

Tutorial: Limit cycle analysis

Answer sheet

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Exercise 1 Matlab code:

```
par.L = 0.132; par.R = 0.06; par.m = 0.07; par.I = 0.00018;  
par.B = 0; par.C = 0.061; par.gamma = 0.005; par.g = 9.81;  
s0 = [0.2; -0.2; -2.5; -2.0];  
t0 = 0;  
  
[s_end, t_end, data] = Step(s0, t0, par);  
  
plot(data.t, data.s)  
legend('phi_st', 'phi_sw', 'phi_st_d', 'phi_sw_d')  
xlabel('time [s]'), ylabel('angle [rad], angular rate [rad/s]')  
  
Animation(data, par)
```

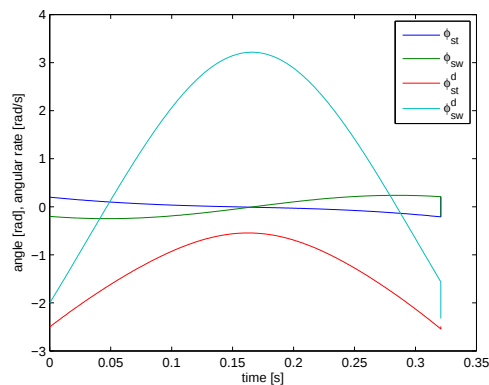


Figure 1: A plot of the four state variables during a step

Exercise 2 Matlab code:

```
nr_steps = 10;  
[s_end, t_end, data] = Walk(s0, t0, par, nr_steps);
```

Exercise 3 Matlab code:

```
optimset('TolFun',1e-10,'TolX',1e-10,'LargeScale','off');
s_star = lsqnonlin(@(s) Step(s,0,par)-s,s0,[],[],options);
```

The state of the limit cycle is:

$$\mathbf{s}^* = \begin{pmatrix} 0.1989 \\ -0.1989 \\ -2.4473 \\ -2.2811 \end{pmatrix}$$

Exercise 4 The limit cycle is unstable and the walker will fall after a number of steps.

Exercise 5 Matlab code:

```
for i=1:4
    s0 = s_star;
    s0(i) = s0(i) + 10e-4;
    J(:,i) = (Step(s0, 0, par)-s_star)'/10e-4;
end
```

The Jacobian is:

$$J = \begin{pmatrix} -1.2920 & -0.0310 & -0.2176 & 0.0258 \\ 1.2920 & 0.0310 & 0.2176 & -0.0258 \\ 2.7426 & 3.1547 & 0.8668 & -0.0808 \\ 1.8657 & 2.2850 & 0.6698 & -0.0854 \end{pmatrix}$$

Exercise 6 The limit cycle is unstable, because one of the eigenvalues has a magnitude larger than 1.

Exercise 7 There are many possible positions for the extra mass on the leg to get a stable limit cycle. The important part of this exercise is that they learn that you cannot just add the extra mass and expect to find a limit cycle. A better approach is to incrementally add the mass and keep tracking the limit cycle solutions.

Exercise 8 One of the eigenvalues is zero, because the definition of the Poincaré Section is that both legs are on the floor. This means that the leg angles at the end of the step are by definition equal and opposite. Any perturbation that violates this definition will completely disappear after one step.