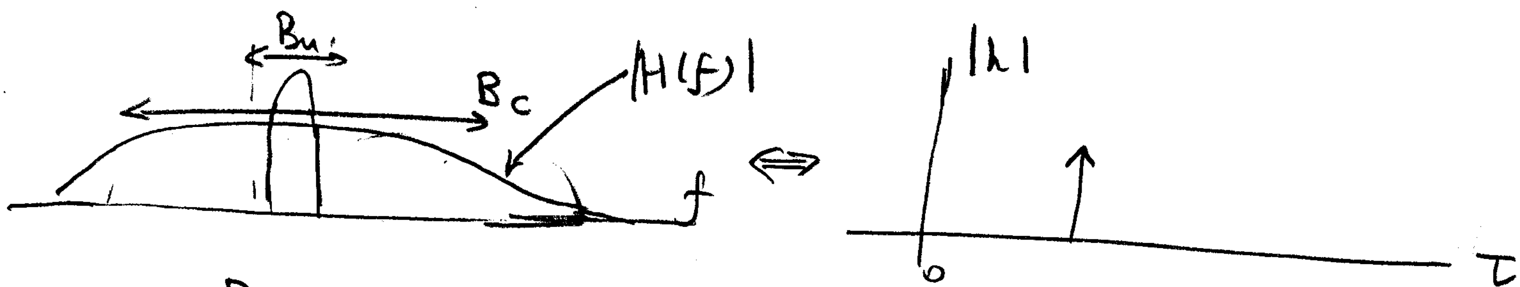
LECTURE #5

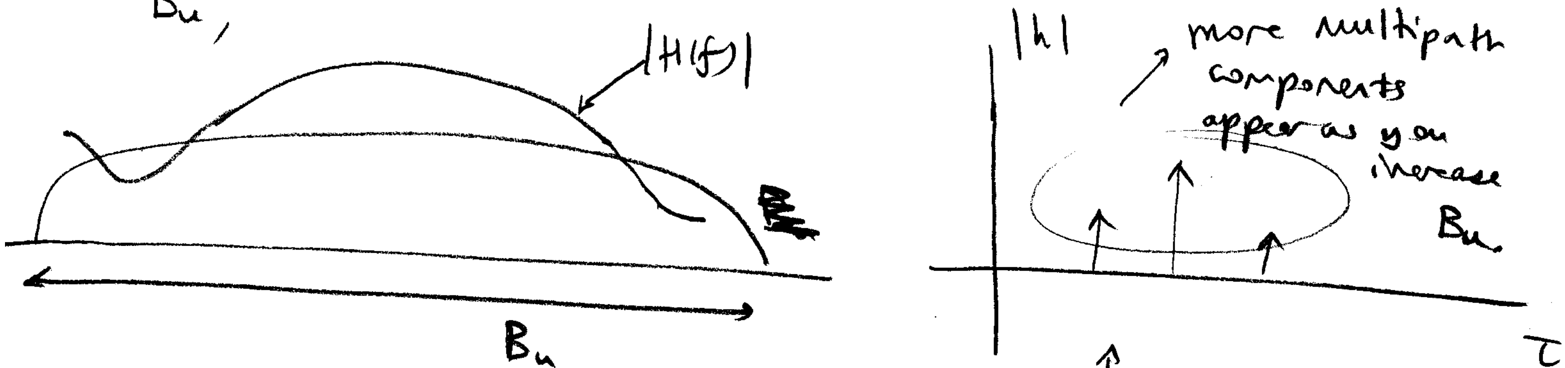
- Last time we talked about the multipath delay profile of a (stationary) wireless channel.
  (static)

• Resolvable multipath components.



$B_u << B_c \Rightarrow$ flat channel

Now, as you increase the B_u,

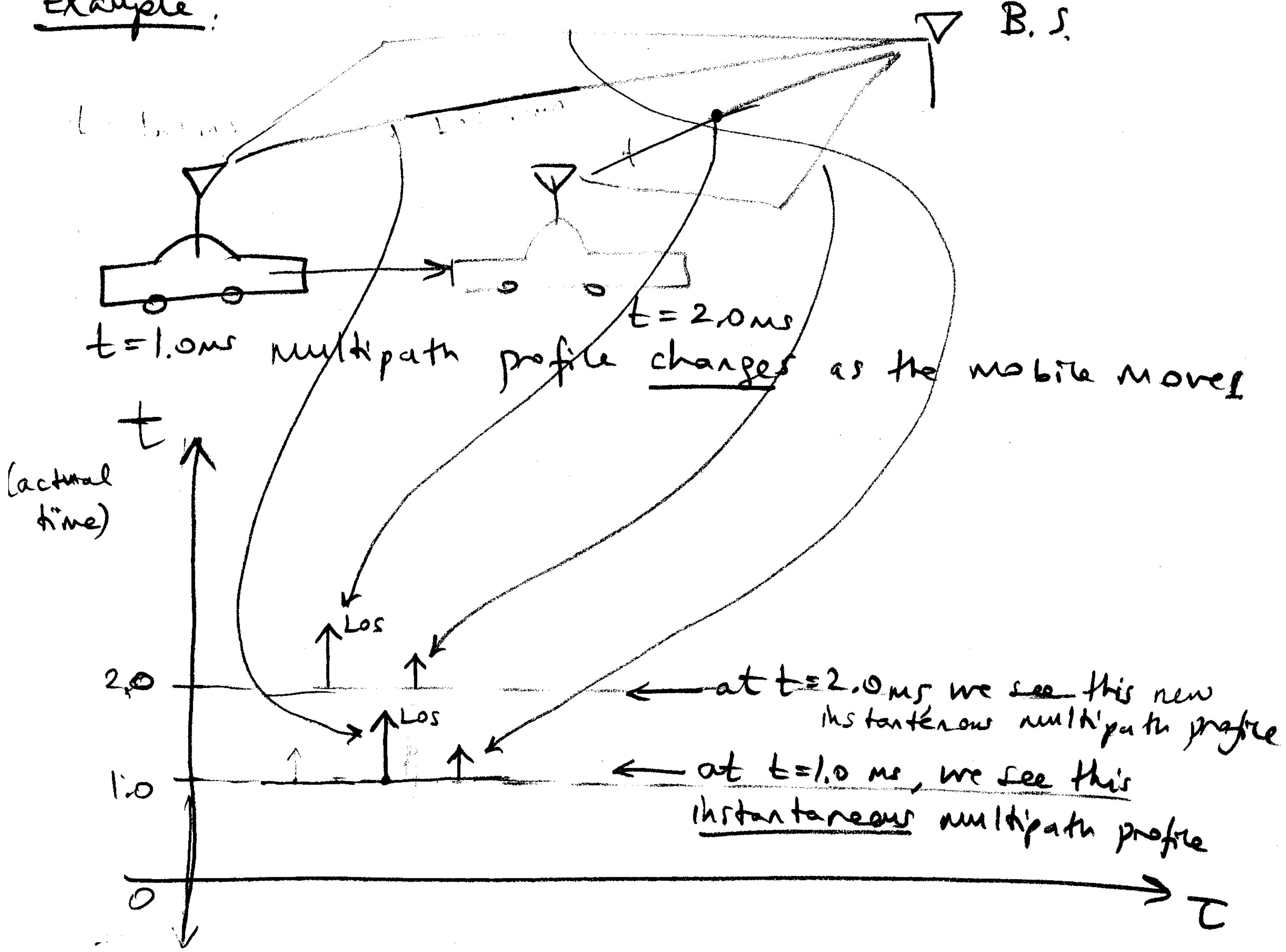


- you will be able to resolve more multipath components.

*It is important that channel sounding is done with at least the B_u that will be used later in the channel

— Now, we introduce <u>mobility</u> of the user into the model. This introduces small-scale variations as the mobile user travels through a range of multipath profiles.

<u>Example</u>:

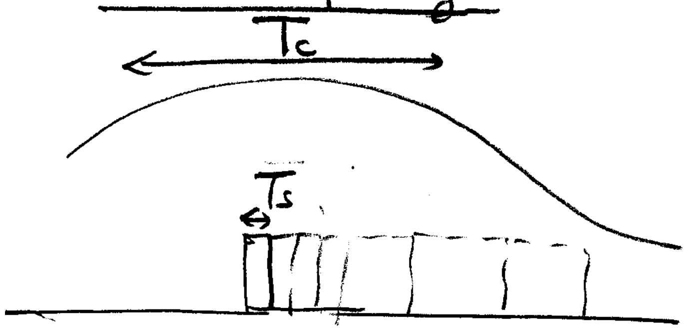


Coherence time of a channel = the amount of duration (in t above not τ!) over which the multipath profile remains roughly constant.

T_c = coherence time.

- Fading : refers to the variations in channel (or in received signal strength) due to mobility. [Fading is a result of the changes in the multipath profile of the channel.]

Slow fading

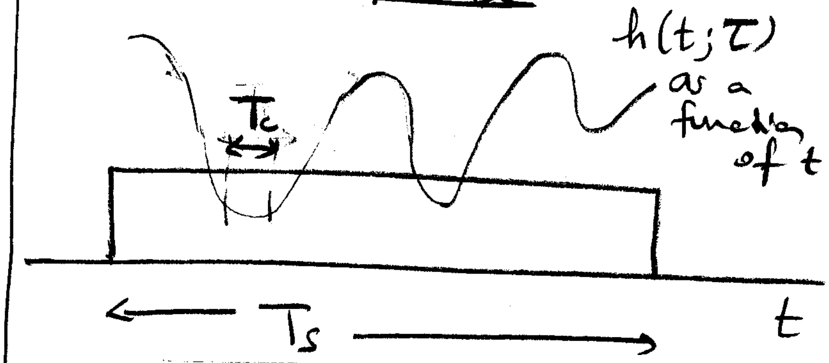


$T_s \ll T_c$

Then each symbol (and many symbols in a row) see the same $h(t;\tau)$ multipath profile.

"SLOW FADING"

Fast fading



(Symbol duration)

If $T_s > T_c$

(symbol duration > coherence time)

- channel impulse response different for different parts of the symbol.

"FAST FADING"

These are 2 distinct regimes that require different techniques.

— Slow fading

1 can actually attempt to track the channel

(& perform equalization across subsequent symbols if the channel is frequency selective.)

2 channel estimation useful.

3 power control on a symbol-by-symbol basis makes sense — based on feedback, can do power control and/or equalization based on feedback.]

Fast fading

— not much you can do.

Try to design robust modulation schemes.

(i.e. if the amplitude $|h(t;\tau)|$ is varying, but phase is remaining same longer, then use angle/frequency modulation schemes rather than amplitude modulation.

[acoustic channel is an example. Bandwidth W limited. $\therefore T_s \simeq 1/W$ must be longer

— Channel changes fast.]

– frequency domain interpretation of fading:

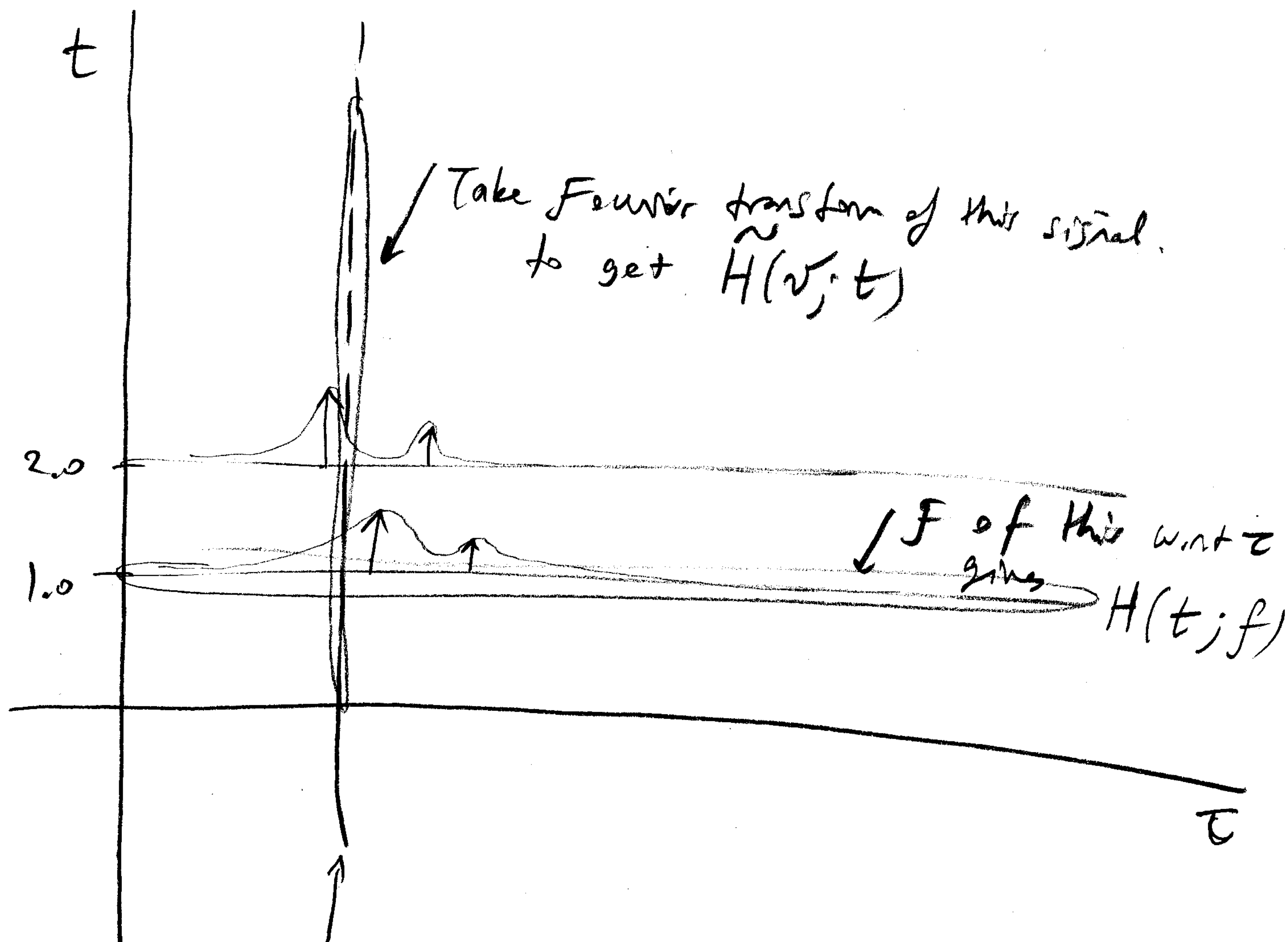
Doppler spread of a channel: $\boxed{B_D \approx \frac{1}{T_c}}$

T_c: coherence time.

Function to look at: Fourier transform w.r.t. t.

$$\tilde{H}(\nu; \tau) = \mathcal{F}_t\{h(t;\tau)\}$$

ν: corresponding frequency domain variable.

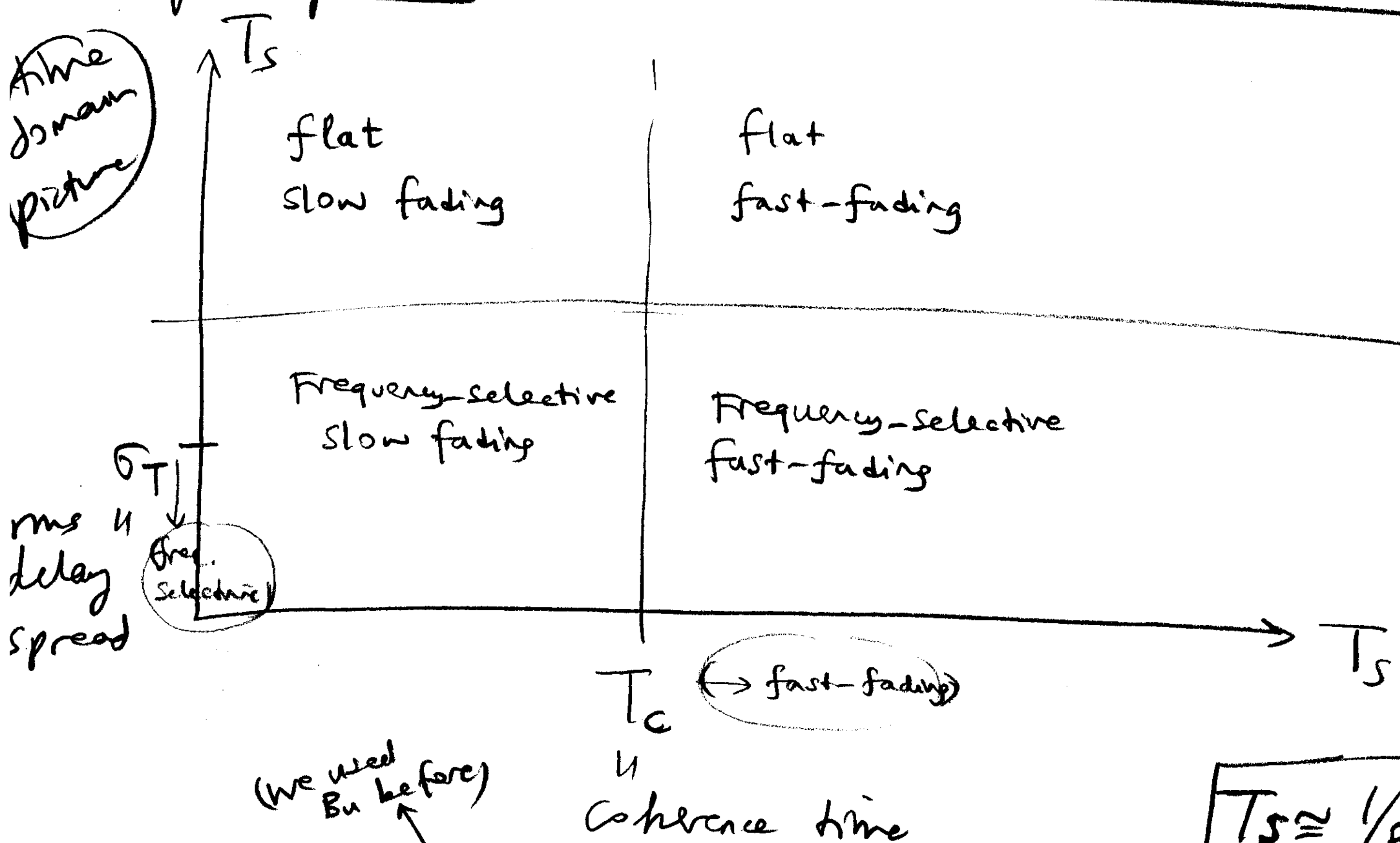


fix τ and take the Fourier transform of the signal you see here (as a fn of t) $t \longrightarrow \nu$.

[Do not confuse ν and f]

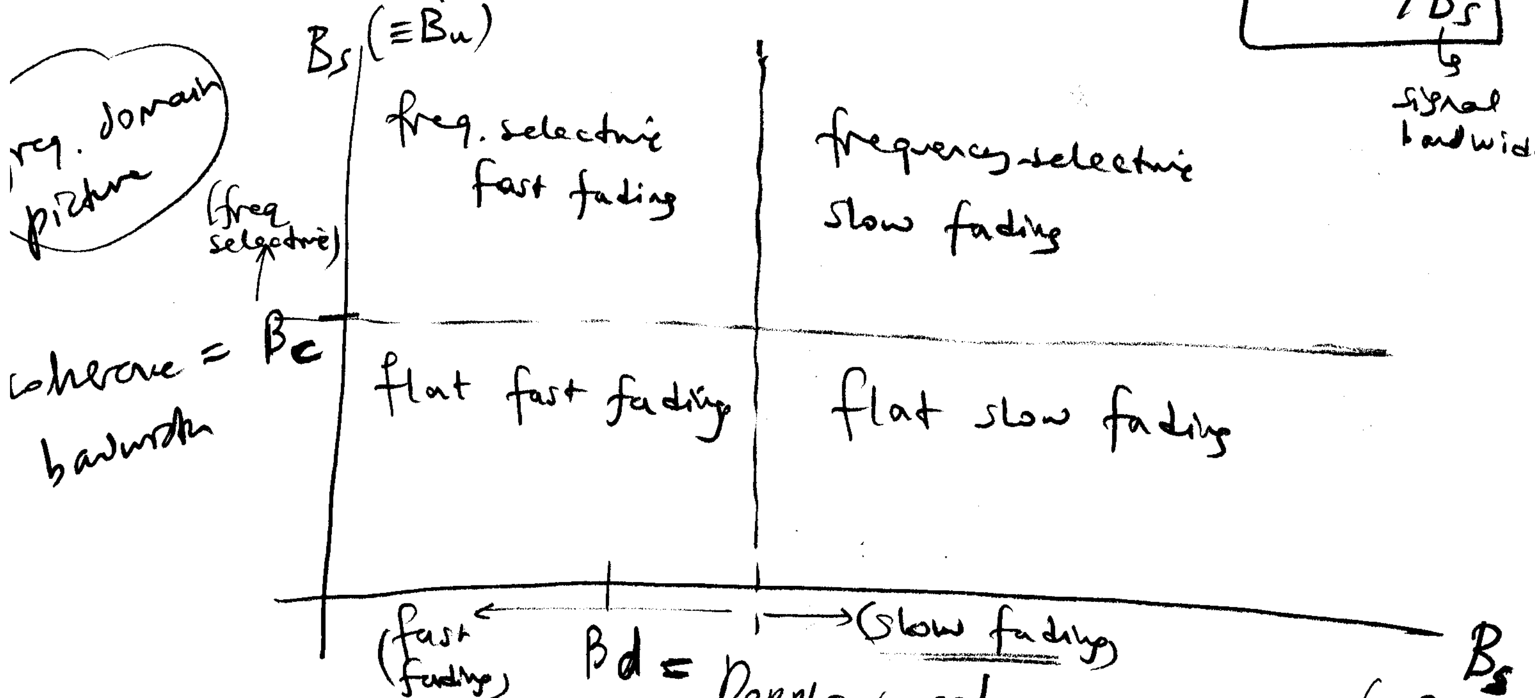
$$\nu \overset{\mathcal{F}}{\longleftrightarrow} t \qquad f \overset{\mathcal{F}}{\longleftrightarrow} \tau$$

Conceptual picture:



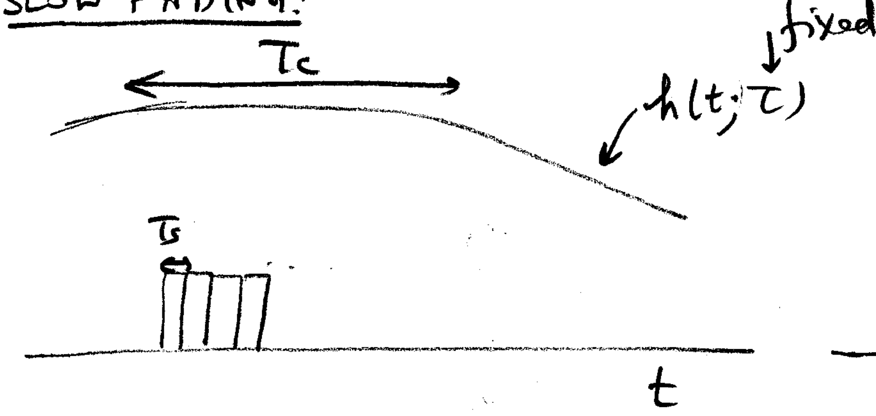
$\boxed{T_S \cong 1/B_s}$

signal bandwidth



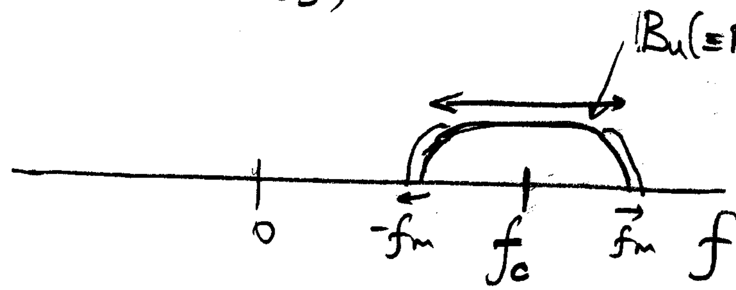
Frequency domain interpretation of Doppler shift:

SLOW FADING:



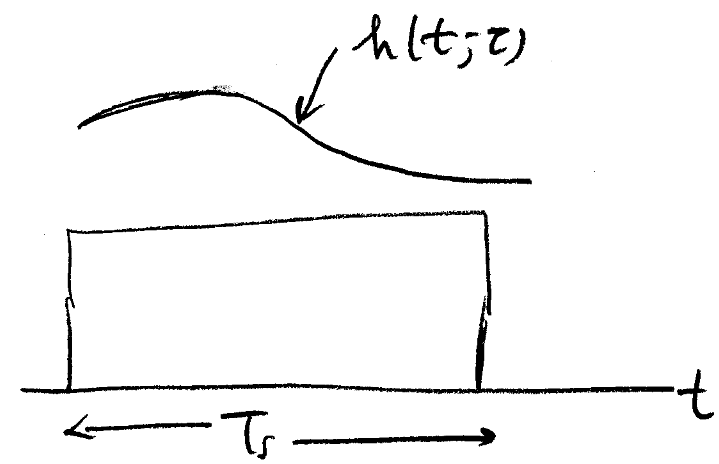
$T_c \gg T_s$

$B_u \gg B_d$
(B_s)



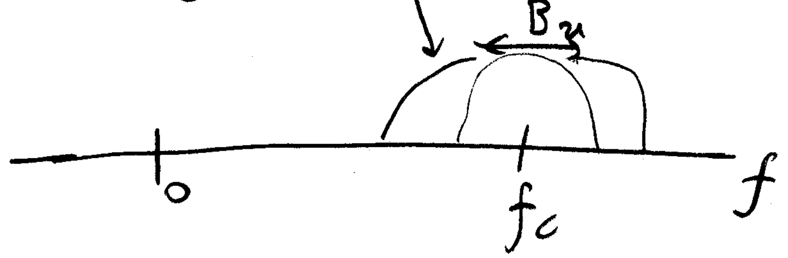
Small spectral broadening and here small smearing by Doppler shift

FAST FADING:



$T_c < T_s$

spectral broadening & smearing comparable to B_u

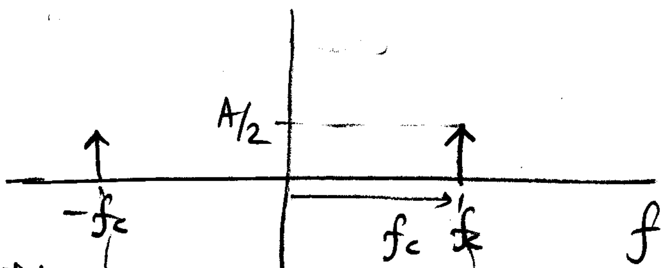


$B_u < B_d$

Question: When the Rx and Tx are both stationary, what is the coherence time?

Understanding Doppler shift:

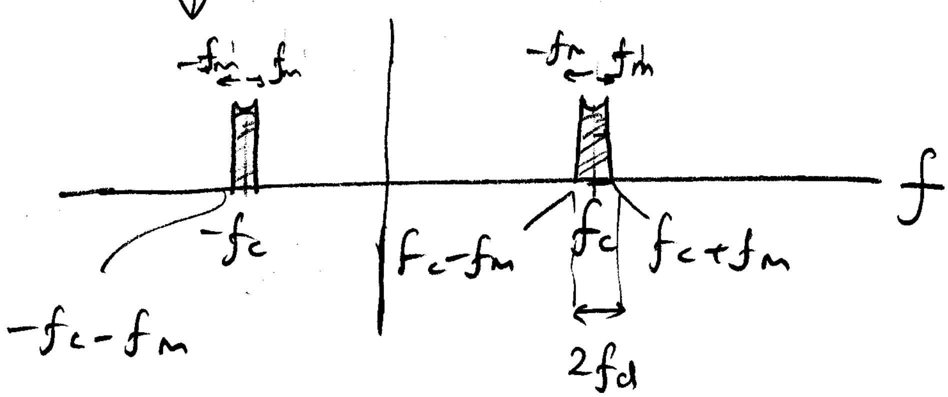
- Doppler shift occurs due to the motion of the tx or rx, we will assume here that the Tx is fixed and Rx is mobile. [



carrier:

$$A\cos(\underbrace{\omega_c}_{2\pi f_c} t) \xleftrightarrow{\mathcal{F}} A\cdot\left[\tfrac{1}{2}\delta(f-f_c)+\tfrac{1}{2}\delta(f+f_c)\right]$$

Send across channel [mobile Rx]
with



- If the Rx antenna is moving, you will see a Spectral broadening of the signal at the receiver.
- The spectral broadening occurs due to the fact that different paths experience different Doppler shifts (based on the angle of arrival at the mobile.

$f_m \equiv$ maximum Doppler shift in any arriving wave (multipath component)

$$\boxed{f_m = \frac{v}{\lambda}}$$

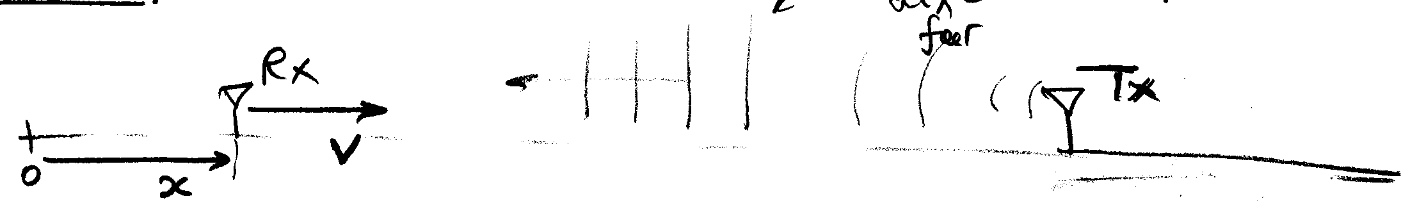
$v \rightarrow$ velocity of the mobile

$\lambda \rightarrow$ wavelength of the signal transmitted.

Why?

<u>Examine</u>:

plane wave approximation at far distances.



• Write down the Electric field at the receiver:

$$\mathcal{E}_{Rx}(x,t) = E_0 \cos(\omega t + \beta x)$$ [electrical field measured at time t and location position x.]

[x: position of the Rx antenna.]

$$\left[\beta \equiv \frac{\omega}{c}\right]$$

$\omega \rightarrow 2\pi f_c$

$c \rightarrow$ speed of light.

Now, $x = v \cdot t$ (assuming $x(0) = 0$.)

$v \uparrow$ velocity of the mobile

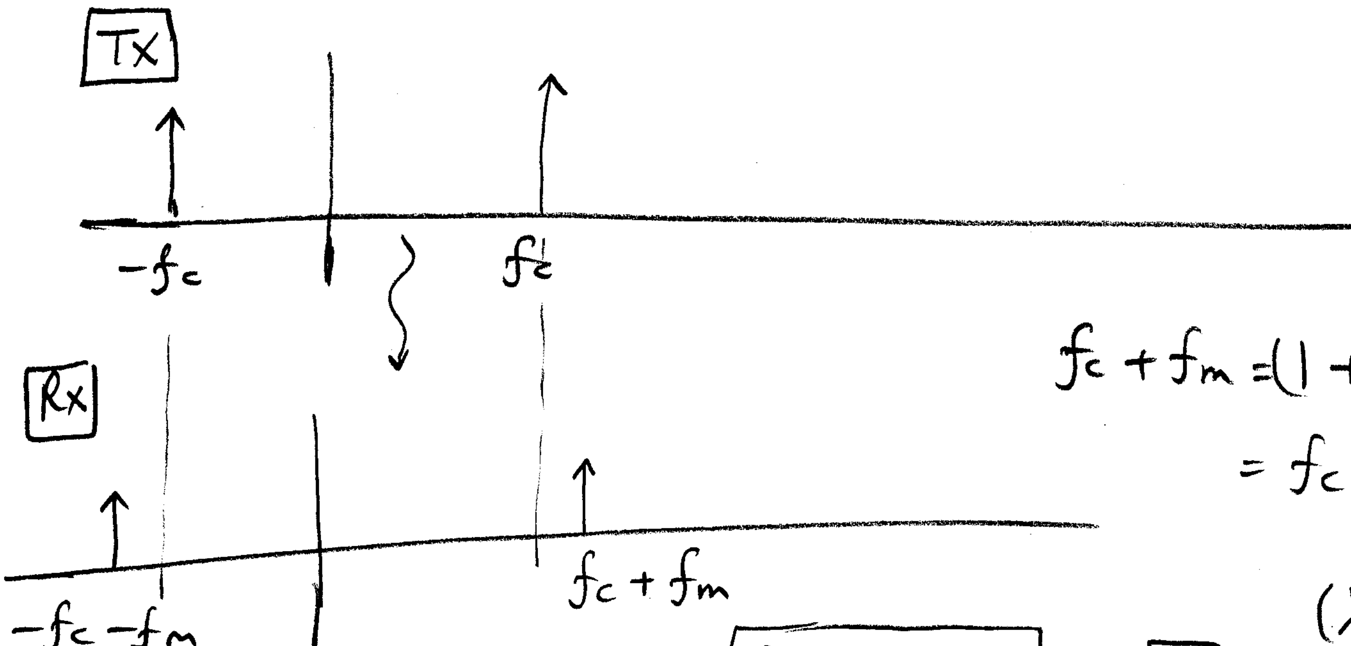
Then:

$$\mathcal{E}_{Rx}(x,t) = E_0 \cos(\underbrace{\omega t + \overset{\omega/c}{\beta} v t}_{\omega\left(1 + \frac{v}{c}\right)t})$$

Hence, at the Rx antenna, the frequency of the received wave is higher by a factor $\left(1+\frac{v}{c}\right)$.

[We expect it to be higher because Rx is moving **towards** the Tx antenna: encounters the wave fronts at a faster rate.]

Then, in frequency domain:



$$f_c + f_m = \left(1+\frac{v}{c}\right) f_c$$
$$= f_c + \frac{v}{c} f_c$$

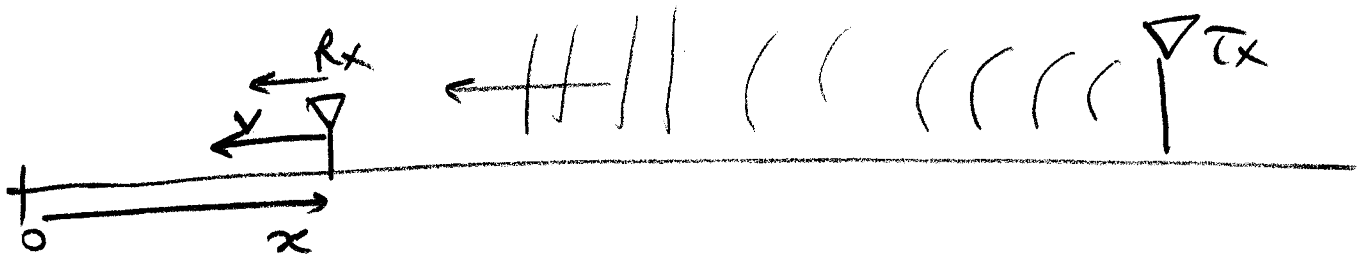
$(\lambda f_c = c)$

where $\boxed{f_m = \frac{v}{c} f_c} = \boxed{\frac{v}{\lambda}}$

- Here, we have a <u>Doppler shift</u> of $+f_m$ Hz.

<u>No</u> spread here. (Spread comes from having <u>multiple</u> wave components experiencing different shifts.), but the maximum shift can be $+\frac{v}{\lambda}$ Hz.

Another extreme condition:



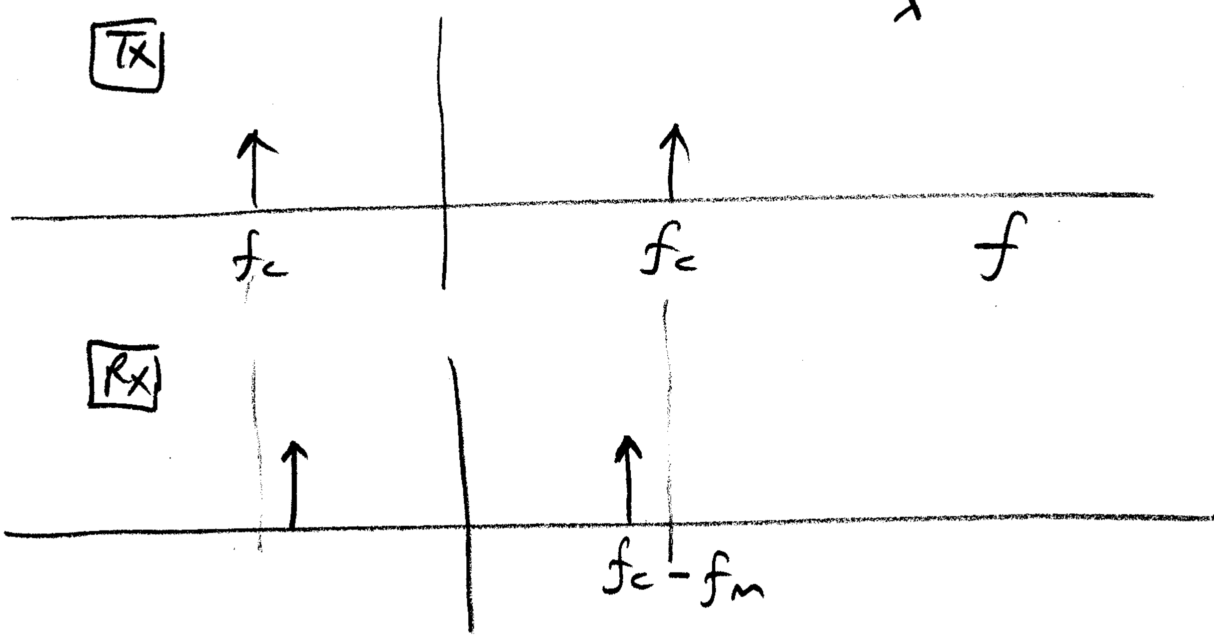
$$E_{Rx}(x,t) = E_0 \cos(\omega t + \beta x),$$

$x = -vt$ (assuming that $x(0) = 0$.)

Then:

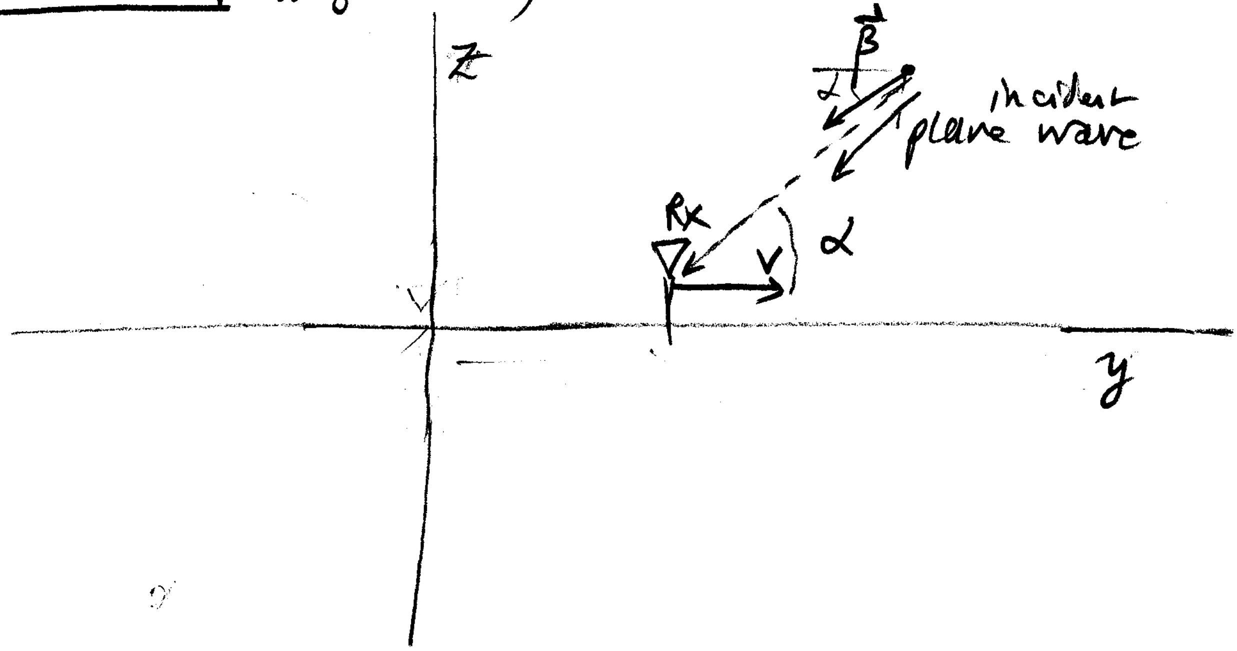
$$= E_0 \cos(\underbrace{\omega t - \overset{\omega/c}{\overset{\|}{\beta}}\, v t}_{\omega(1-\frac{v}{c})t})$$

Then we see a shift of $-f_m = -\frac{v}{\lambda}$ Hz.



$$f_m = \frac{v}{c} \cdot f_c = \frac{v}{\lambda}.$$

general case: (single wave) 1



Then:

$$\vec{E} = E_0 \cdot e^{-j \vec{\beta} \cdot \vec{r}}$$

complex vector notation wave

$$\vec{\beta} = \beta \cdot \left[-(\cos\alpha)\hat{y} - (\sin\alpha)\hat{z} \right]$$

$$= E_0 \cdot e^{-j\left[-(\cos\alpha) y - (\sin\alpha) z\right] \cdot \beta}$$

$$\mathcal{E}_1 = Re\{\vec{E}\, e^{j\omega t}\}$$

$$= E_0 \cdot \cos(\omega t + \beta z \sin\alpha + \beta y \cos\alpha)$$

[This is the electric field measured at location (y, z) at time t.]

Now, at mobile receiver: $z = 0$, and $y = vt$ (assume $y(0) = 0$).

Then

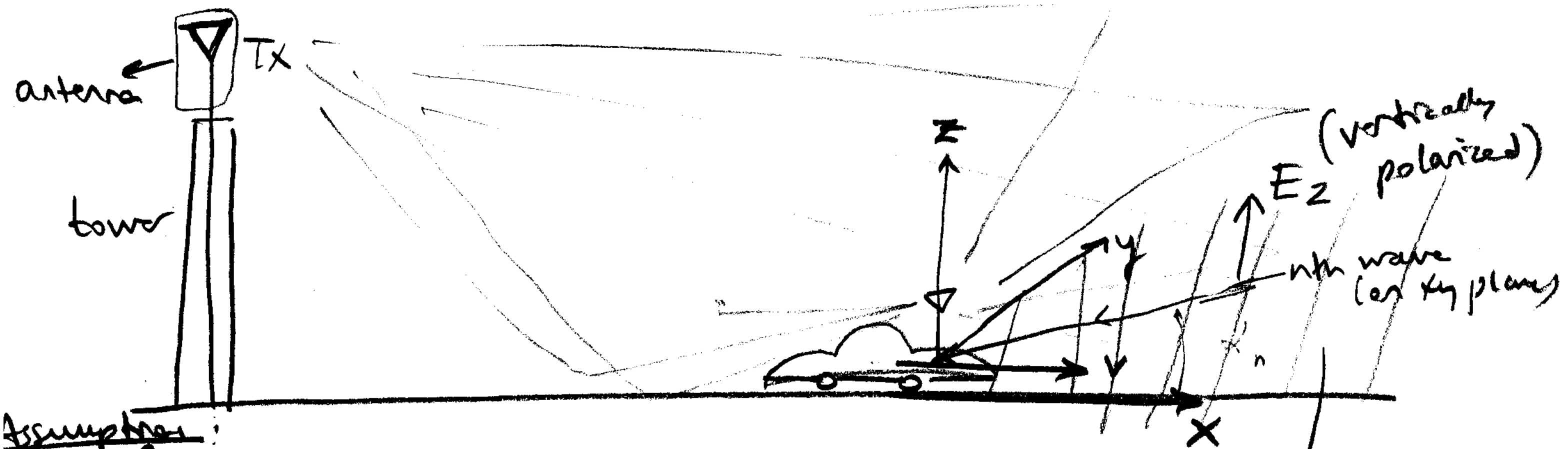
$$\mathcal{E}_{Rx} = E_0 \cos(\omega t + \beta\cos\alpha \; y)$$

$$= E_0 \cos\left(\omega\left(1 + \frac{v}{c}\cos\alpha\right) t\right)$$

$\boxed{\frac{v}{\lambda}\cos\alpha}$ = Doppler shift of

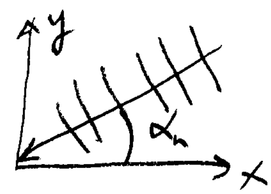
Statistical models for multipath fading channels

- Clarke's model for flat fading:



Assumptions:

① · fixed Tx
· vertically polarized antenna.



②. no LOS path.

- each multipath component that arrives at the mobile's antenna has equal average power (or amplitude)

Assume that there are N waves arriving at the mobile. (multipaths)

– The nth [multipath component/wave] arrives at an angle of α_n degrees to the x axis.

- The Doppler shift that this wave experiences (in frequency) due to the motion of the mobile is:

$$f_n \equiv \frac{v}{\lambda} \cos \alpha_n \quad (Hz)$$

• Superposition of N waves at receiver:

electric field at receiver antenna (vertically polarized)

$$E_z = E_0 \sum_{n=1}^{N} C_n \cdot \cos\left(\boxed{\omega_c \cdot \left(1 + \frac{v}{c}\cos\alpha_n\right)t + \phi_n}\right)$$

phase shift component for nth wave

[comes from:

$$\mathcal{E} = Re\left\{ E\, e^{j\omega(t-\tau_n)} \right\}$$

for nth wave.

①

$$\underbrace{\omega_c t + \underbrace{\overset{2\pi f_c}{\cancel{\omega_c}} \cdot \frac{v}{c}}_{} \cdot \cos\alpha_n t + \phi_n}_{2\pi \cdot \frac{v}{\lambda}\cos\alpha_n t + \phi_n}$$

Recall: $f_n = \frac{v}{\lambda}\cos\alpha_n$

$$= \omega_c t + \underbrace{\boxed{2\pi f_n t + \phi_n}}_{\equiv\, \theta_n}$$

② $E_0 \cdot C_n$: amplitude of electric field (at Rx) of nth wave

$$\sum_{n=1}^{N} \overline{C_n^2} = 1$$

$$\overline{C_n^2} \equiv (E[C_n^2])$$

(i.e., choose E_0 such that this is true.)

Then:

$$E_z = E_0 \sum_{n=1}^{N} C_n \cdot \cos(2\pi f_c t + \theta_n)$$

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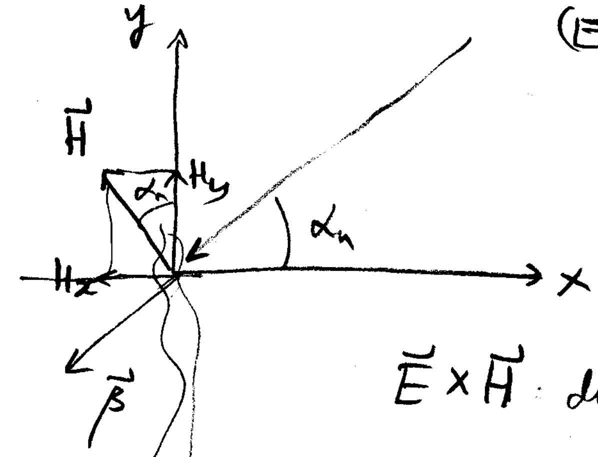
$$E_z = E_0 \cdot \sum_{n=1}^{N} C_n \cdot \cos(2\pi f_c t + \theta_n)$$



(E_z coming out of the plane)

$\vec{E} \times \vec{H}$: direction of motion

$$H_x = -\frac{E_0}{\eta} \sum_{n=1}^{N} C_n \cdot \sin\alpha_n \cdot \cos(2\pi f_c t + \theta_n)$$

(intrinsic impedance of free space)

$$H_y = -\frac{E_0}{\eta} \sum_{n=1}^{N} C_n \cos\alpha_n \cdot \cos(2\pi f_c t + \theta_n)$$

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1) If $f_m \ll f_c$, then the three field components can be modeled as narrowband random processes.

2) If N large, since $\sum (C_n \ldots)$ iid, $\longrightarrow$ Gsn r.v.

if $\theta_n \sim U(0, 2\pi)$

Then

$$E_z \cong \underset{\substack{\uparrow \\ \text{in-phase} \\ \text{component}}}{T_c(t)} \cos(2\pi f_c t) - \underset{\substack{\uparrow \\ \text{quadrature} \\ \text{component}}}{T_s(t)} \sin(2\pi f_c t)$$

where:

$$T_c(t) = E_0 \cdot \sum_{n=1}^{N} C_n \cos(\overbrace{2\pi f_n t + \phi_n}^{\theta_n})$$

and

$$T_s(t) = E_0 \sum_{n=1}^{N} C_n \sin(2\pi f_n t + \phi_n)$$

By LLN, $T_c(t)$ and $T_s(t)$ are Gaussian ~~of~~ random processes.

At any time t, $T_c(t) \overset{\text{uncorrelated}}{\perp} T_s(t)$

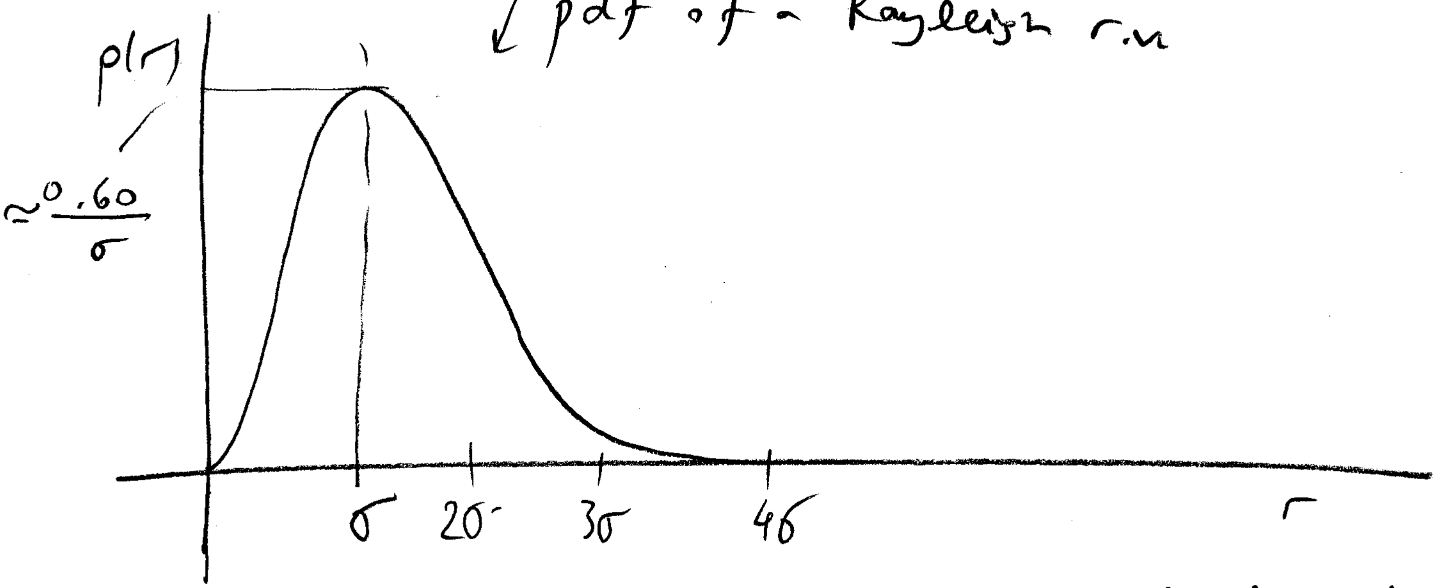
$$E[T_c(t)^2] = E[T_s(t)^2] = \frac{E_0^2}{2}.$$

$$|E_z(t)| = \sqrt{\underset{\substack{\uparrow \\ \text{Gsn r.v.}}}{T_c^2(t)} + \underset{\substack{\uparrow \\ \text{Gsn r.v.}}}{T_s^2(t)}} \equiv \underset{\substack{\uparrow \\ \text{Rayleigh r.v.}}}{r(t)}$$

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} e^{-r^2/2\sigma^2}, & 0 \le r < \infty \\ 0 & r < 0. \end{cases}$$

$$\sigma^2 \equiv \frac{E_0^2}{2}.$$

pdf of a Rayleigh r.v.



<u>Rayleigh fading distribution</u>:

- use envelope detection to detect the signal

$$P(\gamma) \equiv P\{\underset{\substack{\uparrow \\ \text{envelope}}}{R} \le \gamma\} = \int_0^{\gamma} p(r)\, dr = \boxed{1 - e^{-\gamma^2/2\sigma^2}}$$

$$E[\underset{\substack{\uparrow \\ \text{Rayleigh}}}{R}] = \int_0^{\infty} r\, p(r)\, dr = \sigma \cdot \sqrt{\frac{\pi}{2}} \cong \boxed{1.25\, \sigma}$$

$$Var(R) = E(R^2) - (E(R))^2 \simeq \sigma^2(2 - \pi/2) = \boxed{0{,}43\sigma^2}$$

median $(R) = ?$

$$\frac{1}{2} = \int_0^{r_{median}} p(r)\, dr$$

$$\Rightarrow \boxed{r_{median} = 1.177\sigma} \quad (< E(R))$$

<u>Ricean Fading distribution</u>:

— Assumes a non fading LOS component.

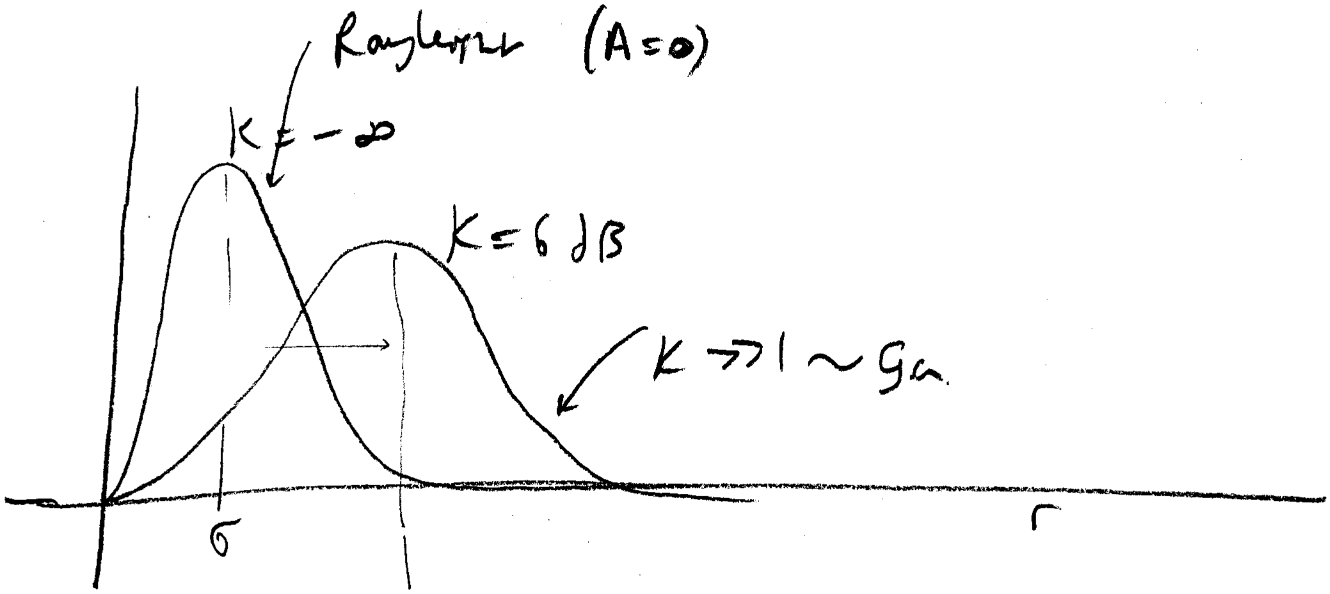
— superimpose on this signal

(dominant component $\to 0$, Ricean $\to$ Rayleigh)

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} e^{-(r^2+A^2)/2\sigma^2} \cdot \underbrace{I_0}_{\text{Bessel 1st kind, 0th order}}\left(\dfrac{Ar}{\sigma^2}\right), & A \geq 0,\ r \geq 0 \\ 0 & r < 0. \end{cases}$$

$A \equiv$ peak amplitude of dominant signal

Define: $K \equiv \dfrac{A^2}{2\underbrace{\sigma^2}_{\text{variance of multipath}}}$ $\qquad 10 \log_{10} A^2/2\sigma^2$. dB



- A shifts it to the ~~left~~ right.

- For $K >> 1$ – $\boxed{\text{Ricean} \rightarrow \text{Gaussian about the \underline{mean}}}$