SAMPLE MIDTERM #6

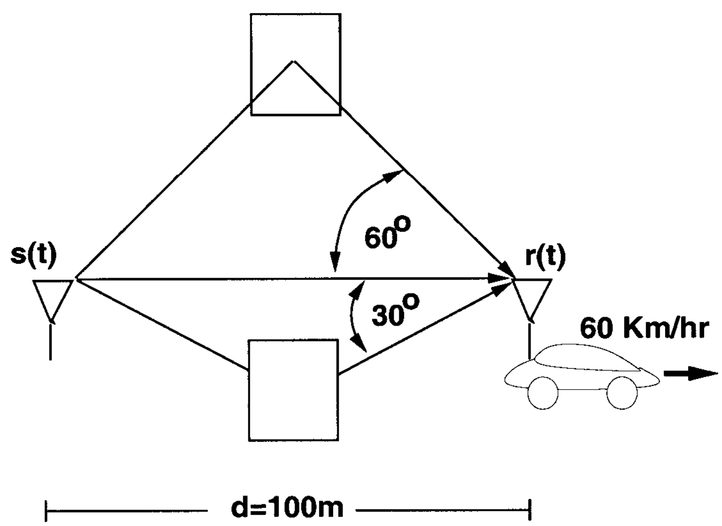
Lecture 13 - EE 359: Wireless Communications - Fall 2001

MIDTERM

This exam is open book and notes, and calculators are needed. Graded exams and solutions will be available next week. Good Luck!

Problem 1 (40 points): Multipath Channels.

This problem illustrates the impact of multipath on wireless communication. Consider the wireless channel shown in the figure below, consisting of one LOS path and two multipath components at angles 30 degrees and 60 degrees, respectively. At the snapshot of time associated with the figure, the distance between transmitter and receiver is 100 m, and the receiver is traveling at 60 Km/hr. Assume a carrier frequency of 1 GHz and nondirectional antennas (0 dB antenna gain) at both transmitter and receiver.



(a) Sketch the scattering function at the snapshot in time associated with the figure. Be sure to label all axes and label all relevent numerical values.

(b) Roughly sketch how your answer to part (a) would change if the scattering function were averaged over a short time period instead of just measured at one snapshot in time.

(c) For the scattering function in part (a), approximate the minimum value of Δf such that if $s(t) = \sin(2\pi f t) + \cos(2\pi(f+\Delta f)t)$ is the channel input, the output $r(t) = \alpha_1 \sin(2\pi f t) + \alpha_2 \cos(2\pi(f+\Delta f)t)$ has the random complex constants α_1 and α_2 approximately independent.

(d) For the scattering function of part (a), assume that the received signal envelope is Rayleigh fading with an average received SNR of 30 dB. Suppose BPSK modulation at 30 Kbps is used with a target BER of 10^{-3}. What is the average length of time that the BER is continuously above its target value? How many bits are affected by this continuous outage?

(e) For the scattering function of part (a), under what conditions will a BPSK signal sent over this channel experience an irreducible error floor?

Problem 2 (30 points): Capacity and Adaptive Modulation.

This problem illustrates that fixed-rate modulation based on truncated channel inversion can come fairly close to channel capacity. This is significant since the fixed-rate modulation is easy to implement and requires no coding. Consider a discrete time-varying AWGN channel with four channel states. Assuming a fixed transmit power $\overline{S}$, the received SNR associated with each channel state is $\gamma_1 = 5$db, $\gamma_2 = 10$db, $\gamma_3 = 15$dB, and $\gamma_4 = 25$dB, respectively. Each state is equally likely with $p_i = .25, i = 1, 2, 3, 4$. Assume both transmitter and receiver have perfect instantaneous estimates of the channel.

(a) Find the optimal power control policy $S(i)/\overline{S}$ for this channel and its corresponding capacity per unit Hertz (C/B bps/Hz).

(b) Armed with your newly acquired EE359 knowledge, you propose to design for this channel a fixed rate modulation strategy with channel inversion power control (and build a startup around this idea). For ease of implementation you use square MQAM modulation, where $M = 2^x$ for any integer x. Given the average power constraint $\overline{S}$, what is the maximum square MQAM constellation size that can be sent over the channel under this power control policy while meeting a target BER of 10^{-3} (use the bound BER $\leq .2e^{-1.5\gamma/(M-1)}$ for $M \geq 4$ and BER $\leq 2e^{-1.5\gamma/(M-1)}$ for $M = 2$.) What is the corresponding spectral efficiency for the channel (bps/Hz), assuming the symbol time $T_s = 1/B$ or, equivalently, that the number of bits/symbol equals the bps/Hz.

(c) You now propose to increase the spectral efficiency by using the same fixed rate square MQAM modulation of part (b), but with truncated channel inversion for the power control. What is the maximum spectral efficiency that can be obtained for this channel if the truncation level is optimized? What is the optimal truncation level and corresponding MQAM modulation that achieves this efficiency.

Problem 3 (30 points): Cellular Systems with Square Cells.

The following problem illustrates how performance can vary significantly at different locations within a cell. Consider a cellular system with square cells where each side is 100 meters. The base station is located in the center of the cell and transmits DPSK modulation. Propagation follows the power falloff with distance model $P_r(d) = P_t(d_0/d)^3$, where $d_0 = 1$m. Mic and Mac, relaxing after the EE359 midterm, decide to test some of the theory by playing around with their cell phones and measurement equipment. Both guys are located at cell boundaries, but Mic is at the corner of the cell (71m from the base station) whereas Mac is in the middle of a cell side (50m from the base station).

(a) Assume no fading or shadowing on the channel. Suppose Mac whips out his handy dandy BER tester and measures a BER of 10^{-6}. If Mic whips out his handy dandy BER tester, what BER will he get?

(b) Repeat part (b) assuming the signals experience Rayleigh fading and no shadowing.

(c) Suppose now that the signals experience slow Rayleigh fading and no shadowing. Suppose that Mac makes some measurements and determines that he has an outage probability of .01 for a target BER of 10^{-3}. What percentage of locations within the cell have an outage probability smaller than .005 for the same target BER?

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Problem 1 (25 points): Wireless Channels.

This problem illustrates the impact of the channel on wireless system performance. Consider a wideband channel with multipath intensity profile

$$A_c(\tau) = \begin{cases} 1 & 0 \leq \tau \leq 10 \ \mu\text{sec.} \\ 0 & \text{else} \end{cases} .$$

Assume the received signal has a Rayleigh fading amplitude with average received SNR of 30 dB.

(a) What is the average delay spread of the channel?

(b) For BPSK modulation and a data rate of 10 Kbps, what is $\overline{P}_b$.

(c) Repeat part (b) assuming the average received SNR is 10 dB.

(d) Again assume an average received SNR of 30 dB. Suppose your cellular provider disconnects your phone call whenever your received SNR falls below 25 dB for more than .01 seconds. Assume that the duration of a fade is always equal to its average value. For a PCS system operating at a carrier frequency of 2 GHz, what is the minimum speed you must be traveling such that you never get disconnected?

Problem 2 (40 points): Capacity of Time-Varying Channels.

This problem illustrates the capacity of time-varying channels under different adaptation policies, and how close we can come to this capacity using uncoded modulation. Consider a discrete time-varying AWGN channel with four channel states. Assuming a fixed transmit power $\overline{S}$, the received SNR associated with each channel state is $\gamma_1 = 5$ db, $\gamma_2 = 10$ db, $\gamma_3 = 15$ dB, and $\gamma_4 = 20$ dB, respectively. The probabilities associated with the channel states are $p(\gamma_1) = .4$ and $p(\gamma_2) = p(\gamma_3) = p(\gamma_4) = .2$.

(a) Find the capacity per unit bandwidth (C/B) and optimal power allocation assuming both transmitter and receiver have perfect channel knowledge.

(b) Find the capacity per unit bandwidth of this channel when both transmitter and receiver have perfect channel knowledge and truncated channel inversion is used. Determine the cutoff value based on maximizing the average rate. What is the received SNR when the system is not in outage?

(c) Find the maximum size MQAM constellation that can be sent over this channel using the truncated channel inversion power control derived in part (b) such that when the system is not in outage, the error probability is less than 10^{-6}. Assume the MQAM is Gray coded and has P_s and P_b in AWGN as given on page 103 of the reader. You can approximate the Q function as $Q(z) \approx e^{(-z^2/2)}/(z\sqrt{2\pi})$ or use the plot at the end of the exam or use your calculator's Q function for your calculation.

(d) What is the data rate per unit bandwidth R/B associated with the MQAM constellation size found in part (c) assuming $T_s = 1/B$? Compare this number to the capacity per unit bandwidth found in part (b) under the same truncated inversion power control.

Problem 3 (35 points): Fading Effects on Error Probability and Outage.

This problem shows that a uniform distribution for received power starting at 0 wreaks havoc on performance. Consider a system with fading of the received power such that the average received SNR is uniformly distributed between 0 and 100 (linear units): $p(\gamma) = .01, 0 \leq \gamma \leq 100$. Suppose the fading decorrelates every T seconds. Throughout this problem you should neglect the error floor due to channel decorrelation.

(a) Find the outage probability for DPSK modulation under this fading distribution for a target error probability of 10^{-3}. For what values of T_b/T is this outage probability a useful performance metric?

(b) Find the average probability of error for DPSK modulation under this fading distribution. For what values of T_b/T is this average error probability a useful performance metric?

(c) Suppose $T << T_b$. What will be the bit error probability of DPSK under this assumption?

(d) Suppose this fading distribution describes the signal variations due to shadowing, on top of which is superimposed Rayleigh fading due to multipath. What is the outage probability for an average P_b target of 10^{-2} for DPSK modulation?

Note that in general DPSK will be affected by the Doppler error floor when fading is relatively fast. We use DPSK here since the calculations under coherent modulation would be intractable on a midterm.

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Problem 1 (40 points): Wireless Channels.

This problem illustrates the impact of channel delay spread on wireless system performance when $T_s \approx T_M$, as well as the impact of Doppler. Consider a channel with impulse response $c(\tau, t) = \delta(\tau) + .5\delta(\tau - 10\mu\text{sec}) + .5\delta(\tau - 40\mu\text{sec})$.

(a) Is the channel time-varying or time-invariant? Explain.

(b) For what data rates does a BPSK signal sent over this channel experience ISI?

(c) Define

$$rect\left(\frac{t}{T_s}\right) = \begin{cases} 1 & |t| < T_s/2 \\ 0 & \text{else} \end{cases} .$$

Suppose we transmit the signal $s(t)$ consisting of two pulses,

$$s(t) = rect\left(\frac{t}{T_s}\right) + rect\left(\frac{t - T_s}{T_s}\right)$$

through the channel with impulse response $c(\tau, t)$. Sketch the received signal for $T_s = 20\mu\text{sec}$. Does the received signal exhibit ISI (why or why not)?

(d) Repeat (c) for $T_s = 2\mu\text{sec}$.

(e) Suppose now that $c(\tau, t) = \alpha_1(t)\delta(\tau) + \alpha_2(t)\delta(\tau - 10\mu\text{sec}) + \alpha_3(t)\delta(\tau - 40\mu\text{sec})$, where the $\alpha_i(t)$ are independent Rayleigh fading processes with average power of 1 Watt. Suppose the average fade duration of $\alpha_i(t)$ below 1 Watt is 8.5 msec, 7.2 msec, and 11 msec, respectively, for $i = 1, 2, 3$. What is the maximum Doppler of this channel?

This problem illustrates that our rule of thumb stating that no ISI is present when $T_s >> T_M$ and that ISI is present when $T_s << T_M$ is just an approximation that does not really indicate what happens when $T_s \approx T_M$. This case must be examined based on the exact channel model to see if ISI is significant.

Problem 2 (30 points): Capacity of Time-Varying Channels.

This problem illustrates the capacity of time-varying channels under different adaptation policies, and compares their capacity with that of an AWGN channel with the same average power. Consider a fading channel with random SNR γ that is uniformly distributed between 1 and 11 (linear units, not dB). Note that for your calculations below, if you don't have a fancy calculator that does integration, you may find the following calculus results useful: $\int \frac{1}{\gamma} d\gamma = \ln(\gamma)$, $log(a/b) = log(a) + log(1/b)$, $\log_2(x) = \ln(x)/\ln(2)$, $\int \log_2(\gamma) d\gamma = (-x + x\ln(x))/\ln(2)$.

(a) Find $\overline{\gamma}$ for this channel and compute the capacity per unit bandwidth (C/B) for an AWGN channel with this average SNR.

(b) Find the capacity per unit bandwidth and optimal power allocation for this channel assuming both transmitter and receiver have perfect channel knowledge. What is the probability that $\gamma < \gamma_0$, the cutoff fade depth in the optimal power allocation strategy?

(c) Find the capacity per unit bandwidth of this channel when both transmitter and receiver have perfect channel knowledge and channel inversion is used. Compare this result to your answer in (a), and explain qualitatively why it is bigger or smaller.

Your answers should yield relatively similar values for capacity. That is because the uniform distribution on SNR used in this problem is relatively benign (no mass at zero), so any reasonable adaptation policy yields similar results, which are in turn similar to the case of no fading.

Problem 3 (30 points): Performance under Fading and Diversity.

This problem shows the superiority of MRC over SC for diversity combining. Consider a receiver with 2 branch diversity, where each branch has i.i.d. fading with SNR γ_i that is uniformly distributed between 0 and 10 (linear units, not dB), i.e. $\gamma_i \sim U[0, 10]$, $i = 1, 2$.

(a) Find the distribution $p(\gamma_\Sigma)$ of the SNR at the combiner output under both SC and MRC. You can plot the distribution or give a formula.

(b) Assume DPSK modulation. What is the outage probability for $P_b = .1$ under both SC and MRC?

(c) Assume that $\gamma_i \sim U[0, 10]$, $i = 1, 2$ corresponds to slow fading (shadowing), and superimposed on top of this shadowing is fast Rayleigh fading. What is $p(\overline{P}_b < .1)$ for DPSK under both SC and MRC?

Note that we have used a very large target BER of .1 in this problem. While this target is easy to meet at low SNRs in AWGN, this problem illustrates that fading makes it much harder to meet even this high target BER.

EXTRA PROBLEM.

combine the output of the two receive antennas, find the SNR of the combiner output. Based on this expression, find the optimal values for c_1 and c_2 to maximize this output SNR. Finally, compute the output SNR for these optimal c_i values, and determine the maximum constellation size M that can be sent over this diversity channel with $P_b = .2e^{-1.5\gamma/(M-1)} = 10^{-3}$ and the corresponding data rate.

(c) Which technique leads to better performance for this system: using antennas for capacity gain or for diversity gain and why?

Problem 3 (30 points): Spread Spectrum and RAKE Receivers

This problem illustrates the benefits of RAKE receivers and the optimal choice of multipath components for combining when the receiver complexity is limited. Consider a multipath channel with impulse response

$$h(t) = \alpha_0\delta(t) + \alpha_1\delta(t-\tau_1) + \alpha_2\delta(t-\tau_2).$$

The α_i are Rayleigh fading coefficients, but their expected power varies due to shadowing such that $E[\alpha_0^2] = 5$ with probability .5 and 10 with probability .5, $E[\alpha_1^2] = 0$ with probability .5 and 20 with probability .5, and $E[\alpha_2^2] = 5$ with probability .75 and 10 with probability .25 (all units are linear). The transmit power and noise power are such that a spread spectrum receiver locked to the ith multipath component will have an SNR of α_i^2 in the absence of the other multipath components.

(a) Assuming maximal linear codes, a bit time T_b, and a spread spectrum receiver locked to the LOS signal component (with zero delay and gain α_0), for what values of τ_1 and τ_2, $0 \le \tau_1 \le \tau_2 < T_b$ will their corresponding multipath components be attenuated by $-1/N$, where N is the number of chips per bit.

For the rest of the problem assume spreading codes with autocorrelation equal to a delta function.

(b) What is the outage probability of DPSK modulation at an instantaneous $P_b = 10^{-3}$ for a single branch spread spectrum receiver locked to the LOS path.

(c) What is the outage probability of DPSK modulation at an instantaneous $P_b = 10^{-3}$ for a 3-branch RAKE receiver where each branch is locked to one of the multipath components and SC is used to combine the paths.

(d) Suppose receiver complexity is limited such that only a 2-branch RAKE with SC can be built. Find which two multipath components the RAKE should lock to in order to minimize the outage probability of DPSK modulation at $P_b = 10^{-3}$ and find this minimum outage probability.