

P_t ; P_A: BS (A) ; $\beta = \left(\frac{\alpha}{d_0^n}\right)$.



$$I = P_B \cdot \beta \cdot \left[\left(\frac{3D}{2}\right)^{-n} + \left(\frac{5D}{2}\right)^{-n}\right].$$

P_t

$$\frac{S}{I} = \frac{(D/2)^{-n}}{\left(\frac{3D}{2}\right)^{-n} + \left(\frac{5D}{2}\right)^{-n}} \qquad \left(\div \frac{D^{-n}}{2^{-n}}\right)$$

1st order model

$$SIR = \frac{1}{3^{-n} + 5^{-n}}$$

(first order interf. model)

[Note that:

$$SIR_{n=2} = \frac{1}{3^{-2}+5^{-2}} = 6.61 \qquad (8.2\,dB)$$

$$SIR_{n=3} = \frac{1}{3^{-3}+5^{-3}} = 22.20 \qquad (13.5\,dB)$$

$$SIR_{n=4} = \frac{1}{3^{-4}+5^{-4}} = 71.71 \qquad (18.6\,dB)$$]

The larger the path loss exponent, the larger the SIR.)

- In general, if M is the frequency reuse factor

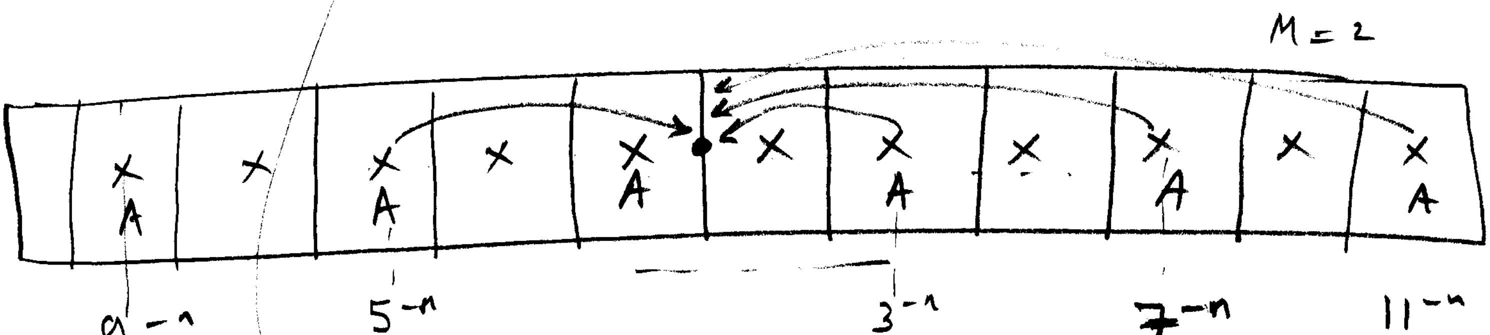
$$\underset{\substack{\uparrow \\ \text{first-order} \\ \text{model}}}{SIR} = \frac{1}{(2M-1)^{-n} + (2M+1)^{-n}} \qquad M \geq 1$$

[When $M=1$: $SIR = \dfrac{1}{1+3^{-n}} < 1$.

This is fine, e.g. DS-SS systems can typically operate at $SIR < 1$.]

- If you want to be rigorous, you can sum over all interferers: $(M=2)$.

$$SIR = \frac{1}{3^{-n} + 5^{-n} + 7^{-n} + 9^{-n} + \ldots}$$



$$= \boxed{\frac{1}{\sum_{r=1}^{\infty} (2r+1)^{-n}}}$$

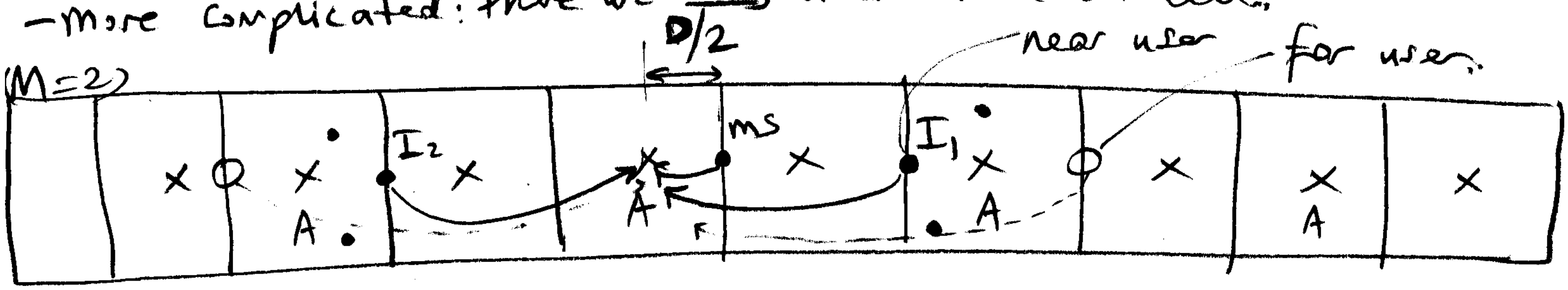
Contributions of other interferers (not first order) can be neglected for $n \geq 3$

Using the 1st order model: [Downlink SIR]

n\M	1	2	3	4
4	−0.05	+18.6	+27.0	+32.5
3	−0.16	+13.5	+19.6	+23.7
2	−0.5	+8.2	+12.2	+14.8

fairly good (SIR > 10 dB)
(M=2)

UPLINK ANALYSIS:
− more complicated: there are many users in each cell.



First-order analysis: NEAR-USER ANALYSIS.

• worst-case: • put MS as far away from B.S. as possible
• put interferers as close as possible.

$$S = P_{ms} \cdot \beta \cdot \left(\frac{D}{2}\right)^{-n}$$

$$I = P_{I_1} \cdot \beta \cdot \left[\left(\frac{3D}{2}\right)^{-n}\right]$$ per near-user interferer.

Assume $P_{ms} = P_{I_1} = P_{I_2}$

$$\underset{\text{(first-order) (near-user)}}{SIR} = \frac{(D/2)^{-n}}{\left(\frac{3D}{2}\right)^{-n}} = \frac{1}{3^{-n}} \quad (M=2)$$

FAR USER ANALYSIS: (first-order model) (M=2).

$$I = P_{I_{far}} \cdot \beta \cdot \left(\frac{5D}{2}\right)^{-n}$$

$$S = P_{MS} \cdot \beta \cdot \left(\frac{D}{2}\right)^{-n}$$

$$\underset{\substack{\text{far user}\\ \text{first order model}}}{SIR} = \frac{1}{5^{-n}}$$

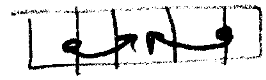
For general M: (first-order model)

$$\underset{\substack{\text{(close/near) user}}}{I} = P_{I_{close}} \cdot \beta \cdot \left[\frac{(2M-1)D}{2}\right]^{-n}$$

$$\underset{\text{far user}}{I} = P_{I_{far}} \cdot \beta \cdot \left[\frac{(2M+1)D}{2}\right]^{-n}.$$

$$\underset{\text{close user}}{SIR} = \frac{1}{(2M-1)^{-n}}$$

$$\underset{\text{far user}}{SIR} = \frac{1}{(2M+1)^{-n}}$$



SIR assuming 2 interferers

UPLINK ~~(per interferer)~~ (dB)

n\M	1		2		3		4	
	close	far	close	far	close	far	close	far
4	−3	+16.1	+16.1	+25.0	+25.0	+30.8	+30.8	+35.2
3	−3	+11.3	+11.3	+18.0	+18.0	+22.3	+22.3	+25.6
2	−3	+6.5	+6.5	+11.0	+11.0	+13.9	+13.9	+16.1

– Note the difference from downlink SIR.

– Even though prop. reciprocal, uplink & downlink co-channel interference not reciprocal.

Channel sets:

Ex)

$M = 2$

$$\begin{matrix} \times & \times & \times & \times & \times & \times \\ \begin{pmatrix} B \\ D \end{pmatrix} & \begin{pmatrix} A \\ C \end{pmatrix} & \begin{pmatrix} A \\ C \end{pmatrix} & \begin{pmatrix} B \\ D \end{pmatrix} & \begin{pmatrix} A \\ C \end{pmatrix} & \begin{pmatrix} B \\ D \end{pmatrix} \end{matrix}$$

two channel sets: $\begin{pmatrix} A \\ C \end{pmatrix}$ & $\begin{pmatrix} B \\ D \end{pmatrix}$

Each channel set here has two channels.

- If each channel: $BW = W$.

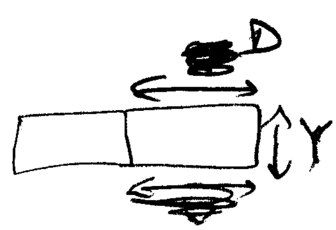
Thus $2W$ per channel set.

- fixed reuse of channel sets
  (1G and 2G)

\# of channel sets $\boxed{C_s = M}$ for 1-D system.

- Distance between B.S.'s that use same channel

$\tilde{D}_s =$ set, $2MD$. $\sim 2MD$



$$C_s = M = \frac{\tilde{D}_s}{2r}$$

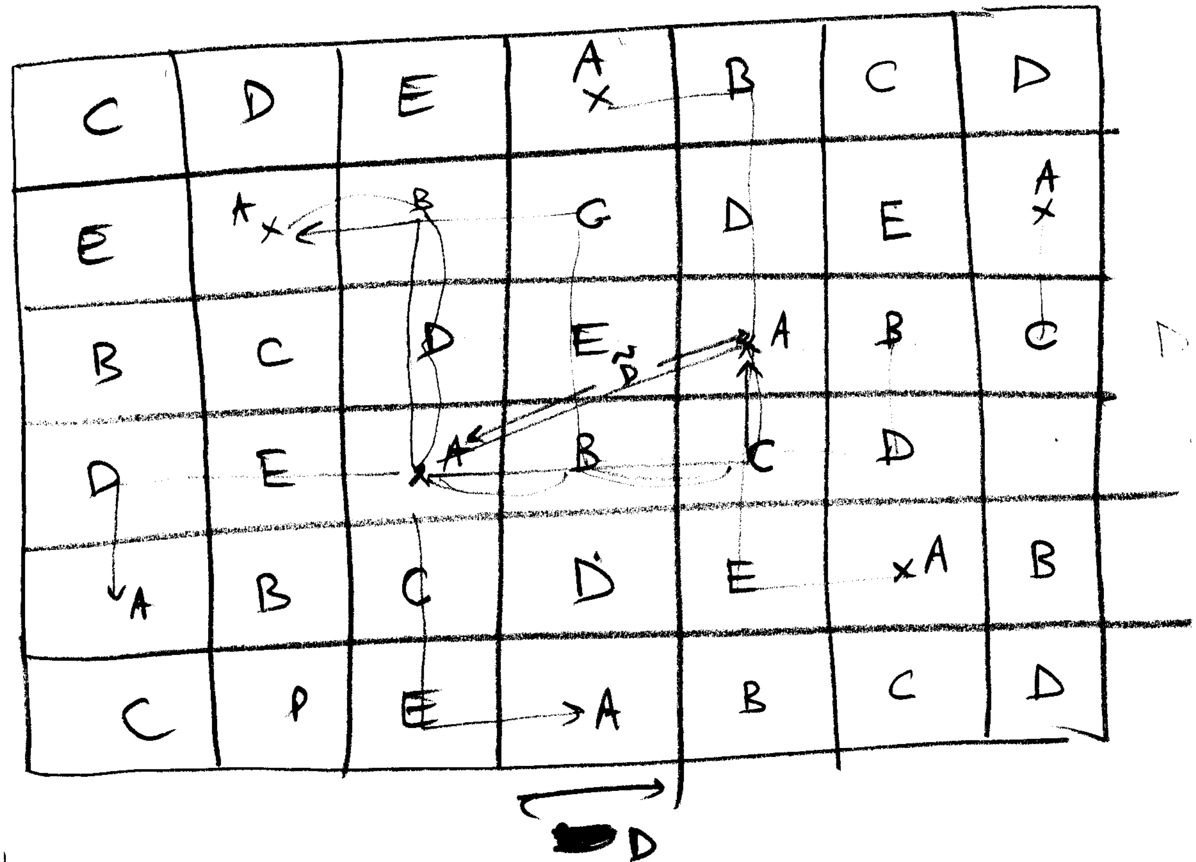
$$\boxed{C_s = \frac{\tilde{D}_s \cdot r}{D r}} = \frac{A_g}{A_c}$$

$A_g \rightarrow$ area covered by all channel sets

$A_c \rightarrow$ area of a single channel cell

2D Systems:

- Square grid:



$k = 2$

$l = 1$

$$C_s = \frac{A_g}{A_{cell}} = \frac{\tilde{D}^2}{D^2} = \frac{(k^2 + l^2)(2r)^2}{(2r)^2}$$

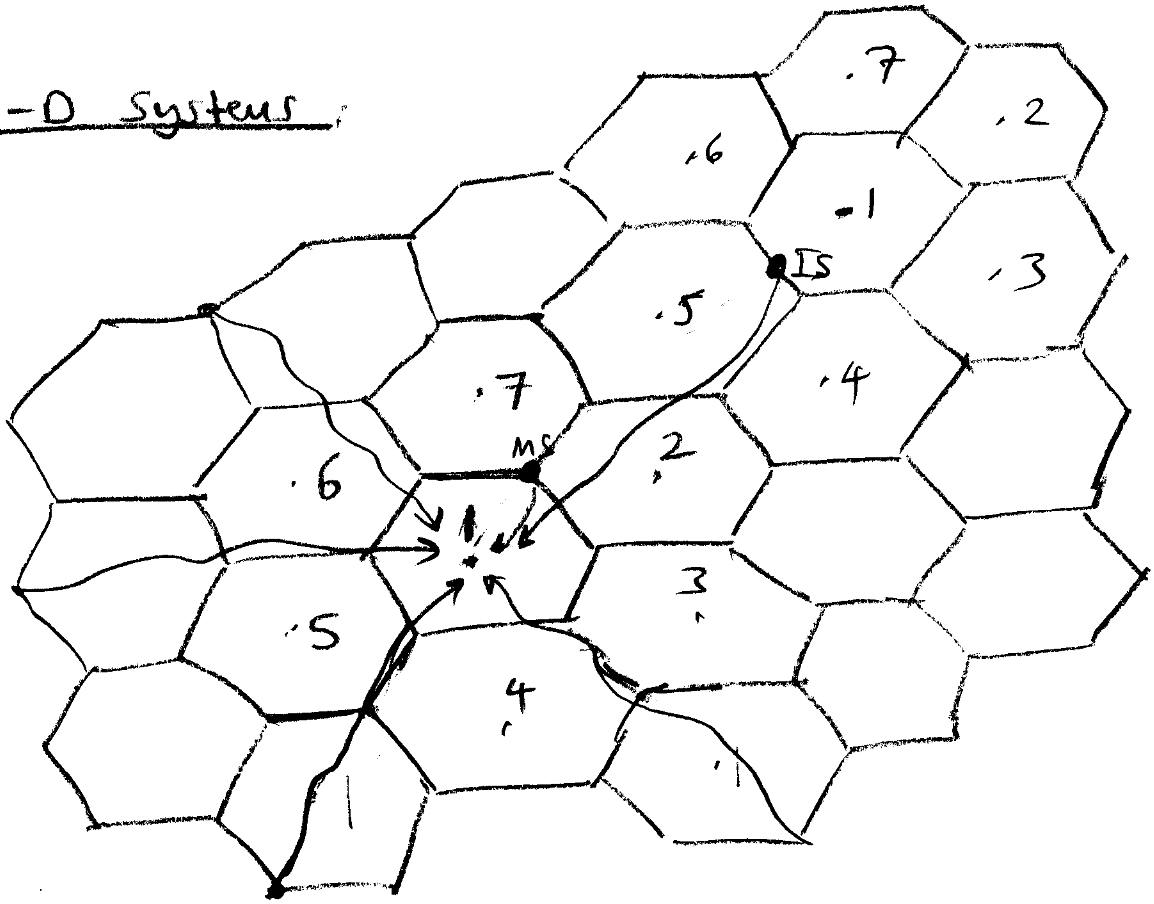
$$= k^2 + l^2$$

$k = 2$, $l = 1$: $\quad C_s = 4 + 1 = \boxed{5} \quad \{A, B, C, D, E\}$

$\boxed{M \equiv \sqrt{C_s}}$ → you will find another BS that uses the same channel set, thus, many D's over

- need for handoff. . .

2-D Systems:

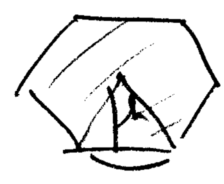


[WN_c ≜ bandwidth occupied by a channel set.

N_c: # of channel sets. per coverage area

Total bw. of system = $(WN_c) \cdot \underbrace{C_s}_{\#\text{ of channel sets}}$]

$\cancel{C_s = \frac{\quad}{(3ah)}}$



Show that: $C_s = k^2 + l^2 + kl$.

$\frac{ha}{2} \cdot 6 = 3ah$

$M = \sqrt{C_h}$.