LECTURE #3

- Path loss model:
  - holds for "far-field" of transmit antenna
    'Fraunhofer region'

d_f : far-field distance.

$$\boxed{d_f = \frac{2D^2}{\lambda}}$$

D: largest linear dimension of the transmitter antenna aperture.

λ: carrier wavelength.

- The reference distance d_0 must be chosen to be in the far field:

$$\left\{ \begin{array}{l} d_0 \geq d_f = \dfrac{2D^2}{\lambda} \quad \text{, and} \\[2ex] d_0 \gg D \quad \text{, and} \\[2ex] d_0 \gg \lambda \end{array} \right\}$$

Example 1:

- Find the far-field distance of an antenna with maximum dimension 1 m, and carrier frequency 900 MHz

Solution: $D = 1$ m.

$f_c = 900$ MHz

$$d_f = \frac{2D^2}{\lambda} = \frac{2(1)^2}{\left[\frac{3\times 10^8 \text{ m/s}}{9\cancel{0}\cancel{0}\times 10^6}\right]} = 2\cdot 3 = \underline{6 \text{ meters}}.$$

Far field: ≥ 6 meters, and

$\gg 1$ meter, and

$\gg \frac{1}{3}$ meter.

$\therefore \approx$ 10 meters will fall in the far-field.

Example 2: Assume $P_t = 50$ watts

omnidirectional tx antenna, $f_c = 900$ MHz; omnidirectional receive antenna

(a) Find received power in dB_m at a distance of 100 m from the antenna in free space.

(b) P_r (10 km) = ?
in free space

Solution:

$$P_r = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2 L}$$

L ← assume 1.

$$\frac{3\times 10^8}{9\times 10^8}$$

$$= \frac{50 \cdot 1 \cdot 1 \cdot \left[\frac{1}{3} \text{ meters}\right]^2}{(4\pi)^2 \cdot (100)^2}$$

$$= 3.5 \mu W$$

$$P_r\,(dBm) = 10 \log_{10}(3.5\times 10^{-3}\, mW) = \underline{\underline{-24.5\, dBm}}$$

(b) Using the path loss model: free space $(n=2)$

$$P_r(10\text{ km}) = P_r(100\text{ meters}) + 20 \log_{10}\left[\frac{100}{10\,000\text{ meters}}\right]$$

100 meter $= (d_0)$; 10 000 meters $=$ 10 km

$$\left[\underset{(dB)}{P_r(10\text{ km})} = \underset{(dB)}{P_r(100\text{ meters})} - \underset{(dB)}{10 \log_{10}\left(\frac{d}{d_0}\right)^2}\right]$$

$$= -24.5\, dBm - 40\, dB$$

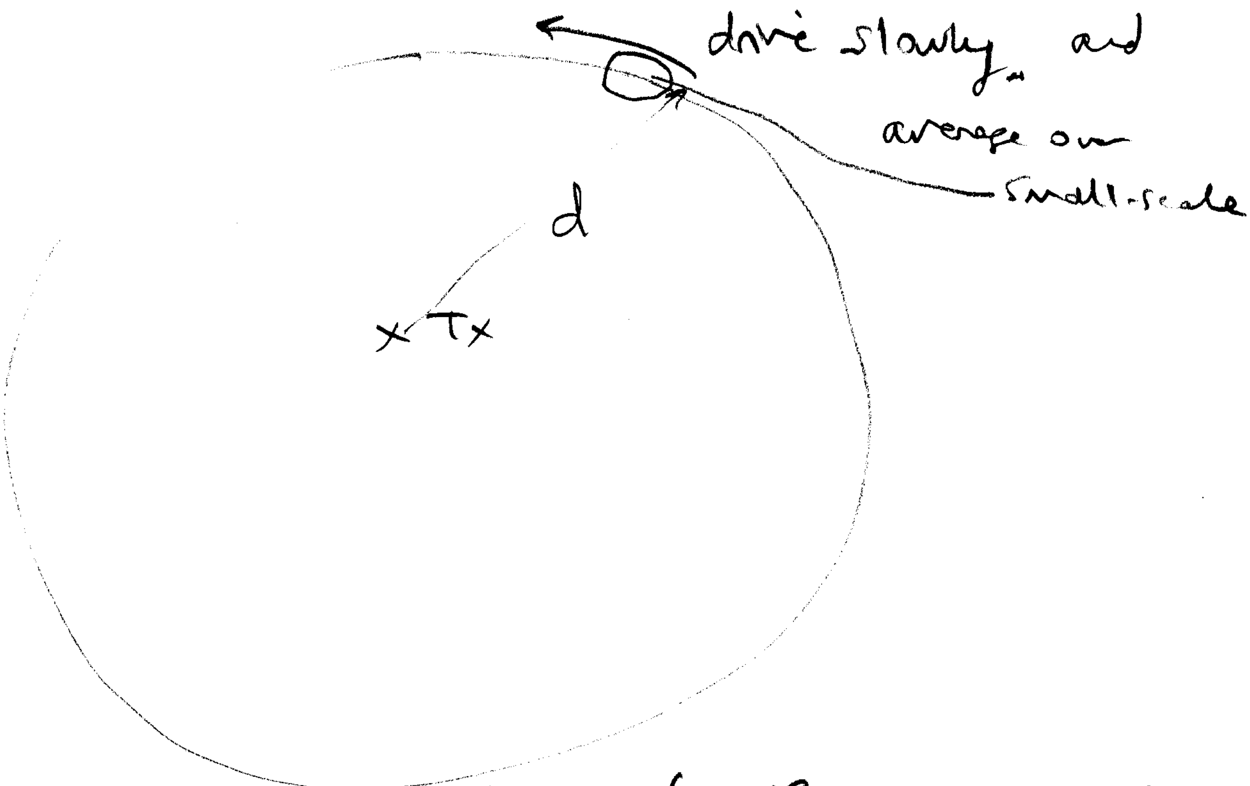
$$= \underline{\underline{-64.5\, dBm}}$$

Lognormal shadowing.

("Large-scale variations")

$$PL(d) = \overline{PL}(d) + X_\sigma$$

$$X_\sigma \sim \mathrm{Gsn}(0, \sigma^2)$$

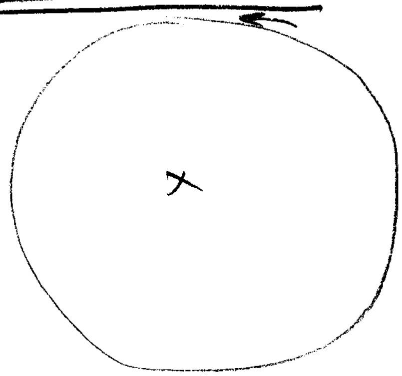


- you'll find that values ~~are~~ (in dB) are normally distributed.

$$\Big[\quad P_r = \underbrace{\alpha \cdot P_t \cdot \left(\frac{d}{d_0}\right)^{-n}}_{\text{path loss deterministic as a fn of } d} \cdot \underbrace{\left(10^{X_\sigma/10}\right)}_{\text{random var — lognormally distributed}}$$

— Arises from obstacles (trees, buildings) between Tx and Rx

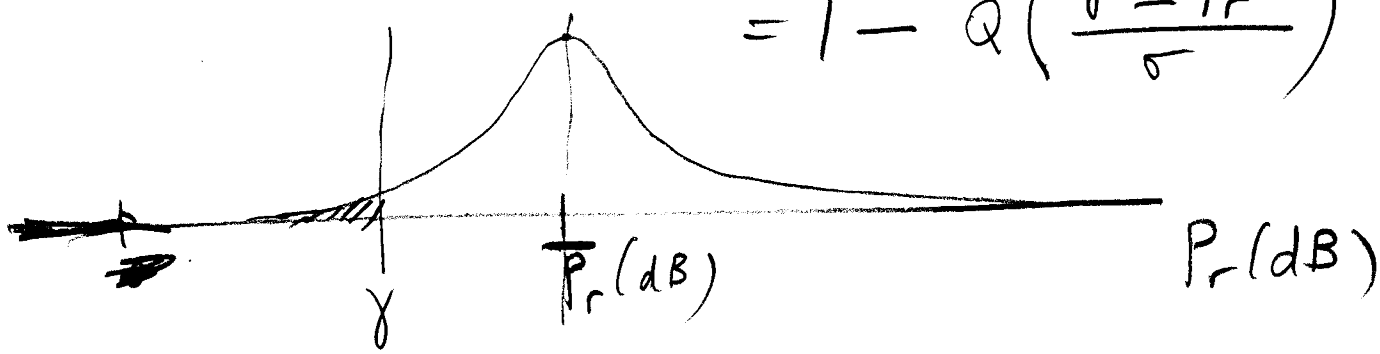
Probability of outage: $= P_r$(the received signal power falls below a certain threshold).



e.g. 5%, 1%

(⇒ call will be dropped)

$$P\{ P_r < \gamma \} = 1 - P_r(P_r > \gamma)$$
$$= 1 - Q\left(\frac{\gamma - \overline{P_r}}{\sigma}\right)$$



$\sigma = 8$–12 dB in macrocells.

[Show slides: Measurements from urban and suburban environments.]

Example: Consider a single cell in a cellular system. Design the system such that $P_{out} = 0.01$ (1%) for the worst-case user (on cell boundary).

SYSTEM PARAMS:

$f_c = 900$ MHz

Antenna dimensions $\simeq 1$ meter.

$d_{max} = 10$ km.
(maximum distance from the B.S.)

$P_t = 50$ Watts

– Assume omnidirectional antennas.

Environment params:

$\sigma = 8$ dB (of shadowing)

$n = 4$.

Solution: First, let's determine the reference distance d_o.

$$d_f = \frac{2D^2}{\lambda} = \frac{2(1)^2}{\left[\frac{3\times10^8 \text{ m/s}}{900\times10^6 \text{ /s}}\right]} \doteq 6 \text{ meters}$$

To be in the far field, $\gg 6$ meters,
$\gg D = 1$ meter
$\gg \lambda = \frac{1}{3}$ meter

take $d_o = 10$ meters

(LOS)
using free space model.

$$P_r(10 \text{ meters}) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2} = \frac{50 \cdot 1 \cdot 1 \cdot [\frac{1}{3}]^2}{(4\pi)^2 \cdot 10^2}$$

$= 0.35$ mW

$\left(\begin{array}{l} 100^2 = 10000 \\ 10^2 = 100 \end{array}\right)$

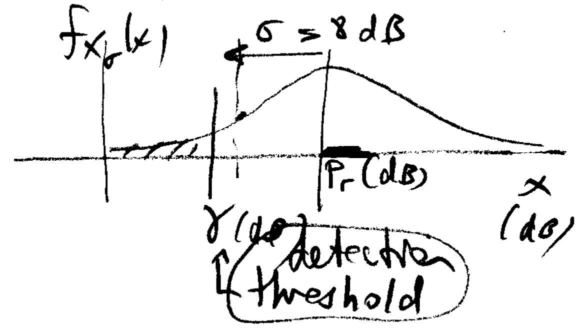
$$\left[\bar{P}_r(d) = \bar{P}_r(d_0)\cdot\left(\frac{d}{d_0}\right)^{-n}\right]$$

Then:

$$\bar{P}_r(10\text{ km}) = (0.35\,mW)\cdot\left(\frac{\cancel{10}\times 10^3}{\cancel{10}}\right)^{-4}$$

$$= \underbrace{0.35\times 10^{-12}\ mW}$$

↑ average power level at 10 km.



Now, $P_{out} = 0.01$, which means

we want:

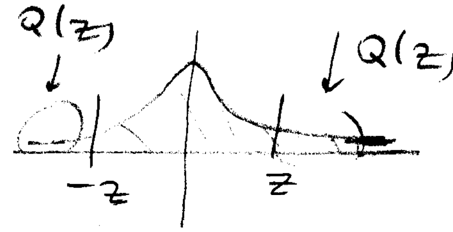
$$\underbrace{P\{P_r^{[dB]}(d_{max}) > \gamma\}} = 0.99$$

$$= P\{\bar{P}_r(d_{max})(dB) + X_\sigma > \gamma\}$$

$$= P\{X_\sigma > \gamma^{(dB)} - \bar{P}_r(d_{max})(dB)\}$$

$$= P\{\sigma Z > \gamma(dB) - \bar{P}_r(d_{max})(dB)\}$$

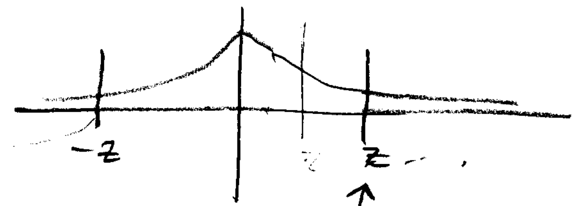
$$= P\left\{Z > \frac{\gamma(dB) - \bar{P}_r(d_{max})(dB)}{\sigma}\right\} = Q\left(\overbrace{\frac{\gamma(dB) - \bar{P}_r(d_{max})(dB)}{\sigma}}^{\equiv\, -z}\right)$$



[useful relationships: $Q(-z) = 1 - Q(z)$
(due to symmetry of Gsn. distribution)

$Q(0) = 1/2$.]

- See Table F.1, on p. 647, Rappaport.
- Note that only $Q(z) \leq 1/2$ values are given.
  (i.e. for $z \geq 0$.)



$$Q\left(\frac{\gamma(dB) - \overline{P}_r(d_{max})(dB)}{\sigma}\right) = 1 - \underbrace{Q\left(\frac{\overline{P}_r(d_{max})(dB) - \gamma(dB)}{\sigma}\right)}_{0.01}$$

$\downarrow$

requires $z \simeq \underline{\underline{2.3}}$
from Table F.1

$$\therefore \quad \overline{P}_r(d_{max})(dB) - \gamma(dB) = 2.3\sigma$$

$$\gamma(dB) = \overline{P}_r(d_{max})(dB) - 2.3\sigma$$

$$= 10 \log_{10}(0.35 \times 10^{-12}\, mW) - 2.3 \cdot (8)$$

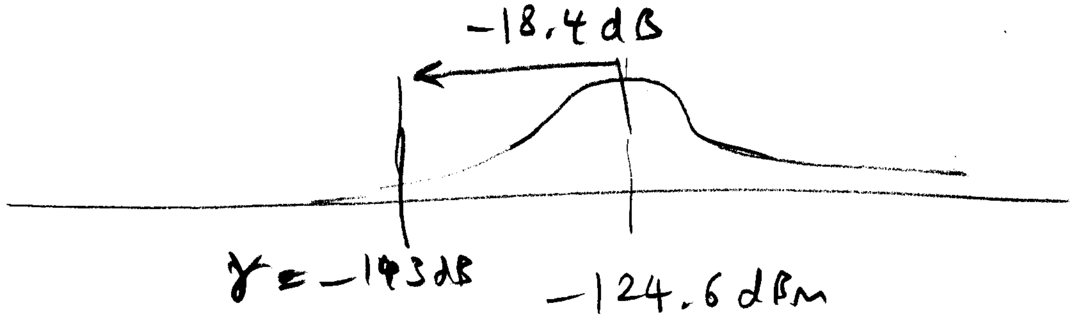
$$= -124.6\ dBm - 18.4\ dB$$

$$= \boxed{-143\ dBm}$$

$\downarrow$

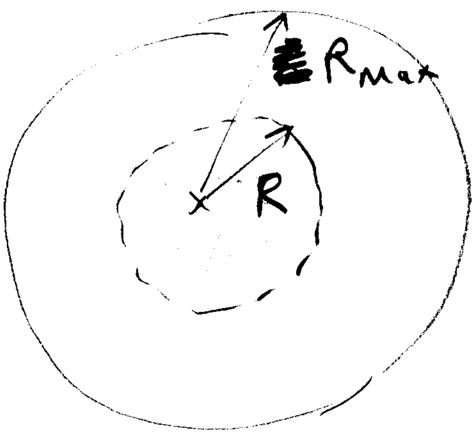
the maximum threshold required (for $P_{out} = 0.01$)

i.e.



- The analysis above was done for the worst-case user. This may be too conservative. A more realistic analysis involves the determination of the coverage area based on $\mathcal{U}(\gamma) \equiv$ the percentage of useful service area.

(≡ the percentage of the area with a received signal $\geq \gamma$.)



$$\mathcal{U}(\gamma) \cong P\{\text{the received signal is} \geq \gamma\}$$

$$= \int_0^{R_{max}} P\{\text{received signal} \geq \gamma \mid \text{mobile user is at a distance } r \text{ from B.S}\} \cdot f_R(r)\,dr$$

condition on r

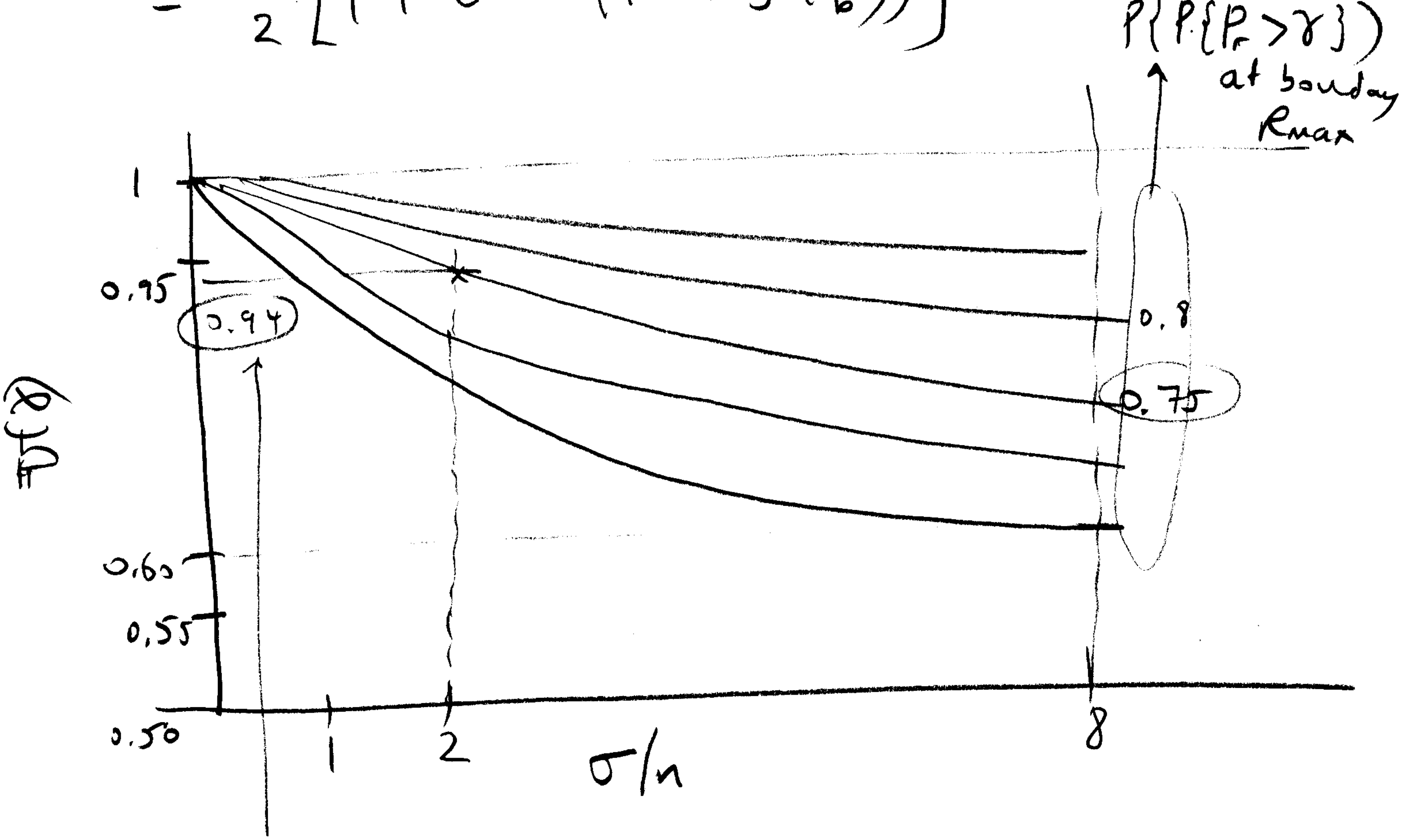
$R \in dr$

$$= \frac{1}{\pi R_{max}^2} \int_0^{2\pi} \int_0^{R_{max}} P\{P_r(r) > \gamma\}\, r\, dr\, d\theta$$

$$P_r(r) > \gamma = Q\left(\frac{\gamma - \bar{P}_r(r)}{\sigma} \right)$$

see Rappaport

$$= \frac{1}{2}\left[1 + e^{1/b^2} \cdot \left(1 - \mathrm{erf}\left(\frac{1}{b}\right)\right)\right]$$



e.g. For $\sigma = 8\,dB$

$n = 4.$

$\sigma/n = \underline{\underline{2}}$: 0.75 = boundary coverage $\Longleftrightarrow$ 94 % area coverage