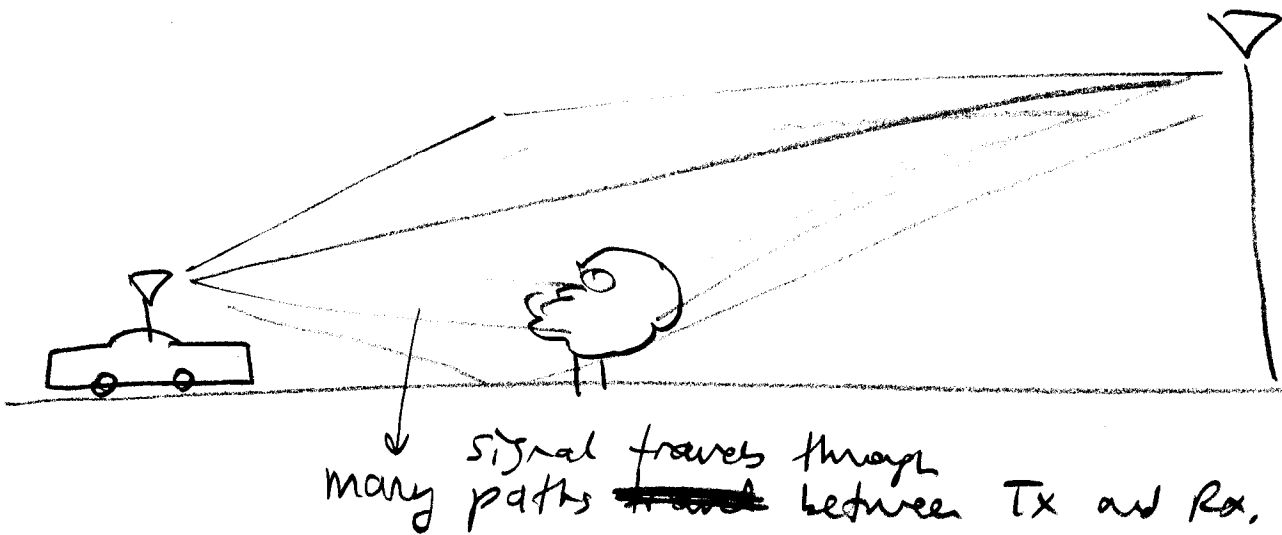


LECTURE #4B

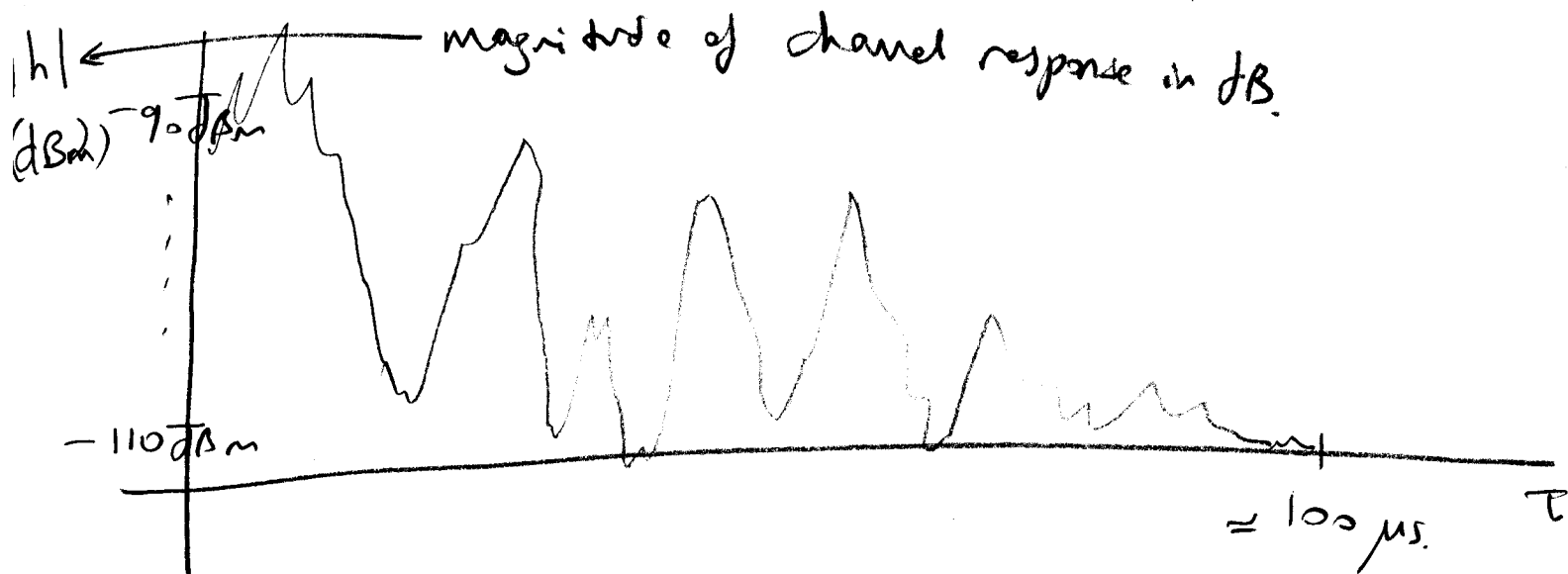
SMALL-SCALE VARIATIONS :

- multipath profile
 - Doppler shifts
- } different effects

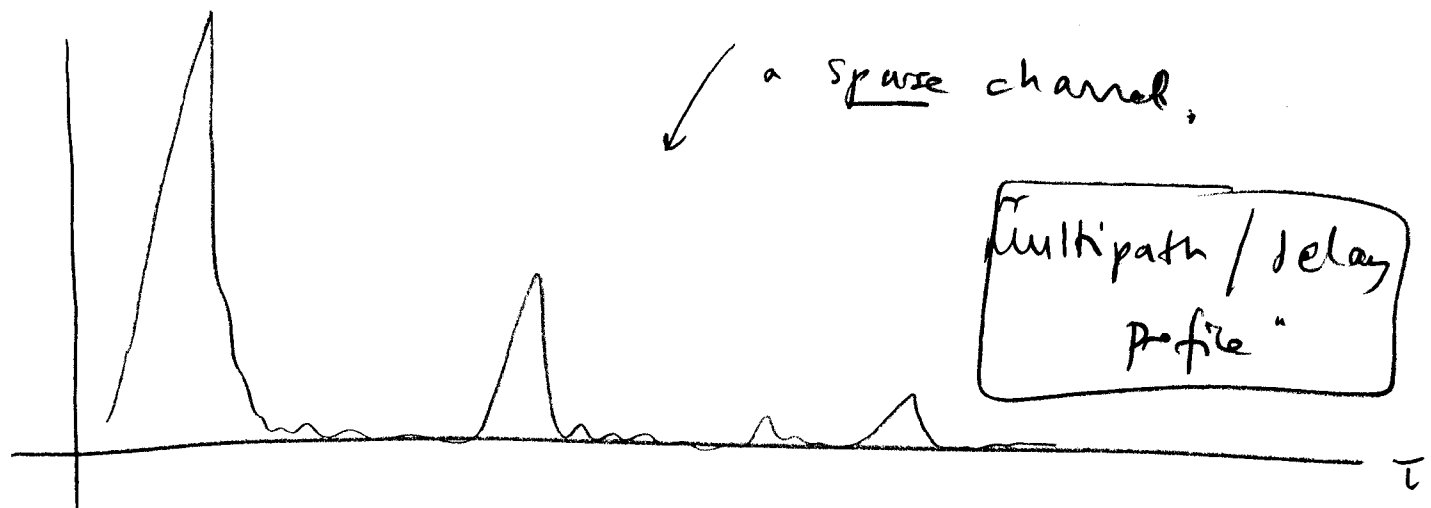


- Assume for now that the user (car) is stationary, (and of course BS stationary).
- we will approximate the channel in between as stationary. (But over the long run, the large-scale variations of obstacles causes the channel to change, so: we take an interval like several seconds over which the channel is constant in time.

- But of course there is still multipath!



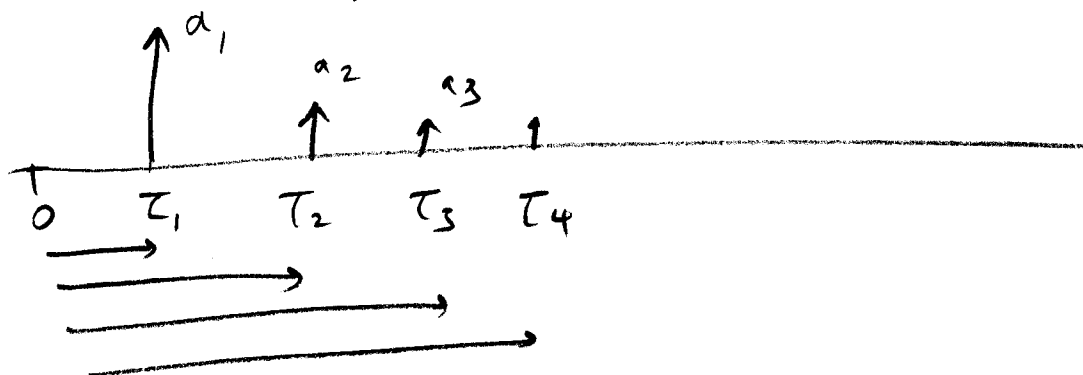
- Send an impulse to the channel and measure its response



- delay spread: is an average measure of how far the energy of the signal ~~extends~~ extends around the mean.

• Mean excess delay:

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k) \cdot \tau_k}{\sum_k P(\tau_k)}$$



• Rms delay spread:

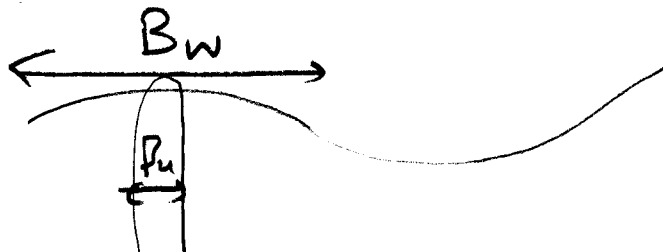
$$\sigma_{\tau} = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

where:

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k) \tau_k^2}{\sum_k P(\tau_k)}$$

Envr.	Freq.	σ_τ
urban	≈ 900 MHz	1.3 - 3.5 μ s (NY) 10 - 25 μ s (SF)
Sub-urban	≈ 900 MHz	200 - 310 ns (why smaller?) = $\leftarrow$ than urban. 1.96 - 2.11 μ s (1.96 - 2.11 μ s)
indoor	1500 MHz	10 - 50 ns
indoor	850 MHz	270 ns
indoor	1.9 GHz	70 - 94 ns

Coherence bandwidth: ^{amount of} BW over which the
freq. response of channel does not vary (much).



If $B_u \ll B_W$: flat channel

If $B_u \gg B_W$: freq.-selective channel

↓
input bandwidth

Roughly.

$$B_c \approx \frac{1}{\sigma_\tau}$$

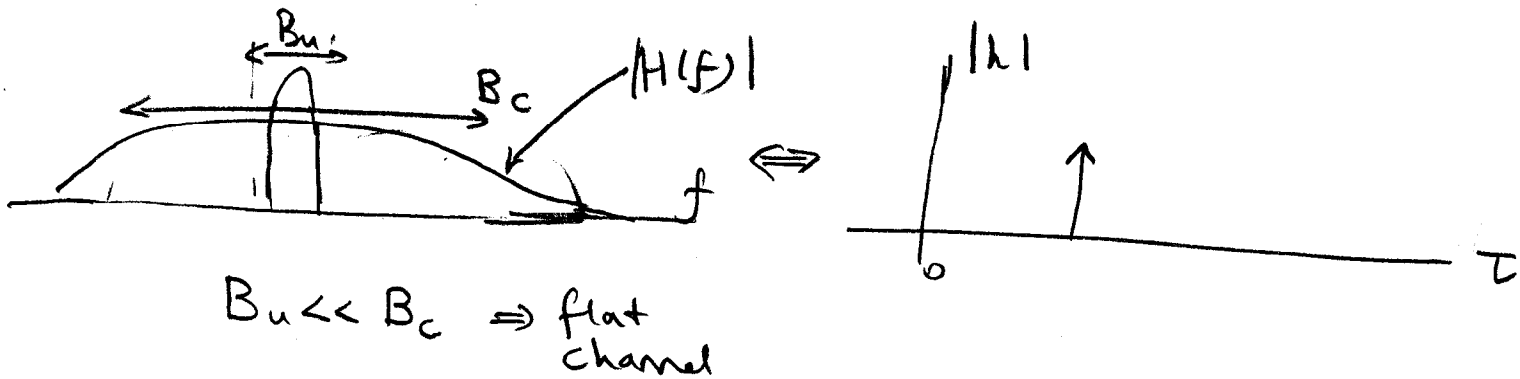
↑
delay spread.

- equalization techniques - OFDM

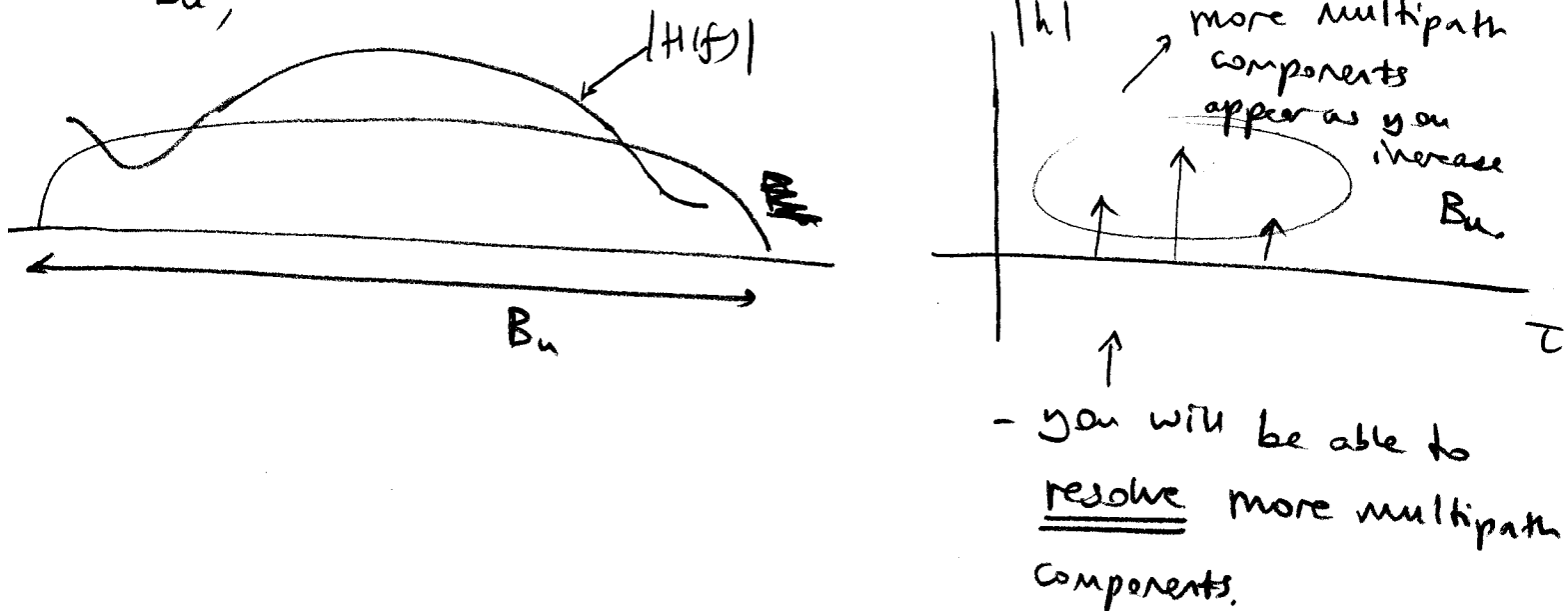
LECTURE #5

- Last time we talked about the multipath delay profile of a (stationary) wireless channel.
(static)

• Resolvable multipath components.



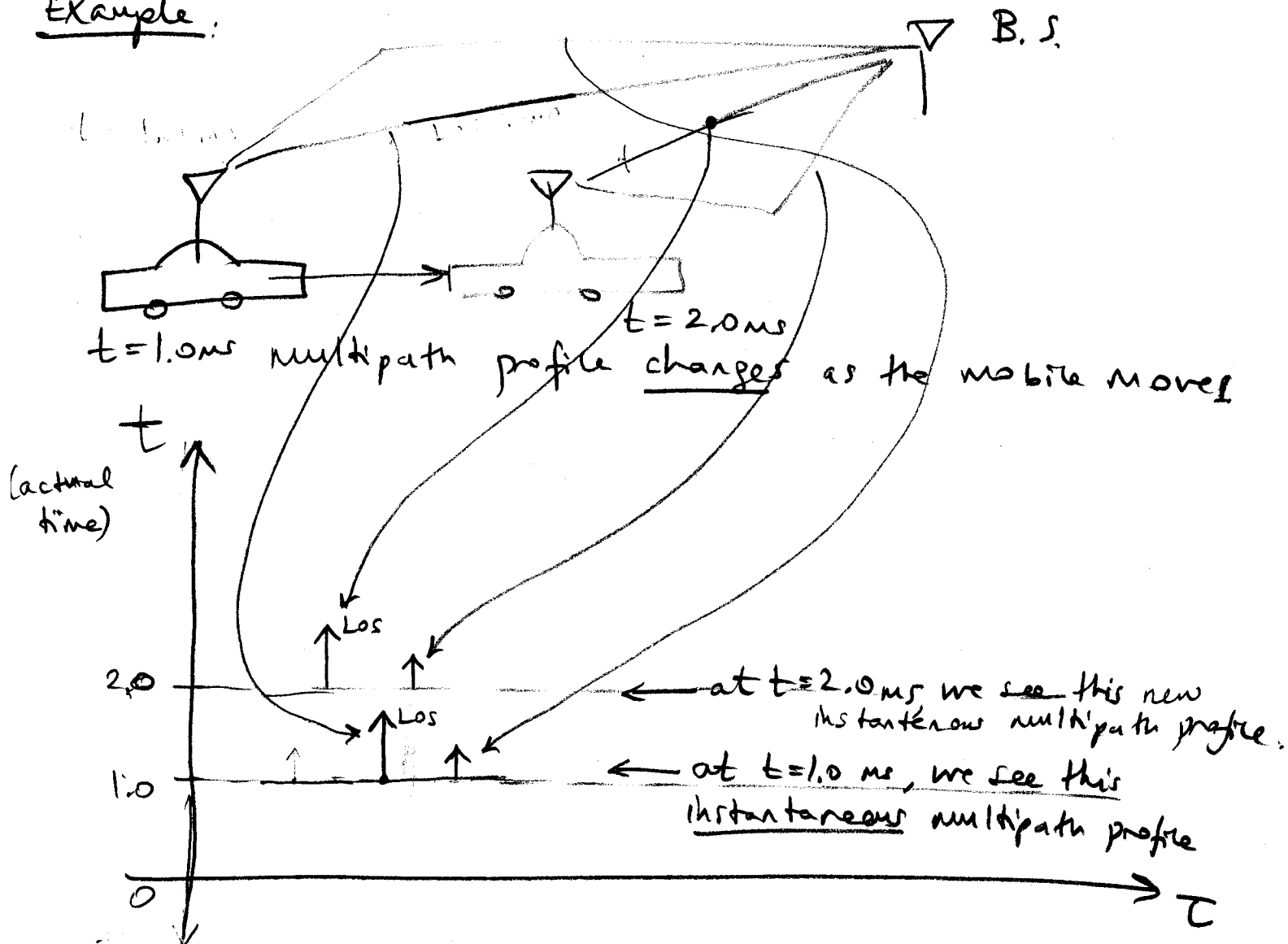
Now, as you increase the B_u ,



*It is important that channel sounding is done with at least the B_u that will be used later in the channel

— Now, we introduce mobility of the user into the model. This introduces small-scale variations as the mobile user travels through a range of multipath profiles.

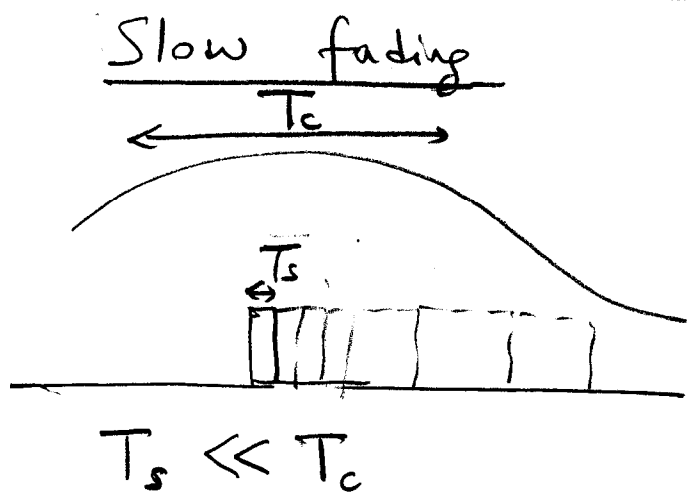
Example:



Coherence time of a channel = the amount of duration (in t above, not T !) over which the multipath profile remains roughly constant.

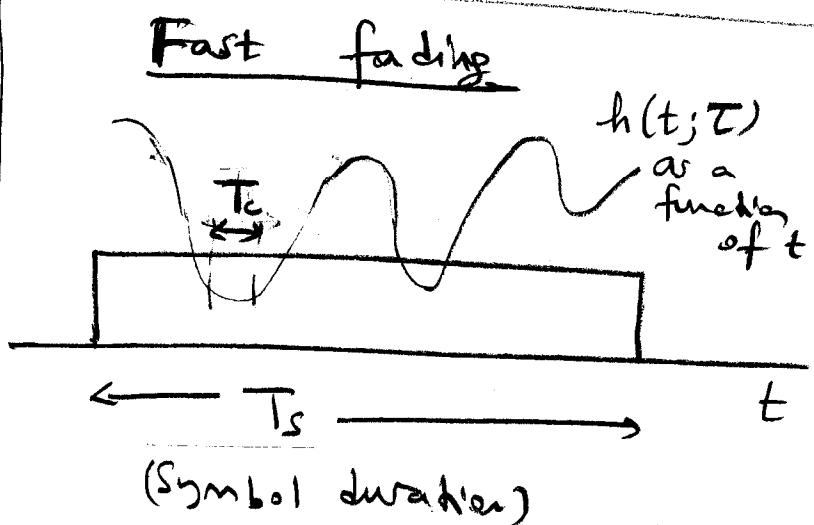
T_c = coherence time.

- Fading: refers to the variations in channel (or in received signal strength) due to mobility. [Fading is a result of the changes in the multipath profile of the channel.]



Then, each symbol (and many symbols in a row) see the same $h(t; T)$ multipath profile.

"SLOW FADING"



If $T_s > T_c$

(Symbol duration > coherence time)

- channel impulse response different for different parts of the symbol.

"FAST FADING"

These are 2 distinct regimes that require different techniques.

Slow fading

- 1 can actually attempt to track the channel
(& perform equalization across subsequent symbols
~~if~~ if the channel is frequency selective.)
- 2 channel estimation useful.
- 3 power control on a symbol-by-symbol basis makes sense — based on feedback, can do power control and/or equalization based on feedback.]

Fast fading

- not much you can do.
- try to design robust modulation schemes.
(i.e. if the amplitude $|h(t; \tau)|$ is varying, but phase is remaining same longer, then use angle/modulation frequency schemes rather than amplitude modulation.)
- [acoustic channel is an example. Bandwidth W limited. $\therefore T_s \approx 1/W$ must be large
- Channel changes fast.]

- frequency domain interpretation of fading:

Doppler spread
of a channel

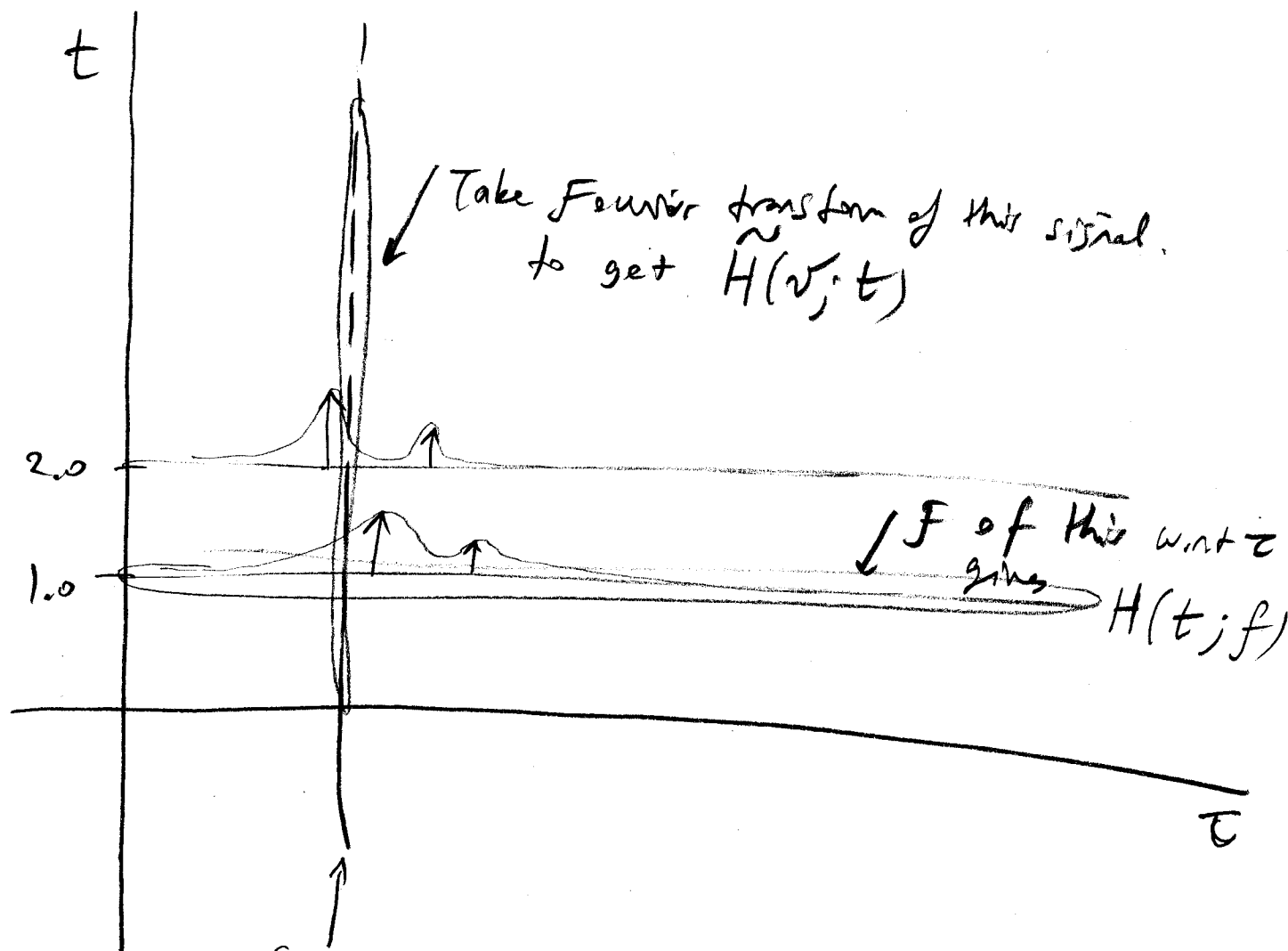
$$B_D \approx \frac{1}{T_c}$$

↑
coherence time.

Function to look at: → Fourier transform w.r.t. t .

$$\tilde{H}(\nu; \tau) = \mathcal{F}_t \{h(t; \tau)\}$$

↑
corresponding frequency domain variable.

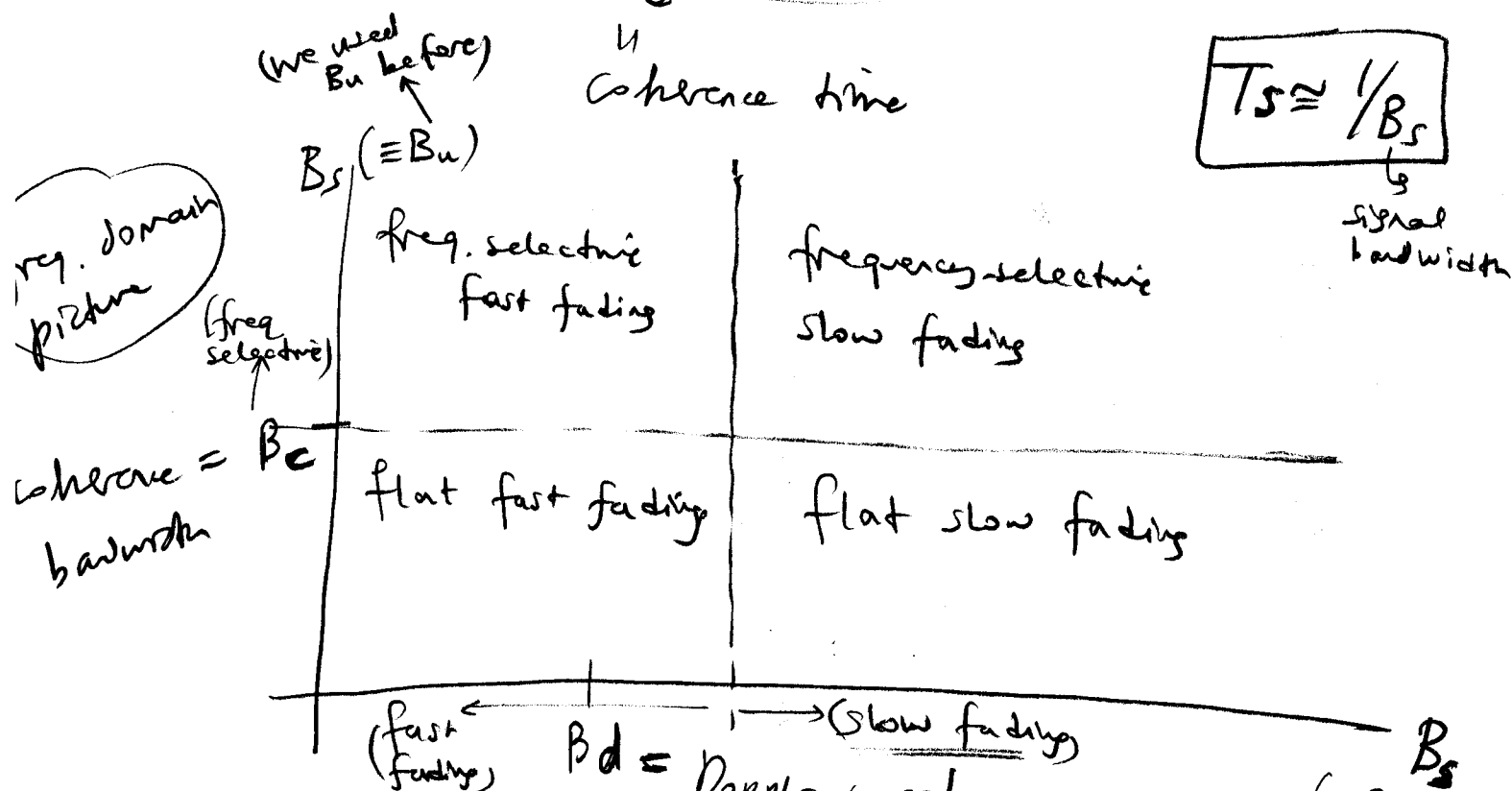
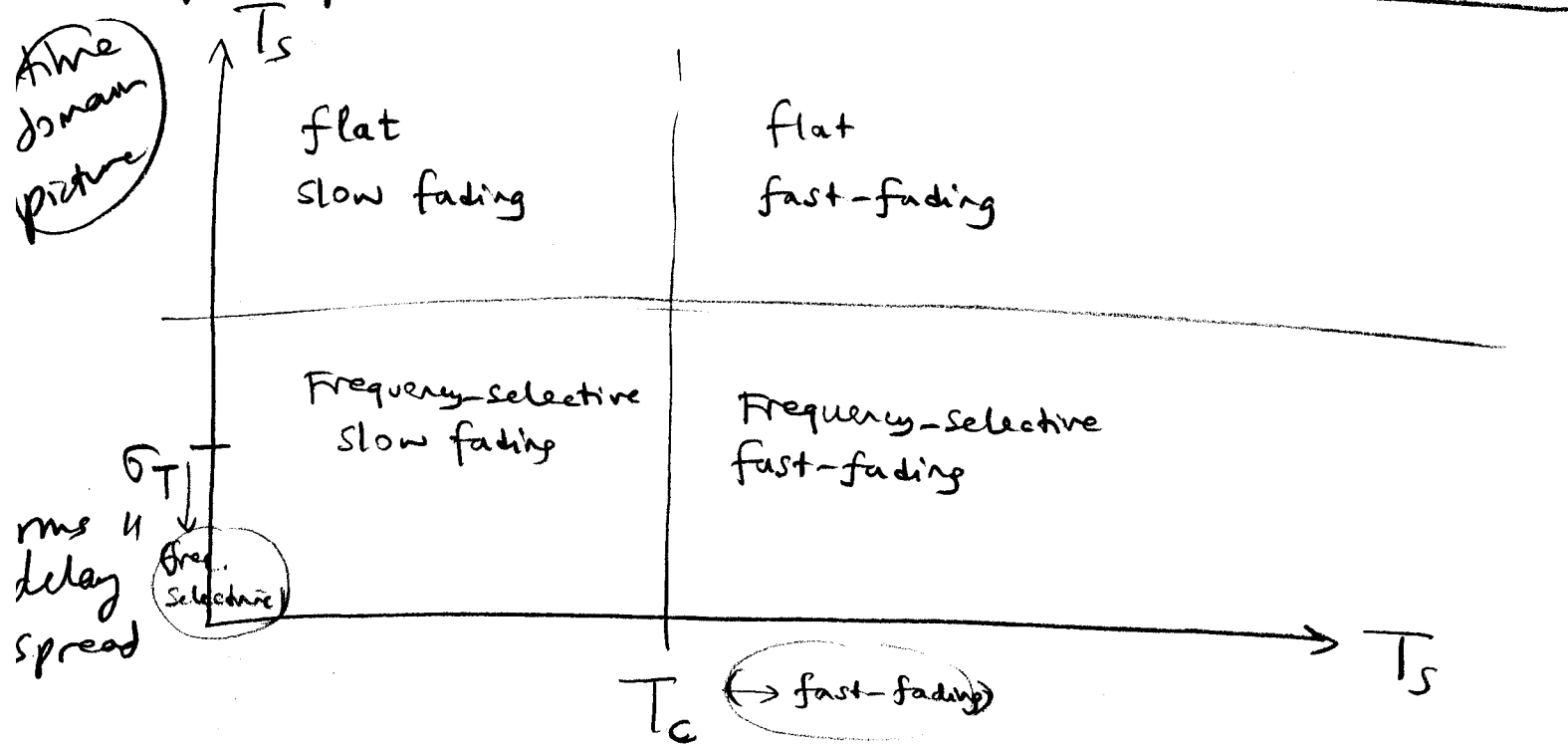


fix τ and take the Fourier transform of the signal you see here (as a fn of t) $t \rightarrow \nu$.

[Do not confuse τ and f]

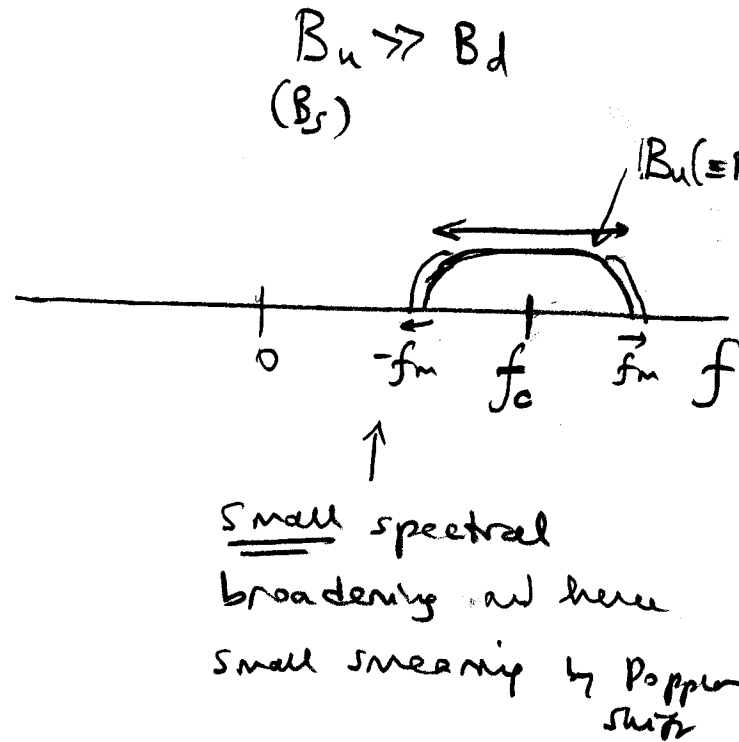
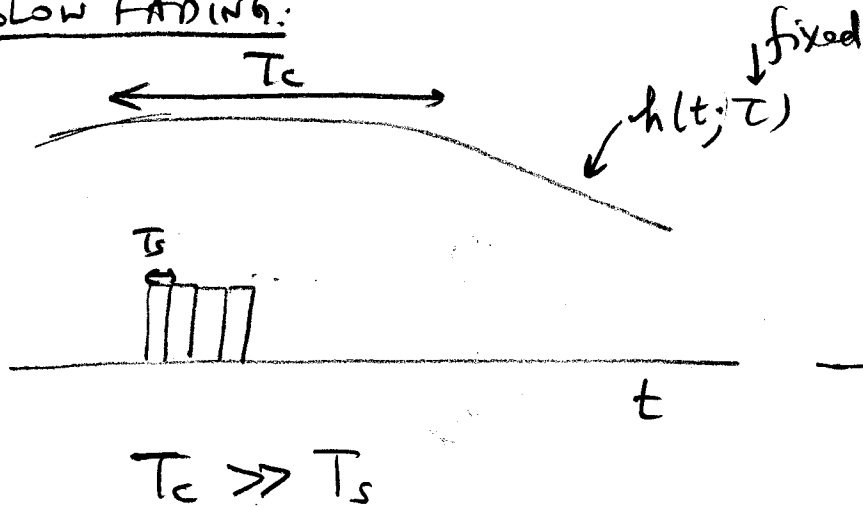


Conceptual picture.

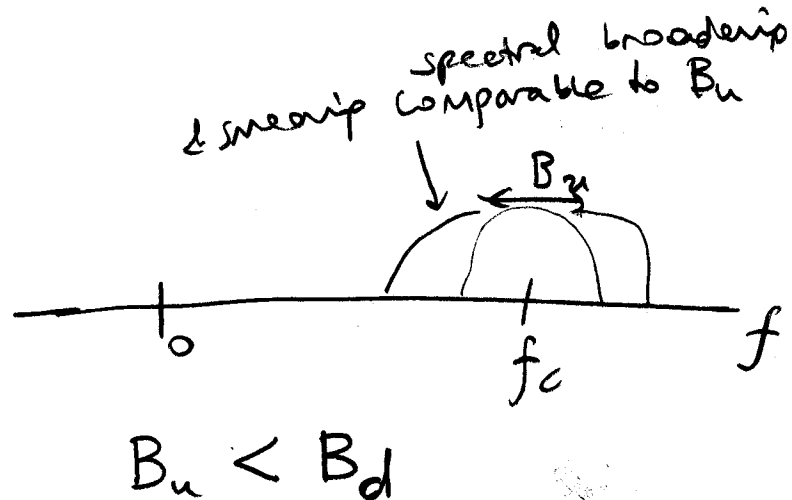
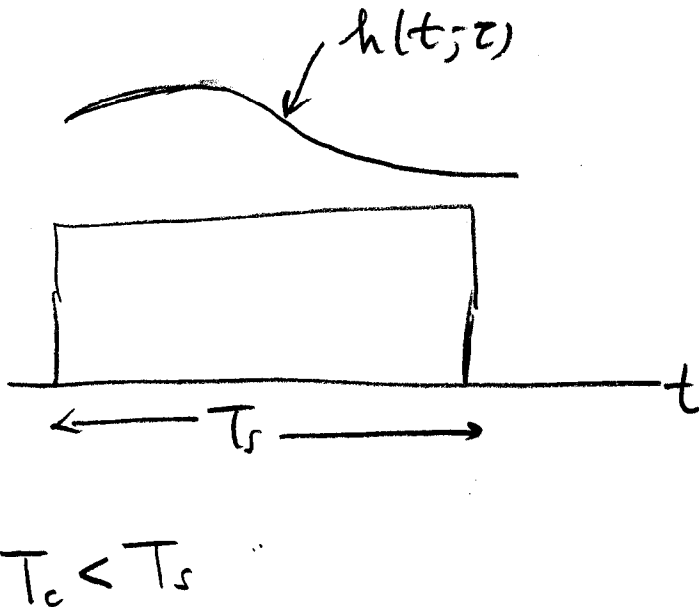


Frequency domain interpretation of Doppler shift:

SLOW FADING:



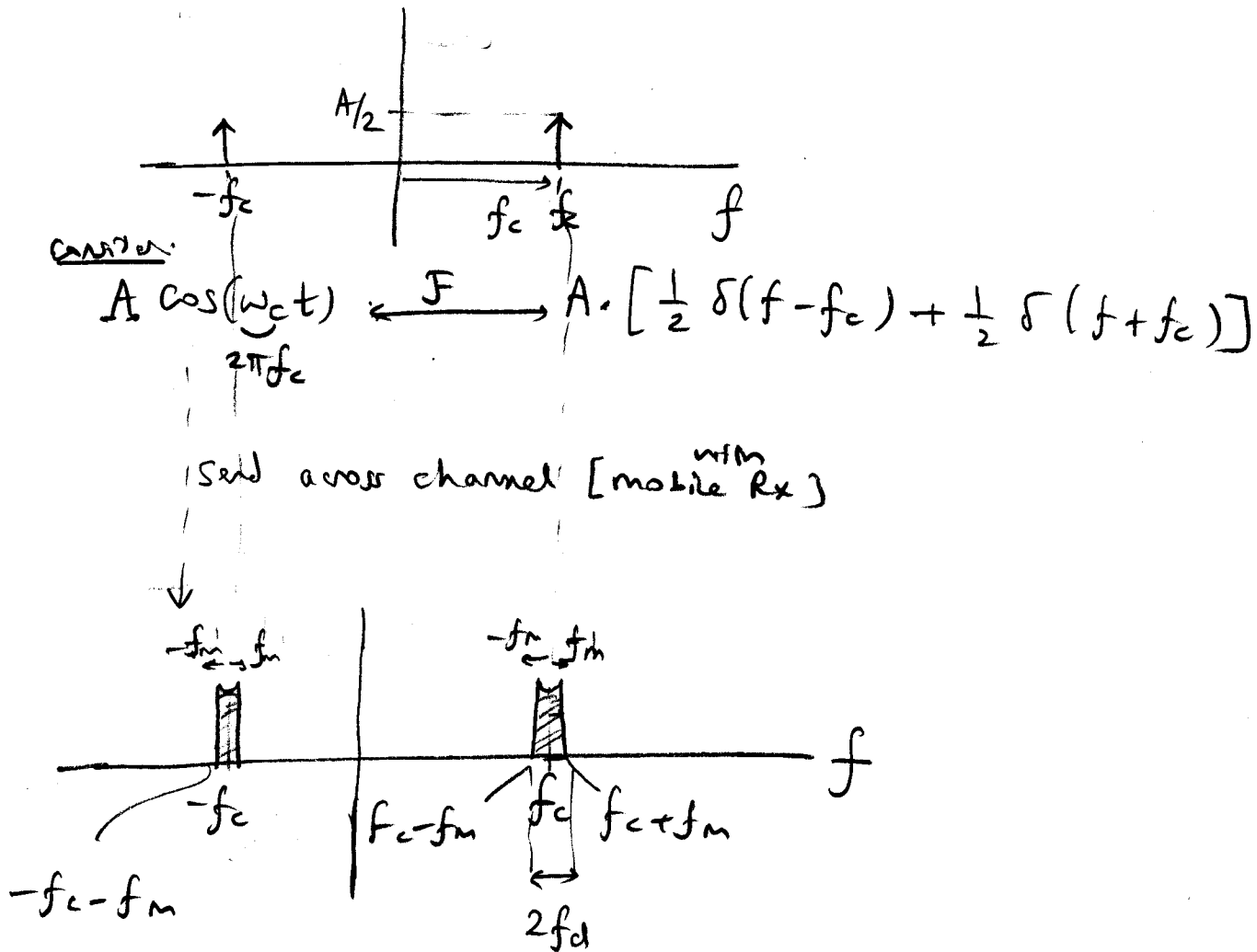
FAST FADING:



Question: When the Rx and Tx are both stationary, what is the coherence time?

Understanding Doppler shift:

- Doppler shift occurs due to the motion of the Tx or Rx, we will assume here that the Tx is fixed, and Rx is mobile.



- If the Rx antenna is moving, you will see a spectral broadening of the signal at the receiver.
- The spectral broadening occurs due to the fact that different paths experience different Doppler shifts based on the angle of arrival at the mobile.

$f_m \equiv$ maximum Doppler shift in any arriving wave (multipath components).

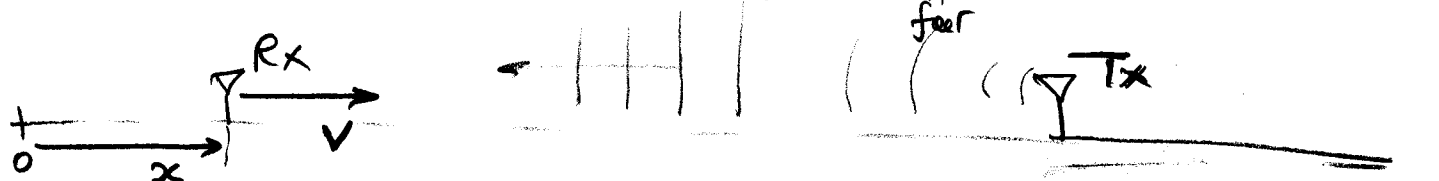
$$f_m = \frac{v}{\lambda}$$

velocity of the mobile

wavelength of the signal transmitted.

Why?

Examine:



• Write down the Electric field at the receiver:

$$E_{Rx}(x, t) = E_0 \cos(\omega t + \beta x)$$

[electrical field measured at time t and position x .]

[x : position of the Rx antenna.]

$$\left[\beta \equiv \frac{\omega}{c} \right]$$

$2\pi f_c$
speed of light.

Now, $x = \underset{\substack{\uparrow \\ \text{velocity of the mobile}}}{v} \cdot t$ (assuming $x(0) = 0$)

Then:

$$E_{Rx}(x, t) = E_0 \cos(\omega t + \beta v t)$$

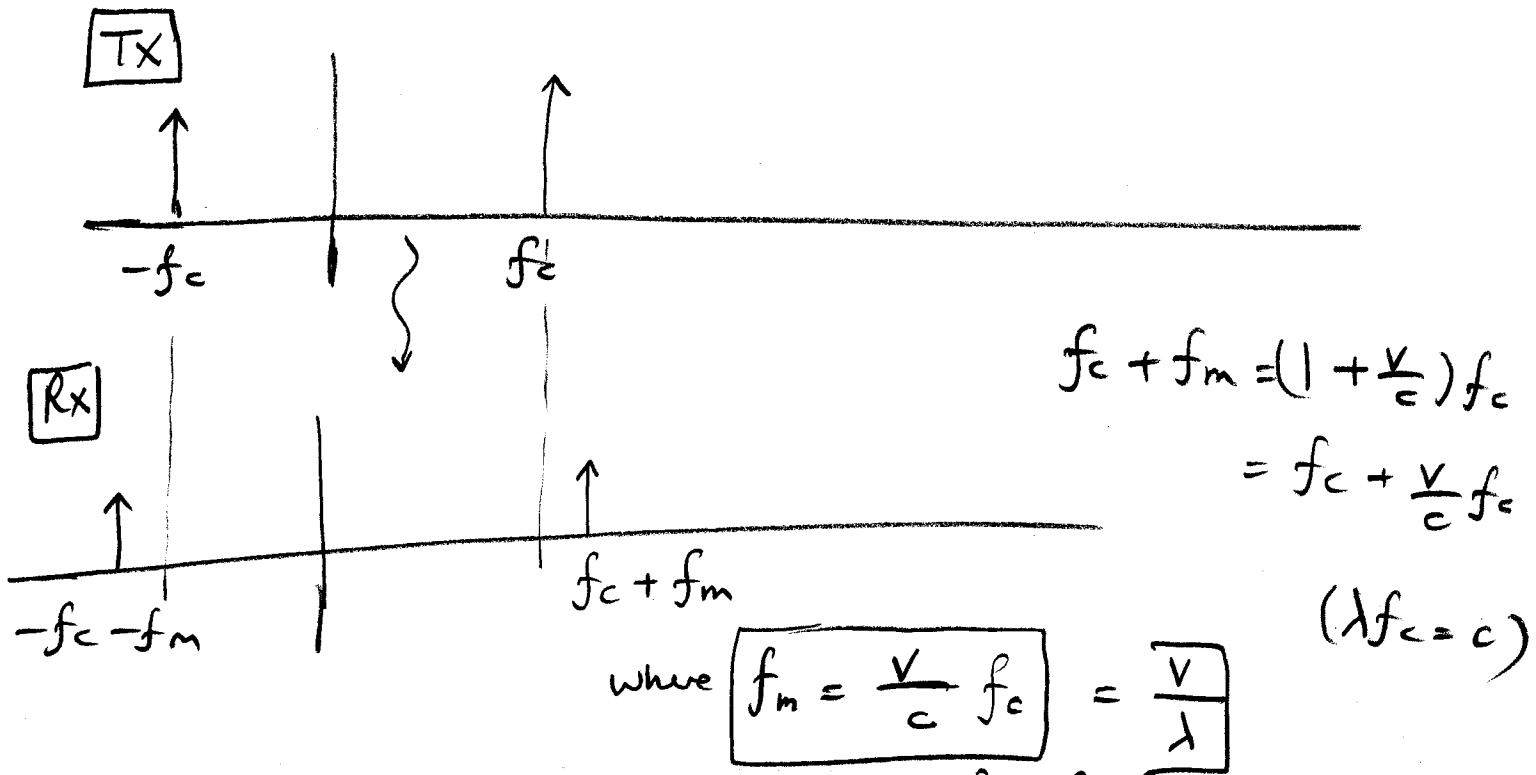
$$\omega \left(1 + \frac{v}{c} \right) t$$

Hence, at the Rx antenna, the frequency of the received

wave is higher by a factor $(1 + \frac{v}{c})$.

[We expect it to be higher because Rx is moving towards the Tx antenna: encounters the wave fronts at a faster rate.]

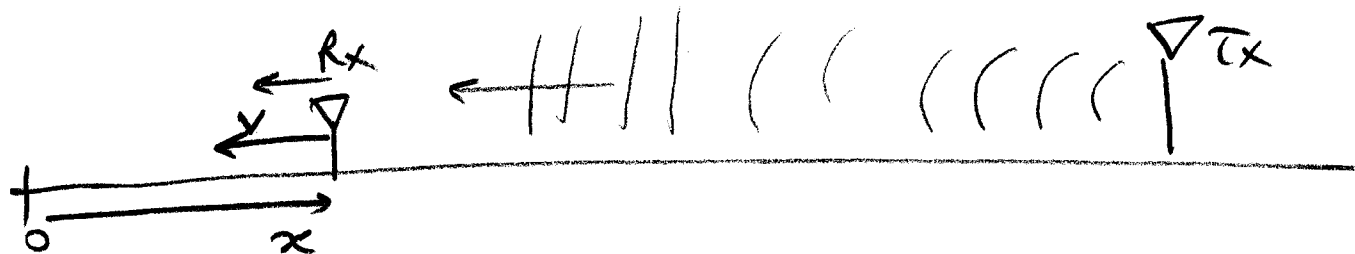
Then in frequency domain:



• Here, we have a Doppler shift of $+f_m$ Hz.

No spread here. (Spread comes from having multiple wave components experiencing different shifts.), but the maximum shift can be $+\frac{v}{\lambda}$ Hz.

Another extreme condition:



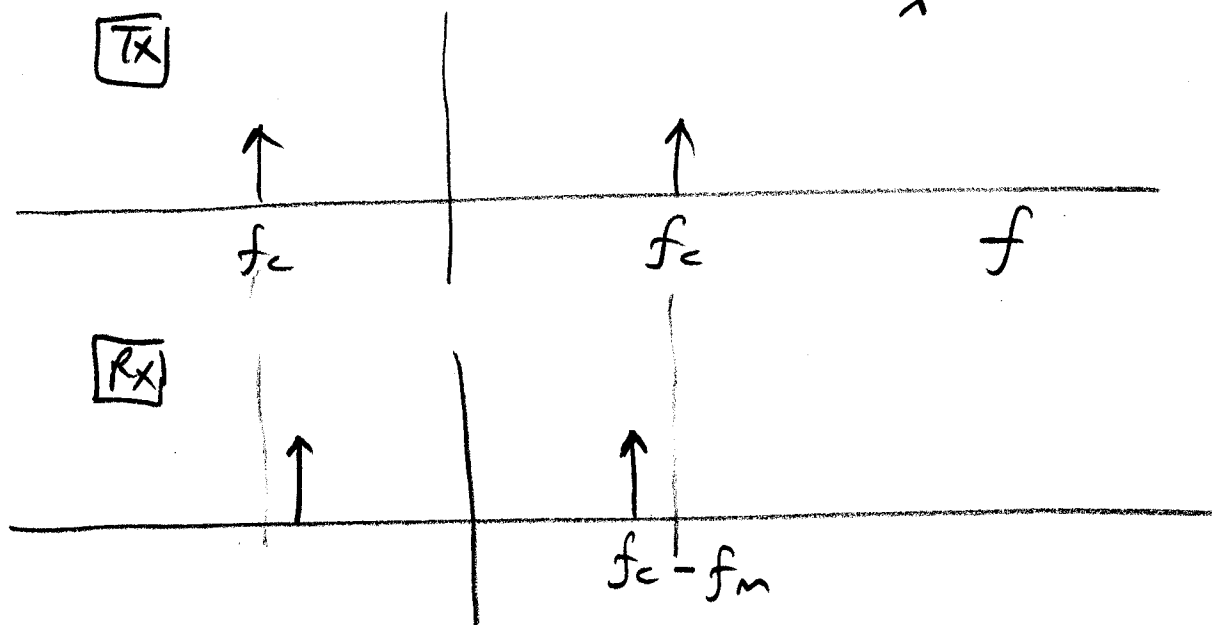
$$E_{Rx}(x, t) = E_0 \cos(\omega t + \beta x),$$

$$x = -vt \quad (\text{assuming that } x(0) = 0.)$$

Then:

$$= E_0 \cos(\underbrace{\omega t - \beta vt}_{\omega(1 - \frac{v}{c})t})$$

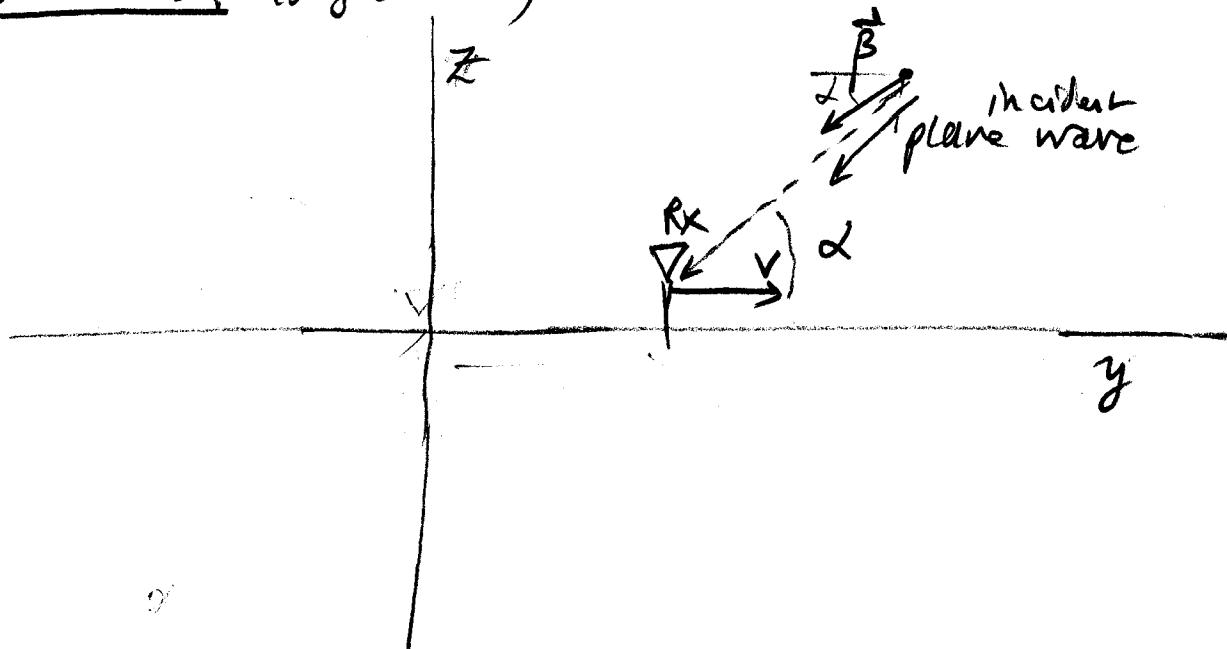
Then we see a shift of $-f_m = -\frac{v}{\lambda}$ Hz.



$$f_m = \frac{v}{c} \cdot f_c = \frac{v}{\lambda}.$$

general case: (single wave)

1



Then:

$$\vec{E} = E_0 \cdot e^{-j \vec{\beta} \cdot \vec{r}}$$

↑
complex vector notation
wave

$$\vec{\beta} = \beta \cdot [-(\cos \alpha) \hat{y} - (\sin \alpha) \hat{z}]$$

$$= E_0 \cdot e^{-j [-(\cos \alpha) y - (\sin \alpha) z] \cdot \beta}$$

$$E_r = \text{Re} \{ \vec{E} e^{j\omega t} \}$$

$$= E_0 \cdot \cos(\omega t + \beta z \sin \alpha + \beta y \cos \alpha)$$

[This is the electric field measured at location (y, z) at time t .]

Now, at mobile receiver: $z=0$, and $y=vt$ (assume $y(0)=0$)

Then

$$E_{rx} = E_0 \cos(\omega t + \beta \cos \alpha \cdot y)$$

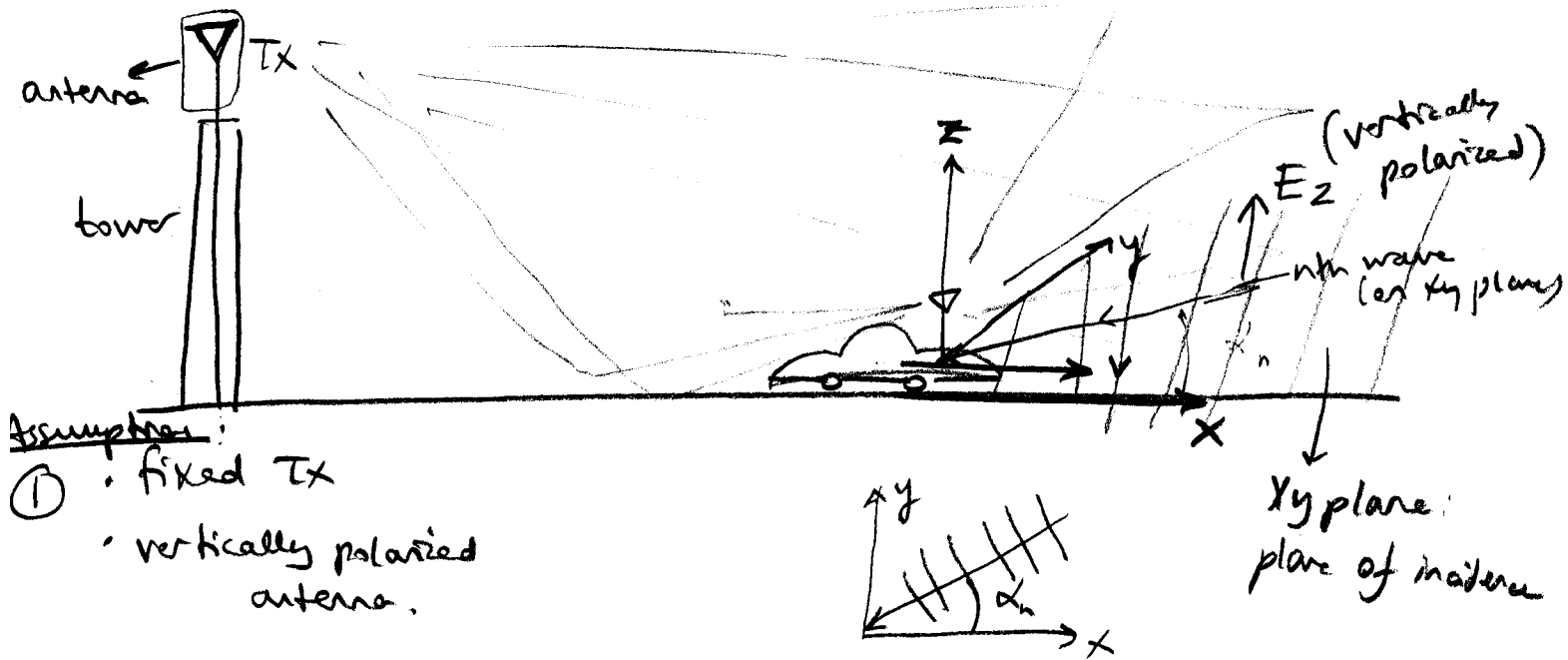
$$= E_0 \cos(\omega (1 + \frac{v}{c} \cos \alpha) t)$$

$$\boxed{\frac{v}{\lambda} \cos \alpha}$$

= Doppler shift

Statistical models for multipath fading channels

Clarke's model for flat fading



Assumptions:

- ① • fixed Tx
- vertically polarized antenna.

②. no LOS path.

- each multipath component that arrives at the mobile's antenna has equal average power (or amplitude).

Assume that there are N waves arriving at the mobile, (multipath)

- The n th [multipath component/wave] arrives at an angle of α_n degrees to the x axis.
- The Doppler shift that this wave experiences (in frequency) due to the motion of the mobile is:

$$f_n \equiv \frac{v}{\lambda} \cos \alpha_n \quad (\text{Hz})$$

• Superposition of N waves at receiver:
electric field at receiver antenna (vertically polarized)

$$E_z = E_0 \sum_{n=1}^N C_n \cdot \cos \left(\omega_c \cdot \left(1 + \frac{v}{c} \cos \alpha_n \right) t + \phi_n \right)$$

①

$$\omega_c t + \underbrace{\cancel{\omega_c} \cdot \frac{v}{c} \cdot \cos \alpha_n t}_{2\pi \cdot \frac{v}{\lambda} \cos \alpha_n t} + \phi_n$$

[comes from:

$$E = \text{Re} \{ E e^{j\omega(t - \tau_n)} \}$$

↑
phase shift component for n th wave
for n th wave.

Recall:

$$f_n = \frac{v}{\lambda} \cos \alpha_n$$

$$= \omega_c t + \underbrace{2\pi f_n t + \phi_n}_{\equiv \theta_n}$$

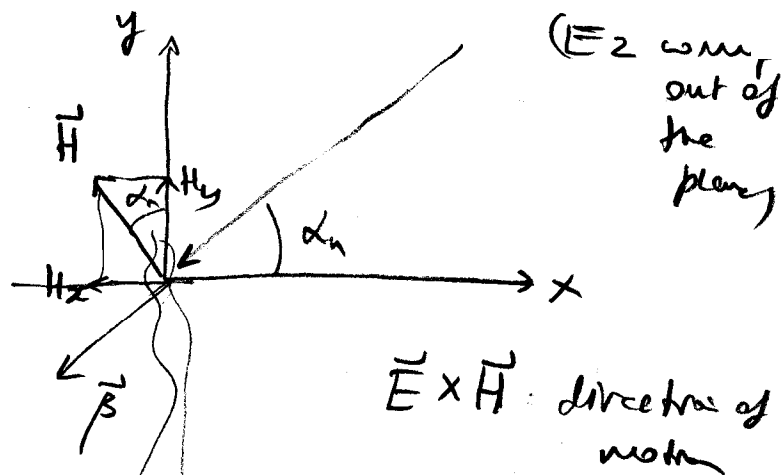
② $E_0 \cdot C_n$: amplitude of electric field (at Rx) of n th wave

$$\sum_{n=1}^N \underbrace{\overline{C_n^2}}_{\equiv E[C_n^2]} = 1 \quad \left(\text{Re. choose } E_0 \text{ such that this is true.} \right)$$

Then

$$E_z = E_0 \sum_{n=1}^N C_n \cdot \cos(2\pi f_c t + \theta_n)$$

$$\vec{E}_z = \vec{E}_0 \cdot \sum_{n=1}^N C_n \cdot \cos(2\pi f_c t + \theta_n)$$



$$H_x = - \frac{E_0}{\uparrow} \sum_{n=1}^N C_n \cdot \sin \alpha_n \cdot \cos(2\pi f_c t + \theta_n)$$

(intrinsic impedance of free space)

$$H_y = - \frac{E_0}{\uparrow} \sum_{n=1}^N C_n \cos \alpha_n \cdot \cos(2\pi f_c t + \theta_n)$$

1) If $f_m \ll f_c$, then the three field components can be modeled as narrowband random processes.

2) If N large, since $\sum_{i=1}^N (C_n \dots)$ iid, $\rightarrow$ Gen. r.v.
if $\theta_n \sim U(0, 2\pi)$

Then

15

$$E_z \cong \underset{\substack{\uparrow \\ \text{in-phase} \\ \text{component}}}{T_c(t)} \cos(2\pi f_c t) - \underset{\substack{\uparrow \\ \text{quadrature} \\ \text{component}}}{T_s(t)} \sin(2\pi f_c t)$$

where:

$$T_c(t) = E_0 \cdot \sum_{n=1}^N C_n \cos(\overbrace{2\pi f_n t + \phi_n}^{\theta_n})$$

and

$$T_s(t) = E_0 \sum_{n=1}^N C_n \sin(2\pi f_n t + \phi_n)$$

By LLN, $T_c(t)$ and $T_s(t)$ are Gaussian random processes.

At any time t , $T_c(t) \overset{\text{uncorrelated}}{\perp} T_s(t)$

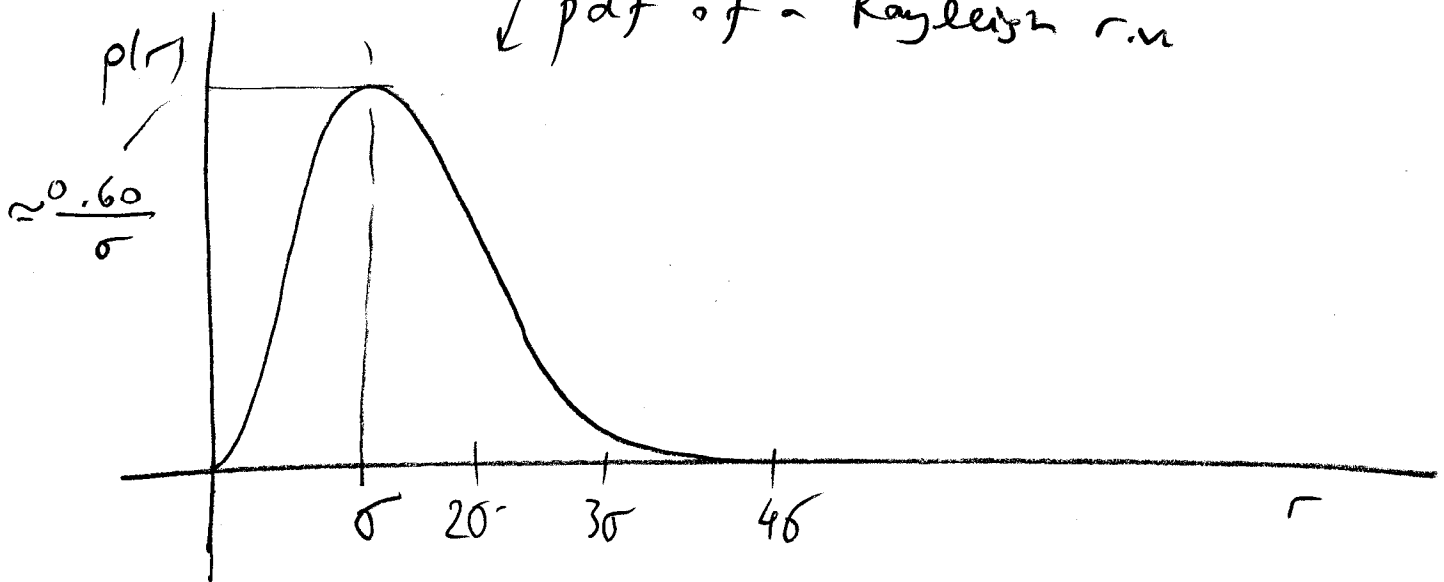
$$E[T_c(t)^2] = E[T_s(t)^2] = \frac{E_0^2}{2}$$

$$|E_z(t)| = \sqrt{\underset{\substack{\uparrow \\ \text{Gn r.}}}{T_c^2(t)} + \underset{\substack{\uparrow \\ \text{Gn r.}}}{T_s^2(t)}} \equiv \underset{\substack{\uparrow \\ \text{Rayleigh r.v.}}}{r(t)}$$

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} & , 0 \leq r < \infty \\ 0 & r < 0. \end{cases}$$

$$\sigma^2 \equiv \frac{E_o^2}{2}$$

↓ pdf of a Rayleigh r.v



Rayleigh fading distribution:

- use envelope detection to detect the signal

$$P(R) \equiv P\left\{ \underset{\substack{\uparrow \\ \text{envelope}}}{R} \leq \gamma \right\} = \int_0^{\gamma} p(r) dr = \boxed{1 - e^{-\gamma^2/2\sigma^2}}$$

$$E[R] = \int_0^{\infty} r p(r) dr = \underset{\substack{\uparrow \\ \text{Rayleigh}}}{R} = \sigma \cdot \sqrt{\frac{\pi}{2}} \approx \boxed{1.25 \sigma}$$

$$\text{Var}(R) = E(R^2) - (E(R))^2 \approx \sigma^2(2 - \pi/2) = \boxed{0.43\sigma^2}$$

Median $(R) = ?$

$$\frac{1}{2} = \int_0^{r_{\text{median}}} p(r) dr$$

$$\Rightarrow \boxed{r_{\text{median}} = 1.177\sigma} \quad (< E(R))$$

Ricean Fading distribution:

— Assumes a non fading LOS component.

— Superimpose on the signal

(dominant component $\rightarrow 0$, Ricean $\rightarrow$ Rayleigh)

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-(r^2 + A^2)/2\sigma^2} \cdot I_0\left(\frac{Ar}{\sigma^2}\right), & A \geq 0, r \geq 0 \\ 0, & r < 0. \end{cases}$$

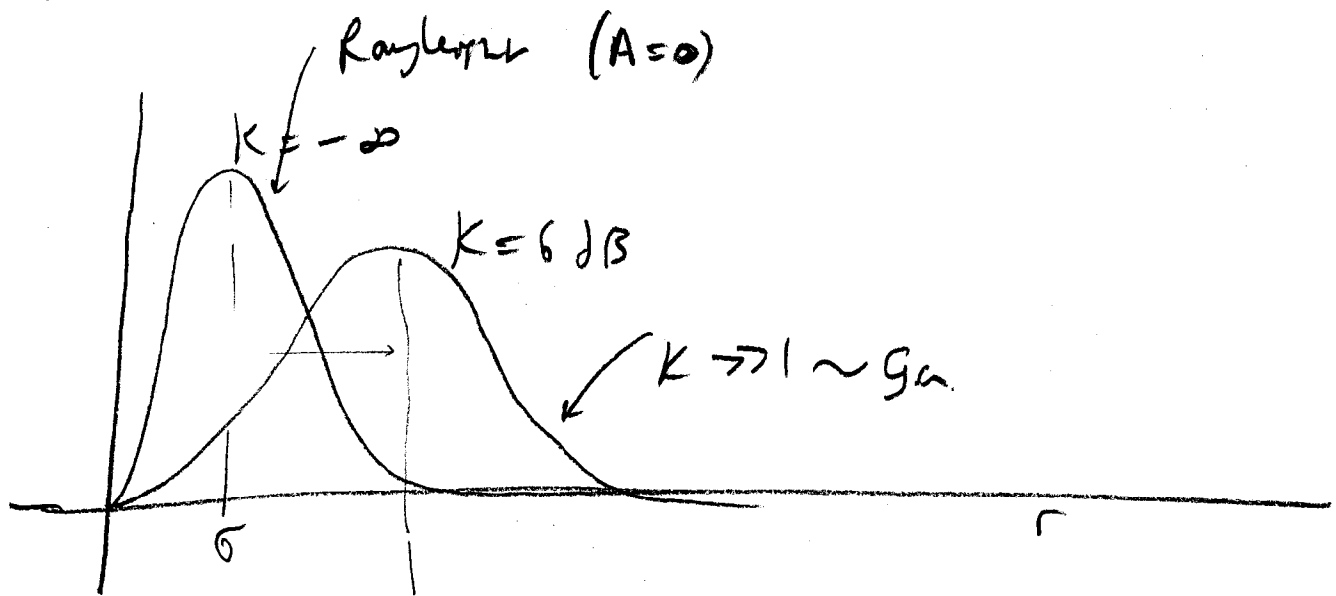
Bessel
1st kind, 0th order

$A \equiv$ peak amplitude of dominant signal

Defns: $K \equiv \frac{A^2}{2\sigma^2}$

$$10 \log_{10} \frac{A^2}{2\sigma^2} \quad dB$$

variance of multipath



- A shifts it to the ~~set~~ right.

For $K \gg 1$ - Rician $\rightarrow$ Gaussian about the mean