Name:____

ECE 250-Spring 2005

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FINAL EXAMINATION

INSTRUCTIONS:

1. READ THIS PAGE THOROUGHLY WHEN YOU RECEIVE IT, BUT DO NOT START TURNING TO THE OTHER PAGES UNTIL YOU ARE INSTRUCTED TO DO SO.

2. On other problems, PARTIAL CREDIT will be given only to true statements that make progress towards the correct answer. Partial credit may be given to correct reasoning in developing structures (such as tables, etc.) in which the variables are clearly labeled. NO partial credit will be given for incorrect statements or statements to which no truth value can be assigned (such as a bunch of numbers or algebraic expressions). NO partial credit will be given for statements that use symbols that the problem statement or you have not defined. NO credit will be given for any work that is not clearly labeled with the part and problem number to which this work provides an answer.

3. All the exam rules in the course syllabus apply to this exam.

4. You may remove the staple from the exam pages, if is more convenient. We will provide a stapler at the end of the exam.

5. There is a total of 100 points on this exam: The distribution of points is as follows:

Problem 1:	10 points
Problem 2:	20 points
Problem 3:	20 points
Problem 4:	20 points
Problem 5:	30 points

6. The exam is open-book. You may use calculators, but laptops are not allowed.

7. You have 3 hours to complete this exam.

8. The Honor Code will be strictly observed.

9. Turn in your scratch work SEPARATELY from the paper you want graded. Each has to be stapled, and your name should appear on each. The scratch work should say "SCRATCH WORK" on it.

Problem 1

[redacted]pts

10

(1) Assume that a vertically polarized omnidirectional receiving antenna is moving at a uniform speed of 50 ft/sec through a multipath propagation environment that has a very large number of waves arriving from angles uniformly distributed in azimuth in the horizontal plane. Assume that all arrival angles are confined to the horizontal plane and the amplitudes of the waves are random such that the received signal envelope is Rayleigh distributed. Assume that the transmitted signal is a single frequency sinusoid at frequency f that has a wavelength $\lambda = 1$ foot. The fading signal envelope at the output of the receiver connected to the moving receiving antenna is sampled every 0.0276 seconds. Write an expression for the probability that one or more of the samples out of k samples is greater than 6 dB below the mean of the fading envelope. Explain your solution in a few notes. Make reasonable assumptions and note your assumptions.

Problem 2 ([redacted] points): Frequency-Selective Fading Channels.

This problem illustrates how multicarrier modulation with adaptive loading can achieve significantly higher data rates on FS fading channels than can be attained with equalization. Consider a communication system where the modulated signal $s(t)$ has power 10 mW, carrier frequency f_c, and passband bandwidth $B_s = 40$ MHz. The signal $s(t)$ passes through a frequency-selective fading channel with frequency response

$$H(f) = \begin{cases} 1 & f_c - 20\text{MHz} \leq f < f_c - 10\text{MHz} \\ .5 & f_c - 10\text{MHz} \leq f < f_c \\ 2 & f_c \leq f < f_c + 10\text{MHz} \\ .25 & f_c + 10\text{MHz} \leq f < f_c + 20\text{MHz} \\ 0 & \text{else} \end{cases}$$

The received signal is $y(t) = s(t) * h(t) + n(t)$, where $n(t)$ is AWGN with PSD $N_0 = 10^{-12} W/Hz$.

Determine a multicarrier modulation scheme (FDM: nonoverlapping subchannels) for this channel with optimal power and rate adaptation across subchannels to maximize the total data rate under the total transmit power constraint of 10 mW and an error probability $P_b < 10^{-3}$. Assume no restrictions on the constellation size or power level associated with each subchannel, and assume the subchannel modulation has a symbol duration $T_s = .5/B$, where B is the subchannel bandwidth. What is the maximum data rate of this FDM scheme?

3. Partitioning – 20 [redacted] pts.

A discrete-time channel has response $P(D) = 1 + 2D + D^2 = (1+D)^2$, with input energy per dimension equal to $\bar{\mathcal{E}}_x = 2$, and AWGN variance per dimension $\sigma^2 = 1$.

a). What is the smallest length of a guard period in samples to avoid interference between successive symbols in a DMT system? $\nu = ?$ (1 pts)

b). What should be the symbol length in samples if the excess bandwidth is to be less than 5%? (1 pt)

(rate-adaptive)

c). Estimate to within 20% the subchannels you expect to be used after RA loading is complete [redacted] for the symbol length in part b and with uncoded QAM transmission at $P_e \leq 10^{-6}$? (3 pts)

d). Repeat part c if the channel were changed to $(1-D)^2$. (2 pt)

e). Let T be fixed and let N go to infinity and compute the limiting performance/SNR of the DMT system if all tones are used with equal energy and no bandwidth optimization is used – does this look familiar? Note the the following expression $\frac{1}{6}(1+D)^2(1+D^{-1})^2 + \frac{1}{12}$ has one root with magnitude 1.8415 and phase 2.5657 radians. (3 pts).

[Problem 4] (20 points)

Consider a direct-sequence spread-spectrum CDMA system with bandwidth expansion $B_{exp} = G_s = T_b / T_c$ where G_s is the spreading gain and T_b is the bit period of bit streams used to modulate binary phase shift keyed carriers that are spread by different CDMA codes that have chip periods T_c. Assume a single base station configuration with all CDMA signals received at a power level of P_r, i.e. either on downlinks or power controlled uplinks. Assume that the receiver also has additive Gaussian white noise present at the input and in the despread receiver output the equivalent input *rf* noise power is $N_o W_{eq}$, that is, the equivalent despread signal-to-noise power ratio at the correlator output is $\left(\frac{S}{N}\right)_{rf} = P_r / N_0 W_{eq}$. The number of CDMA signals that can be supported at the same carrier frequency is N_s.

Since CDMA signals with different spreading codes appear like noise at the correlator output, the total correlator output noise is $I + N_0 W_{eq}$ where I is the total interference power in the correlator output due to the $N_s - 1$ other users. The ratio of (signal power) to (noise and interference power) at the correlator output required to detect the signal with the required bit error ratio is $S/N|_{req}$.

The equation for the number of users, N_s, was derived in class with no Gaussian noise present, i.e. for only other user interference.

3a) Derive an equation for N_s in terms of $\left(\frac{S}{N}\right)_{rf}$, $S/N|_{req}$, and B_{exp}.

3b) For low signal-to-receiver noise ratio of, $\left(\frac{S}{N}\right)_{rf} = \left.\frac{S}{N}\right|_{req}$, what is N_s?

3c) For high signal-to-receiver noise, $\left(\frac{S}{N}\right)_{rf} \to \infty$, what is N_s?

3d) If $\left(\frac{S}{N}\right)_{rf.5}$ is the value that reduces the number of users, N_s, to 1/2 the number of users for ∞ value of $\left(\frac{S}{N}\right)_{rf}$, derive an equation for the value of $\left(\frac{S}{N}\right)_{rf.5}$ in terms of B_{exp} and $\left.\frac{S}{N}\right|_{req}$.

3e) What is the relationship between $\left(\frac{S}{N}\right)_{rf.5}$ derived in part 3d) and $\left.\frac{S}{N}\right|_{req}$ if $B_{exp} >> \left.\frac{S}{N}\right|_{req}$?

3f) For conditions in 3e) what is the ratio of (noise power) to (total interference power) in the output of the correlator?

PROBLEMS (30 POINTS)

You are designing a wireless communications system to work under the following conditions:

a) Propagation:

- $\frac{P_r}{Pt} = (k/d^4)L$ where, for d in meters, 10 log$k$=-1.3 dB; k includes antenna gains, etc.; d is distance in meters; and L accounts for the large scale variation of the median of the small scale Rayleigh distributed fading.
- the factor L is log normally distributed with a standard deviation of $\sigma = 10dB$

b) System Specifications:

- $P_t = 0.1W$
- 3 branch selection microscopic diversity will be used for parts B) and D)
- TDMA is used, and N users can be served simultaneously per base station.
- calls are blocked by the base station if one of the following conditions occurs:
  - i) user is in a location where the envelope of the small scale signal variation after the diversity combiner is <-100 dBm for more than 2% of the time
  - ii) all N channels are occupied.
- Blocked calls are cleared in either case, that is, blocked calls are removed from the system.
- Calls are exponentially distributed in length, lasting $\left(\frac{1}{\mu}\right)$hr, on the average.

A) With everything else being equal, how much less power needs to be transmitted when using 3 branch selection microscopic diversity than if you were to use only 2 branch selection microscopic diversity? Express your answer in dB for the ratio of power required for 2-branch to power required for 3-branch.

B) Suppose $N = \infty$. A user is randomly located (uniformly distributed) on the circumference of a circle 200m from the base station. What is the probability, P_B, of a call being blocked? (in this case, calls are never blocked because all N channels are occupied)

C) Suppose now N=4, and all users are in locations where the small scale signal envelope is greater than -100 dBm for 99% of the time. That is, for this case, calls are never blocked because of poor signal quality. Call attempts are Poisson distributed, with an arrival rate λ attempts/hr. Express P_B in terms of λ and μ. Now solve for the specific numerical case of λ=5.22, μ=6.

D) Again, N=4. In this case, all users are randomly, independently, and uniformly distributed along the circumference of a circle of radius 200m, around the base station. Poisson distributed call attempts arrive at λ attempts/hr. What is the overall P_B due to both poor signal and no available channel? Express first in terms of λ and μ and the probability of blocking due to propagation. Give the specific numerical value of P_B for λ=5.22, μ=6.