LECTURE #3

- Path loss model:
  - holds for "far-field" of transmit antenna
    'Fraunhofer region'

d_f : far-field distance.

$$\boxed{d_f = \frac{2D^2}{\lambda}}$$

D: largest linear dimension of the transmitter antenna aperture.

λ: carrier wavelength.

- The reference distance d_0 must be chosen to be in the far field:

$$\left\{ \begin{array}{l} d_0 \geqslant d_f = \dfrac{2D^2}{\lambda} \quad , \text{ and} \\ \\ d_0 \gg D \quad , \text{ and} \\ \\ d_0 \gg \lambda \end{array} \right\}$$

Example 1:

- Find the far-field distance of an antenna with maximum dimension 1 m, and carrier frequency 900 MHz

Solution: $D = 1$ m.

$f_c = 900$ MHz

$$d_f = \frac{2D^2}{\lambda} = \frac{2(1)^2}{\left[\frac{3\times10^8 \text{ m/s}}{900\times10^6}\right]} = 2\cdot 3 = \underline{6 \text{ meters}}.$$

Far field: ≥ 6 meters, and

$\gg 1$ meter, and

$\gg \frac{1}{3}$ meter.

$\therefore \approx \underline{10 \text{ meters}}$ will fall in the far-field.

Example 2: Assume $P_t = 50$ watts

omnidirectional tx antenna, $f_c = 900$MHz; omnidirectional receive antenna

(a) Find received power in dB_m at a distance of $\underline{100 \text{ m}}$ from the antenna in free space.

(b) $P_r(10 \text{ km}) = ?$ in free space

Solution:

$$P_r = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2 L}$$

L ← assume 1.

$$\frac{3\times 1\cancel{0}^{8}}{9\times 1\cancel{0}^{8}}$$

$$= \frac{50 \cdot 1 \cdot 1 \cdot [\frac{1}{3} \text{ meters}]^2}{(4\pi)^2 \cdot (100)^2}$$

$$= 3.5\,\mu W$$

$$P_r\ (dBm) = 10 \log_{10} (3.5 \times 10^{-3}\ mW) = \underline{\underline{-24.5\ dBm}}$$

(b) Using the path loss model: ($n=2$) free space

$$P_r(10\ km) = P_r(100\ \text{meters}) + 20 \log_{10} \left[\frac{\overset{\overset{\text{100 meter } (d_o)}{\|}}{100}}{\underset{\underset{10 km}{\|}}{10\,000 \text{ meters}}} \right]$$

$$\left[\underset{(dB)}{P_r(10\ km)} = \underset{(dB)}{P_r(100\ \text{meters})} - \underset{(dB)}{10 \log_{10} \left(\frac{d}{d_o}\right)^2} \right]$$

$$= -24.5\ dBm - 40\ dB$$

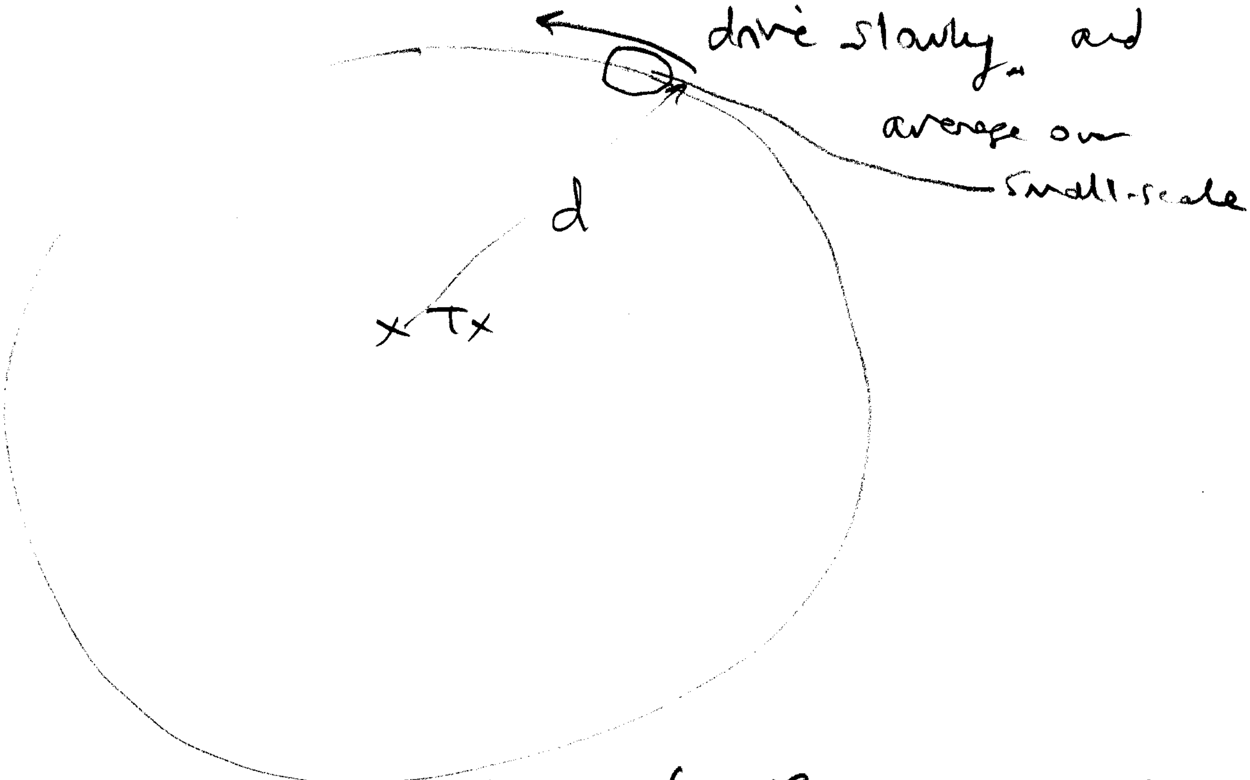
$$= \underline{\underline{-64.5\ dBm}}$$

Lognormal shadowing.

("Large-scale variations")

$$PL(d) = \overline{PL}(d) + X_\sigma$$

$$X_\sigma \sim \mathrm{Gsn}(0, \sigma^2)$$

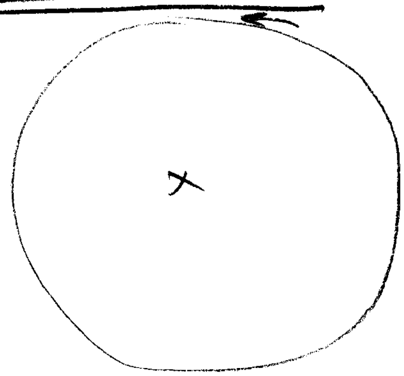


- you'll find that values ~~are~~ (in dB) are normally distributed.

$$\left[\quad P_r = \underbrace{\alpha \cdot P_t \cdot \left(\frac{d}{d_0}\right)^{-n}}_{\substack{\text{path loss}\\ \text{deterministic}\\ \text{as a fn of } d.}} \cdot \underbrace{\left(10^{X_\sigma/10}\right)}_{\substack{\text{random var}\\ \text{— lognormally}\\ \text{distributed.}}}\right.$$

— Arises from obstacles (trees, buildings) between Tx and Rx

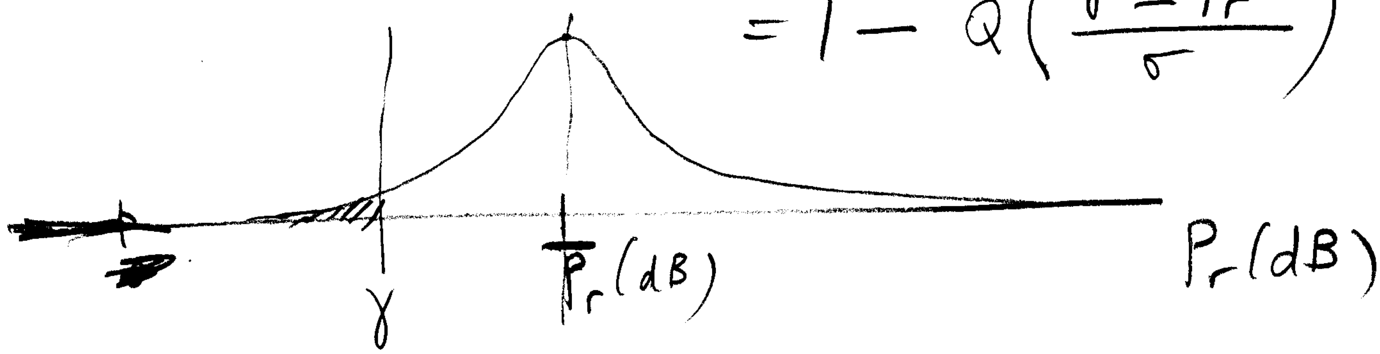
Probability of outage: $= P_r$(the received signal power falls below a certain threshold).



e.g. 5%, 1%

($\Rightarrow$ call will be dropped)

$$P\{P_r < \gamma\} = 1 - P_r(P_r > \gamma) = 1 - Q\left(\frac{\gamma - \overline{P_r}}{\sigma}\right)$$



$\sigma \doteq 8-12$ dB in macrocells.

[Show slides: Measurements from urban and suburban environments.]

Example: Consider a single cell in a cellular system. Design the system such that $P_{out} = 0.01$ (1%) for the worst-case user (on cell boundary).

SYSTEM PARAMS: $f_c = 900$ MHz

Antenna dimensions $\simeq 1$ meter.

$d_{max} = 10$ km.

(maximum distance from the B.S.)

$P_t = 50$ Watts

– Assume omnidirectional antennas.

Environment params: $\sigma = 8$ dB (of shadowing)

$n = 4$.

Solution: First, let's determine the reference distance d_o.

$$d_f = \frac{2D^2}{\lambda} = \frac{2(1)^2}{\left[\frac{3\times10^8 \text{ m/s}}{900\times10^6 \text{ /s}}\right]} \doteq 6 \text{ meters}$$

To be in the far field, $\gg 6$ meters, $\gg D = 1$ meter, $\gg \lambda = \frac{1}{3}$ meter } take $d_o = 10$ meters

(LOS)

using free space model.

$$P_r(10 \text{ meters}) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2} = \frac{50 \cdot 1 \cdot 1 \cdot [\frac{1}{3}]^2}{(4\pi)^2 \cdot 10^2}$$

$$= 0.35 \text{ mW}$$

$\left(\begin{array}{l} 100^2 = 10000 \\ 10^2 = 100 \end{array}\right)$

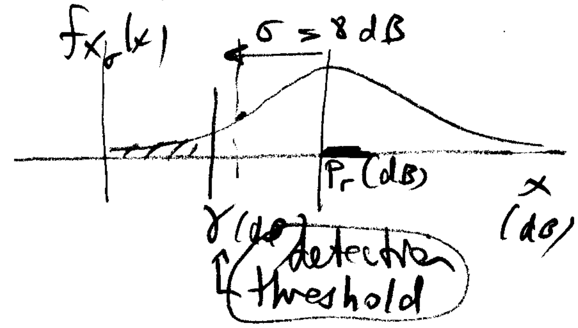
$$\left[\,\overline{P_r}(d) = \overline{P_r}(d_0)\cdot\left(\frac{d}{d_0}\right)^{-n}\right]$$

Then:

$$\overline{P_r}(10\text{ km}) = (0.35\,\mathrm{mW})\cdot\left(\frac{\cancel{10}\times 10^{3}}{\cancel{10}}\right)^{-4}$$

$$= \underbrace{0.35\times 10^{-12}\ \mathrm{mW}}$$

average power level at 10 km.



Now, $P_{out} = 0.01$, which means

We want:

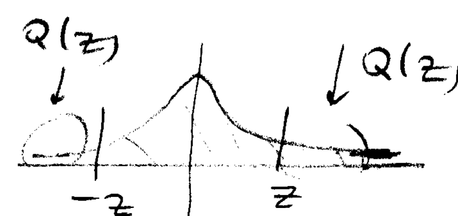
$$\underbrace{P\{P_r(d_{max})^{[dB]} > \gamma\}} = 0.99$$

$$= P\{\overline{P_r}(d_{max})(dB) + X_\sigma > \gamma\}$$

$$= P\{X_\sigma > \gamma(dB) - \overline{P_r}(d_{max})(dB)\}$$

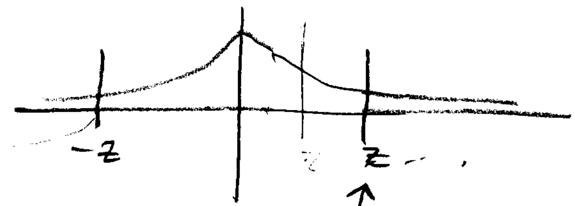
$$= P\{\sigma Z > \gamma(dB) - \overline{P_r}(d_{max})(dB)\}$$

$$= P\left\{Z > \frac{\gamma(dB) - \overline{P_r}(d_{max})(dB)}{\sigma}\right\} = Q\left(\overbrace{\frac{\gamma(dB) - \overline{P_r}(d_{max})(dB)}{\sigma}}^{\equiv\, -z}\right)$$



[useful relationships: $Q(-z) = 1 - Q(z)$
(due to symmetry of Gsn. distribution)
$Q(0) = 1/2$.]

- See Table F.1, on p. 647, Rappaport.
- Note that only $Q(z) \leq 1/2$ values are given.
  (i.e. for $z \geq 0$.)



$$Q\left(\frac{\gamma(dB) - \overline{P}_r(d_{max})(dB)}{\sigma}\right) = 1 - \underbrace{Q\left(\frac{\overline{P}_r(d_{max})(dB) - \gamma(dB)}{\sigma}\right)}_{0.01}$$

$\downarrow$

requires $z \simeq \underline{\underline{2.3}}$
from Table F.1

$$\therefore \quad \overline{P}_r(d_{max})(dB) - \gamma(dB) = 2.3\sigma$$

$$\gamma(dB) = \overline{P}_r(d_{max})(dB) - 2.3\sigma$$

$$= 10 \log_{10}(0.35 \times 10^{-12}\, mW) - 2.3 \cdot (8)$$

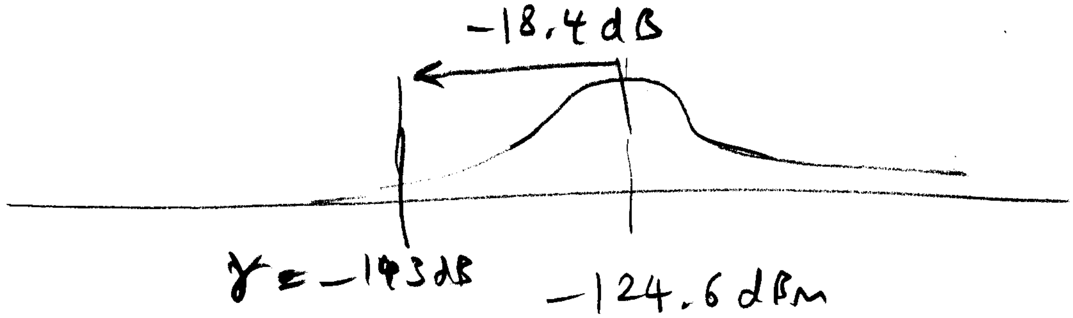
$$= -124.6\ dBm - 18.4\ dB$$

$$= \boxed{-143\ dBm}$$

$\downarrow$

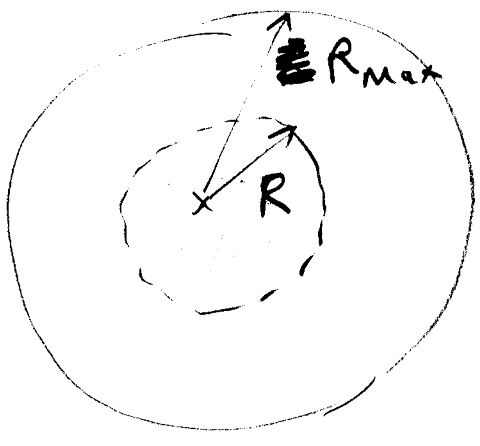
the maximum threshold required (for $P_{out} = 0.01$)

i.e.



— The analysis above was done for the worst-case user. This may be too conservative. A more realistic analysis involves the determination of the coverage area based on $U(\gamma) \equiv$ the percentage of useful service area.

($\equiv$ the percentage of the area with a received signal $\geq \gamma$.)



$U(\gamma) \cong P\{$ the received signal is $\geq \gamma\}$

$R \in dr$

$= \int_0^{R_{max}} P\{$ received signal $\geq \gamma \mid$ mobile user is at a distance r from B.S$\}$ $\cdot f_R(r)\,dr$

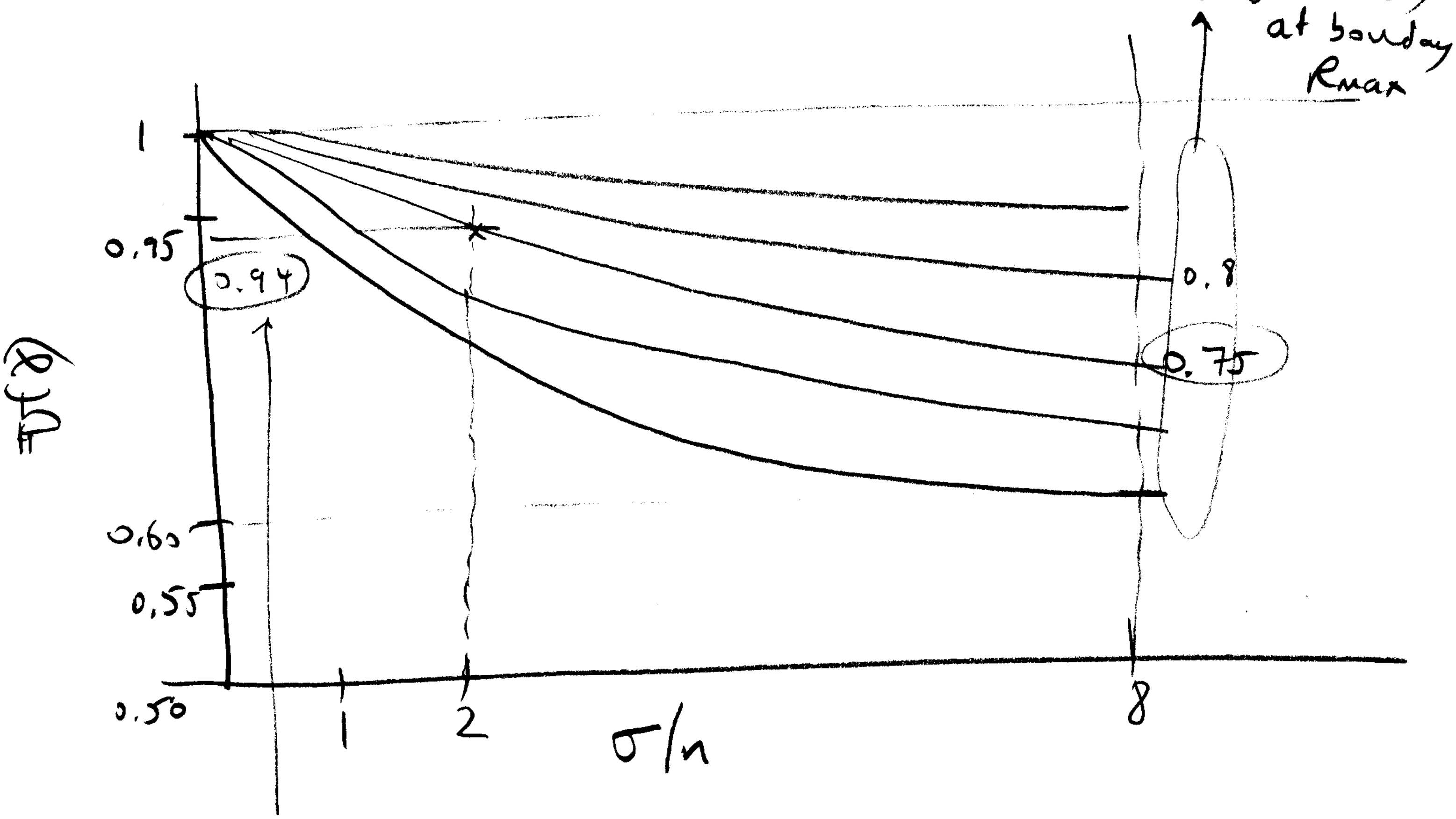
↑ condition on r

$$= \frac{1}{\pi R_{max}^2} \int_0^{2\pi} \int_0^{R_{max}} P\{P_r(r) > \gamma\}\, r\, dr\, d\theta$$

$$P_r(r) > \gamma = Q\left(\frac{\gamma - \overline{P}_r(r)}{\sigma}\right)$$

see Rappaport

$$= \frac{1}{2}\left[1 + e^{1/b^2} \cdot \left(1 - \operatorname{erf}\left(\frac{1}{b}\right)\right)\right]$$



e.g. For $\sigma = 8\,dB$

$n = 4.$

$\sigma/n = 2$: 0,75 = boundary coverage $\Longleftrightarrow$ 94% area coverage

Part II: TRAFFIC and QUEUING

- Independent of multiplexing technique, traffic/queuing theory characterizes the efficiency of channel usage.

Assumptions: Assume that:

(A) 1 The number of users is very large

2 The rate of service requests from each user is small.

Then, the service requests ("arrivals") can be considered to be random, and independent, and approximated by a Poisson process.

(B) The holding times (i.e. how long you are on the phone) is also assumed to be random & independent.

(C) Assume that the ~~[illegible]~~ traffic process is stationary in time. ~~[illegible]~~ & assume that it is ergodic.

- Erlang: (a dimensionless unit); Measure of traffic (intensity) ~~[illegible]~~ carried.

Ex) A radio channel that is occupied for 30 minutes during 1 hour carries 0.5 Erlangs of traffic.

[- used mainly for voice traffic.]

- Trunked radio system:

  ≡ Each user is allocated a channel on a per-call basis

  [upon termination, the channel is returned to the pool of available channels.]

- Grade of service: ≡ Measure of the ability of a user to access a trunked system during the busiest hour.

- ↕ inversely proportional to

  the probability that a call is blocked. (i.e. a call is ~~[illegible]~~ placed, but due to 0 # of channels being available, user cannot ~~[illegible]~~ make the call.)

- Traffic intensity: (offered by a user)

  = call request rate × Holding time

$$\underbrace{A_u}_{\text{Erlangs}} = \mu \cdot H$$

Erlangs — dimensionless

μ — average # of call requests per second or minute

H — (seconds or minute)

- System with U users, the total traffic intensity

$$A = U A_u = U \cdot \underbrace{\mu \cdot H}$$

(assuming same for each user)

- Traffic intensity per channel.

$$A_c = \frac{U A_u}{C}$$

C — # of channels

- Maximum amount of traffic carried by a system is C Erlangs. (C is the # of channels)

Ex AMPS (analog, 1G): $\overbrace{\text{GOS}}^{\text{grade of service}}$ of 2% blocking.

(during busiest hour)

Trunked systems

(Lost) Blocked calls cleared (LCC)

Blocked calls queued. (Lost calls delayed) (LCD)

• user may retry the call.

GOS: $\boxed{\text{blocking probability.}}$

GOS: P{calls will have delays > t seconds.}

Ⓐ LCC systems: (Blocked calls cleared).

— Assume that service time's (holding times) are exponentially distributed.

— Finite # of channels (C)

A: total load offered.

— calls arrive ~ Poisson process

C channels

U users — over all users

λ: total mean call arriving rate (per unit time) [arrival rate]

H: average holding time.

A: total offered load.

A_U: ~~avg~~ average load (per user)

$(A_U = \lambda_1 \cdot H)$

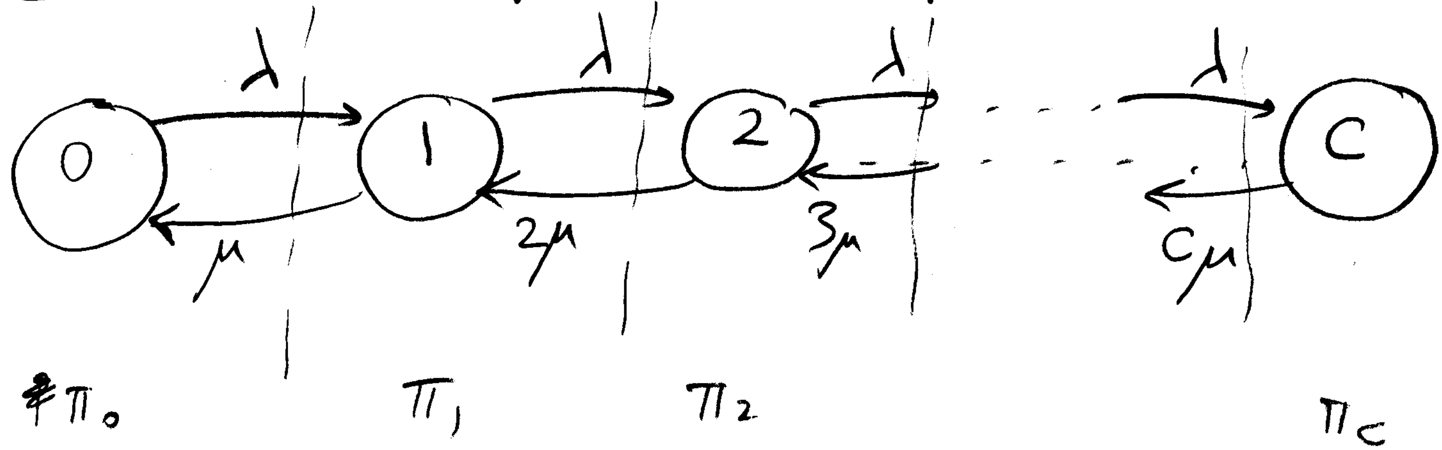
$\uparrow$ arrival rate of user 1.

$$\boxed{A = U A_u = \lambda H}$$

$P\{\text{blocking}\} = P\{\text{none of the channels free } \cancel{\text{you access}}\ \cancel{\text{(post)}}\}$

- Markov process:

each state $\longleftrightarrow$ # of channels occupied in system:



Then $P\{\text{blocking}\} = P\{\text{state } c\}$.

1 Solve $\Pi P = \Pi$ (steady state probabilities) and look at Π_c..

2 or write the balance equations for each cut in the flow graph.

$$\lambda \pi_0 = \mu \cdot \pi_1 \qquad \longrightarrow \pi_1 = \left(\frac{\lambda}{\mu}\right)\pi_0$$

$$\lambda \pi_1 = 2\mu \cdot \pi_2 \qquad \longrightarrow \pi_2 = \left(\frac{\lambda}{2\mu}\right)\pi_1 = \frac{\lambda^2}{2\mu^2}\pi_0$$

$$\lambda \pi_2 = 3\mu \pi_3 \qquad \longrightarrow \pi_3 = \left(\frac{\lambda}{3\mu}\right)\pi_2 = \frac{\lambda^3}{3.2.1.\mu^3}\pi_0$$

$$\vdots$$

$$\lambda \pi_{i-1} = i\mu \cdot \pi_i \quad 1 \le i \le c \qquad \longrightarrow \pi_i = \frac{\lambda^i}{i!\,\mu^i}\pi_0$$

and $\sum_{i=0}^{c} \pi_i = 1$

Now, $$\sum_{i=0}^{c}\left[\frac{\lambda^i}{i!\,\mu^i}\right]\pi_0 = 1.$$

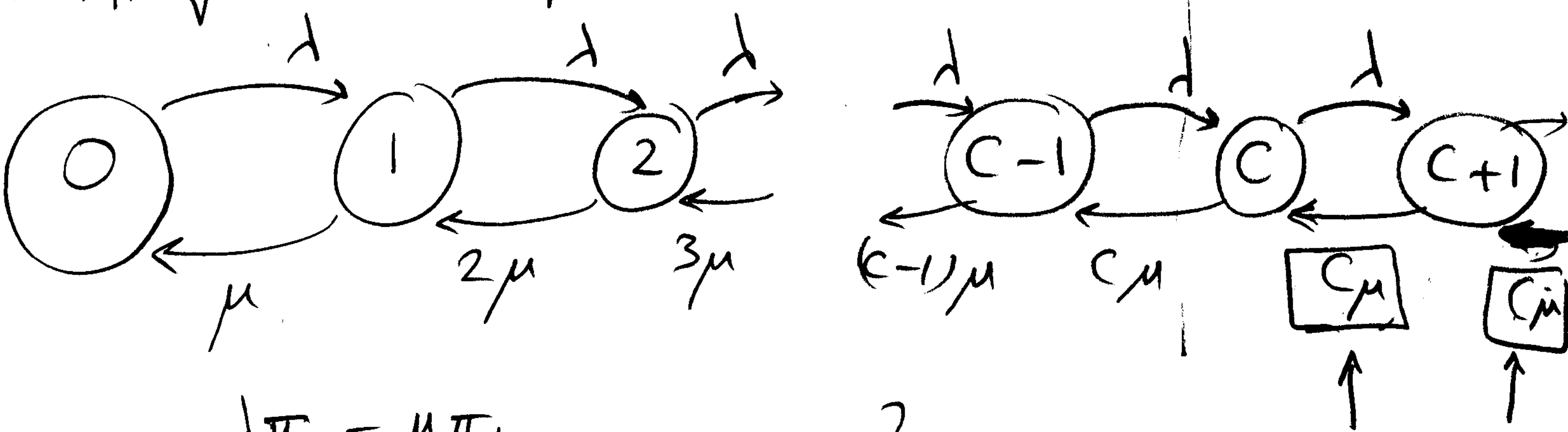
$$\pi_0 = \frac{1}{\sum_{i=0}^{c}\frac{\lambda^i}{i!\,\mu^i}}.$$

Erlang B formula

$$\boxed{\pi_c = \frac{\frac{\lambda^c}{c!\,\mu^c}}{\sum_{i=0}^{c}\frac{\lambda^i}{i!\,\mu^i}}} = \frac{\left(\underbrace{\frac{\lambda}{\mu}}_{A}\right)^c \cdot \frac{1}{c!}}{\sum_{i=0}^{c}\left(\underbrace{\frac{\lambda}{\mu}}_{A}\right)^i \frac{1}{i!}}$$

(B) Lost call delayed (LCD ~~[illegible]~~ discipline)

mk :: # of channels occupied.



$$\lambda\pi_0 = \mu\pi_1$$

$$\lambda\pi_1 = 2\mu\pi_2$$

$$\lambda\pi_2 = 3\mu\pi_3$$

$$\vdots$$

$$\lambda\pi_{c-2} = (c-1)\mu\cdot\pi_{c-1}$$

$$\lambda\pi_{c-1} = c\mu\cdot\pi_c$$

$$\boxed{\lambda\pi_{i-1} = i\mu\cdot\pi_i \qquad 1 \le i \le c.}$$

$$\lambda\pi_c = \mu\cdot c\cdot\pi_{c+1}$$

$$\boxed{\lambda\pi_i = \mu\cdot c\cdot\pi_{i+1} \qquad c \le i < \infty.}$$

$$\vdots$$

$$\sum_{i=0}^{\infty}\pi_i = 1$$

Solving for π_0: $$\pi_0 = \frac{1}{\sum_{i=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^i\frac{1}{i!} + \frac{1}{i!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{1}{1-\frac{\lambda}{\mu c}}\right)}$$

$$\boxed{\pi_i = \begin{cases} \left(\frac{\lambda}{\mu}\right)^i \frac{1}{i!}\, \pi_0 & 0 \le i \le c \\ \frac{1}{c!} \left(\frac{\lambda}{\mu}\right)^i \frac{1}{c^{i-c}}\, \pi_0 & i \geq c \end{cases}}$$