Lecture 2 - Some Topics Common in Robot Locomotions

"preliminaries", part 1

This Week: Toy Examples

- Rimless Wheel
- ~~Cart-pendulum system~~

Announcements

- HW1, out tonight (or tomorrow)

  Due ~~Oct. 10 (Thurs.)~~ $\boxed{\text{Oct. 11 (Friday)}}$

  HW1 → HW3 are the basis for Midterm.

- Midterm Nov. 5 (Tues.)

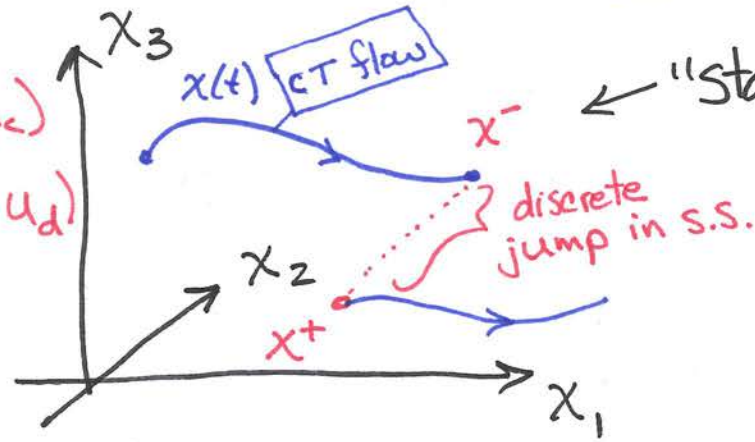
→ email me katiebyl@gmail.com if not on GauchoSpace.

Common to Robot Locomotion:

1. Hybrid systems: Combine both continuous-time (CT) and discrete-time (DT) dynamics.

CT: $\dot{X} = f_c(X, u_c)$

DT: $X^+ = f_d(X^-, u_d)$



← "state space" (s.s.)

← state vector

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

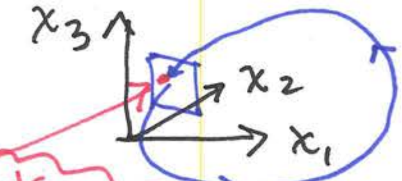
↑ eg. positions & vel's

q: joint angle

DOF: degree of freedom

2. Impacts (e.g., hopping, taking a step, etc. → velocity not continuous)
   - DT part of hybrid system (typically)
3. Limit cycles (& similar) behaviours
   - Poincaré maps (return maps)

   (-cobweb plots)



Snapshots, once per cycle

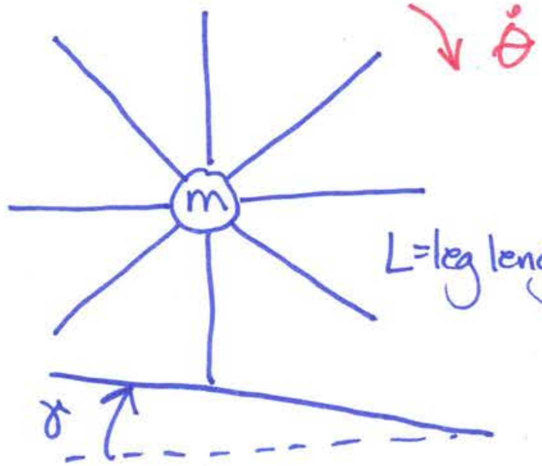
4. Nonlinear dynamics - Lagrangian method, to derive
5. Underactuation (often) - For our Newtonian systems, we cannot arbitrarily pick all $\ddot{q}$'s give our actuators.

   ↖ DOF accel's.

Rimless Wheel (McGeer 90)

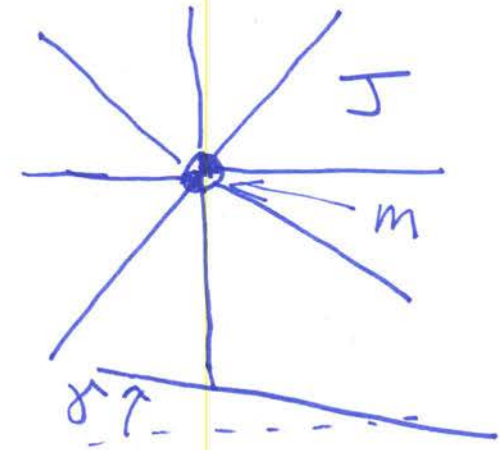
or "I"

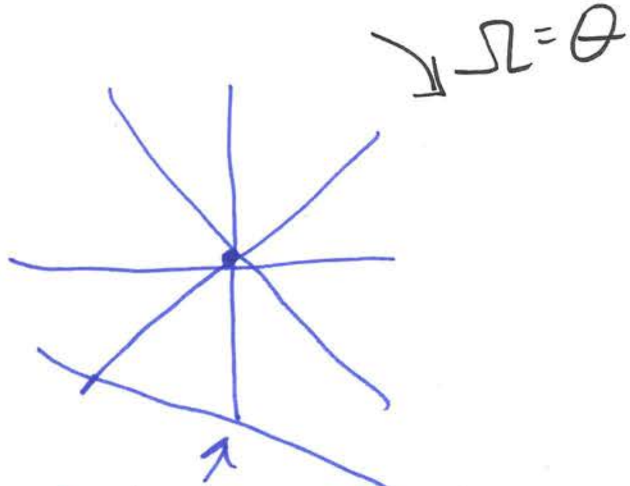
a) point mass, only, m



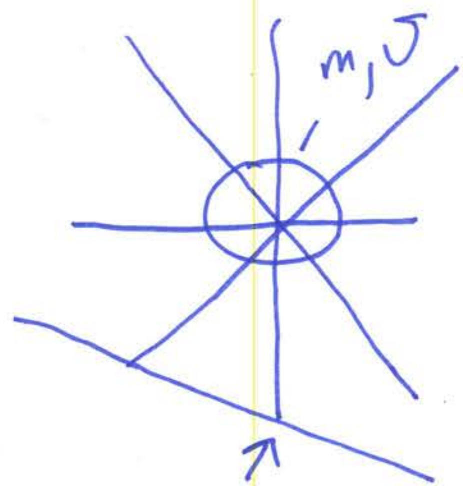
b) distributed inertia, J

McGeer 90 case





balance angular momentum

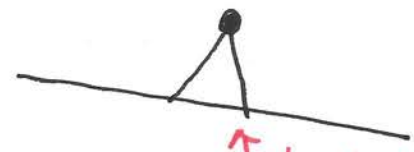


$(J \neq 0)$

Angular momentum __before__ impact $= H^- = H^+$ (after)

$$H = \sum_i \vec{r}_i \times m_i \vec{v}_i$$

(particles)

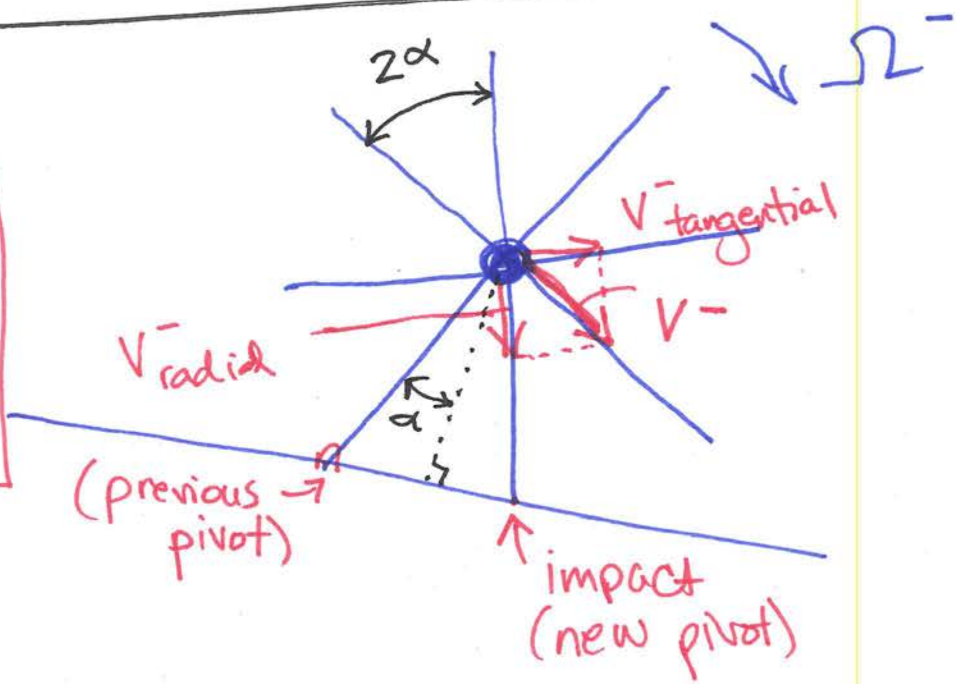


• for a rigid body, we use J (or I) [momentum of inertia], to write more concisely:

$$H \equiv J\Omega = J\dot{\theta}$$

Before impact.

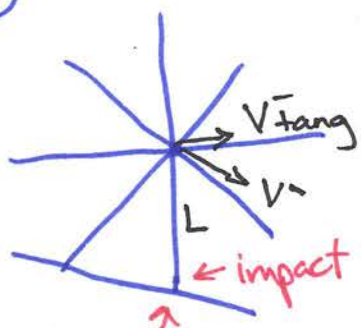
$$\boxed{\begin{aligned} V^-_{tan} &= V^- \cos(2\alpha) \\ V^-_{rad} &= V^- \sin(2\alpha) \end{aligned}}$$



After impact,

- V^-_{radial} is assumed to "disappear", b/c we assume collision is "inelastic".
- Also assume instantaneous collision (impulsive).
- V^-_{tang} contributes to H^- about (new pivot) impact pt.

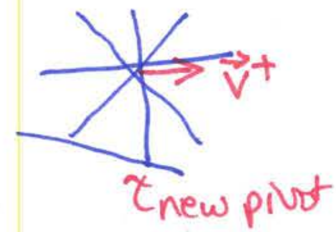
$J=0$



$H^+ = H^-$ wrt this point

$$H^- = \vec{r} \times m\vec{V}^-$$
$$= L m V^-_{tang}$$
$$\boxed{= L\cdot(mL\Omega^- \cos(2\alpha))}$$

$$H^+ = \vec{r} \times m\vec{V}^+$$
$$= L m V^+$$
$$\boxed{= L(mL\Omega^+)}$$



$$H^- = H^+$$

$$mL^2\Omega^- \cos(2\alpha) = mL^2\Omega^+ \rightarrow \boxed{\Omega^+ = \Omega^- \cos(2\alpha)}$$

$J \neq 0$

$$H^- = Lmv^-_{tang} + J\Omega^-$$
$$= (L^2 m\cos(2\alpha) + J)\,\Omega^-$$

$$H^+ = Lmv^+ + J\Omega^+$$
$$= (mL^2 + J)\,\Omega^+$$

$$\Omega^+ = \Omega^- \left(\frac{\cos(2\alpha) + \frac{J}{mL^2}}{1 + \frac{J}{mL^2}} \right)$$

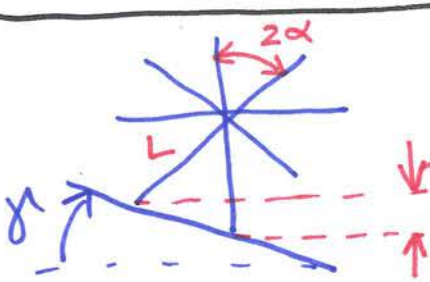
← non-dimensional "radius of gyration"

Note: normally, $r_{gyr} \equiv \sqrt{\frac{J}{m}}$ ← a length.

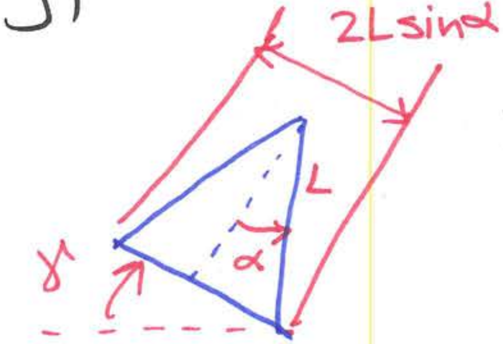
in McGeer90, $r_{gyr} \equiv \sqrt{\frac{J}{mL^2}}$ ← unitless.

For the C.T. dynamics of rolling, we have

<u>conservation of energy</u>:



$$\boxed{\Delta Z = 2L\sin\alpha\sin\gamma}$$



At steady-state,

we reach the same post-impact velocity, for each step...

$\Omega_B^- = \Omega_C^- = \Omega_D^- \dots$ ← sequential impacts.

& $\Omega_B^+ = fn(\Omega_B^-)$ ← last page.

$E_B^+ = E_C^-$ ← cont. dyn. (see notes on web... & HW1)

$$\Omega_{ss}^+ = \eta\,\Omega_{ss}^- = \sqrt{\frac{4mgL\sin\alpha\sin\gamma\,\eta^2}{(2J+mL^2)(1-\eta^2)}}$$