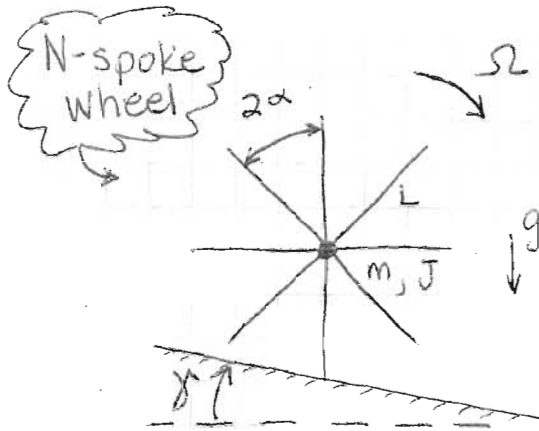


Rimless Wheel

see also: { McGeeer 90
Coleman 02



Ω : angular velocity
 2α : interleg angle

$$\alpha = \pi/N$$

m : (point) mass

J : moment of inertia

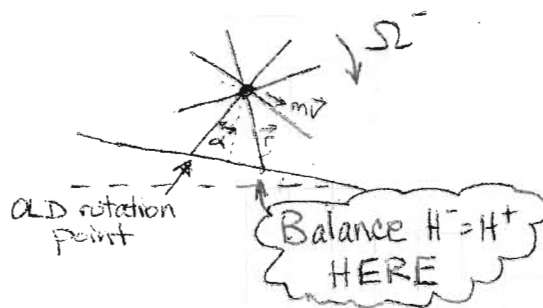
Note - Usually, $J=0$,
and m is a point mass.

γ : ground slope

L : leg length

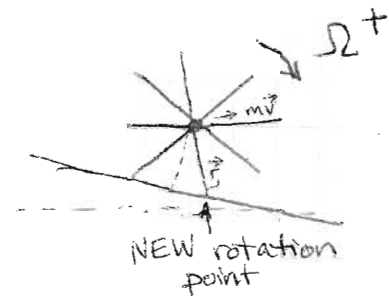
Case "A": Both step $n \frac{1}{2}$ and $n+1$
are rolling DOWNHILL...

1. At impact, angular momentum is conserved:



$$H^- = \sum \vec{r} \times m \vec{v}^-$$

$$= J \Omega^- + L (m L \Omega^- \cos(2\alpha))$$



$$H^+ = \sum \vec{r} \times m \vec{v}^+$$

$$= J \Omega^+ + L (m L \Omega^+)$$

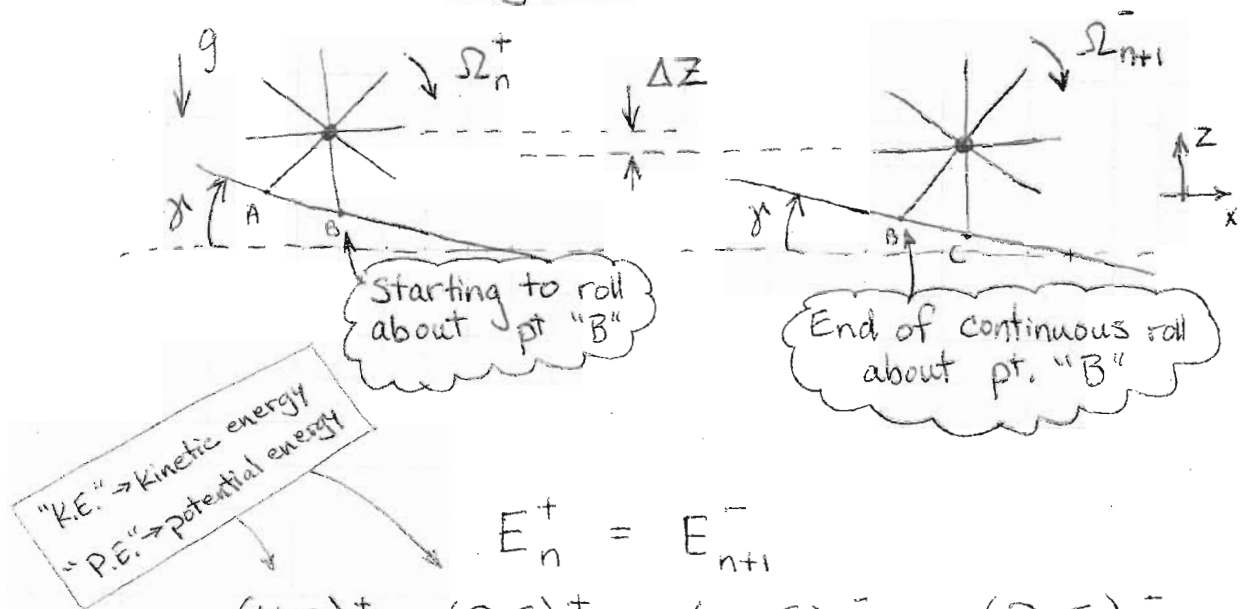
$$\therefore H^- = J \Omega^- + m L^2 \cos(2\alpha) \Omega^- = J \Omega^+ + m L^2 \Omega^+ = H^+$$

So,

$$\frac{\Omega^+}{\Omega^-} = \frac{J + m L^2 \cos(2\alpha)}{J + m L^2} \equiv \eta$$

for $J=0$,
 $\eta = \cos(2\alpha)$
p.1

2. During continuous (rolling) phase,
energy is conserved:

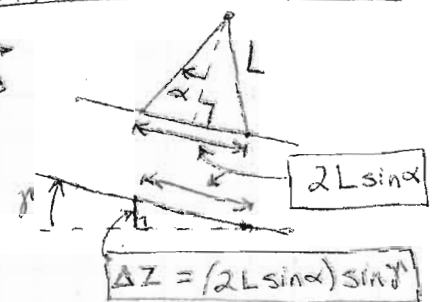


$$E_n^+ = E_{n+1}^-$$

$$(K.E.)_n^+ + (P.E.)_n^+ = (K.E.)_{n+1}^- + (P.E.)_{n+1}^-$$

$$\frac{1}{2}(J + mL^2)(\Omega_n^+)^2 + mg\Delta Z = \frac{1}{2}(J + mL^2)(\Omega_{n+1}^-)^2$$

Calculate ΔZ
from geometry



So,

$$\Omega_{n+1}^- = \sqrt{(\Omega_n^+)^2 + \left(\frac{4mgL \sin \alpha \sin^2 \alpha}{J + mL^2} \right)}$$

3. Combining 1. & 2. :

$$\Omega_{n+1}^+ = \frac{J + mL^2 \cos(2\alpha)}{J + mL^2} \sqrt{(\Omega_n^+)^2 + \frac{4mgL \sin \alpha \sin^2 \alpha}{J + mL^2}}$$

c_n

3. (continued...)

When $\boxed{J=0}$, $\Omega_{n+1}^+ = \cos(2\alpha) \sqrt{(\Omega_n^+)^2 + 4 \frac{g}{L} \sin\alpha \sin\delta}$

Case "B" → What if: " $mg\Delta Z_{\max} > \frac{1}{2}(J+mL^2)(\Omega_n^+)^2$ "?

• Teeters & falls "backward".

• $\Omega_{n+1}^- = -\Omega_n^+ \leftarrow (\text{no } \Delta P.E.)$
(reverses direction)

• $\boxed{\Omega_{n+1}^+ = -\eta \Omega_n^+}$

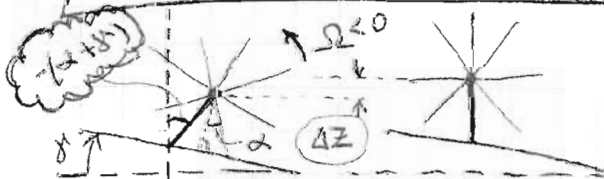
Case "C" What if: " $\Omega_n^+ \dot{\epsilon} \Omega_{n+1}^+$ are both uphill"?

• Loses potential energy, instead of gaining.

$\Omega_{n+1}^+ = -\eta \sqrt{(\Omega_n^+)^2 - \frac{4gL \sin\alpha \sin\delta}{J+mL^2}}$

uphill is negative velocity

Finally, what are $(\Omega_{\text{crit}}^+)_{\text{downhill}}$ and $(\Omega_{\text{crit}}^+)_{\text{uphill}}$, which bound the range of "teetering" (Case "B")?



$\Delta Z_{\max} = L - L \cos(\alpha + \delta)$

K.E. = ΔP.E.,

$\frac{1}{2}(J+mL^2)(\Omega_{\text{crit}, \text{up}}^+)^2 = mgL(1 - \cos(\alpha + \delta))$

$(\Omega_{\text{crit}}^+)_{\text{up}} = \sqrt{\frac{2mgL(1 - \cos(\alpha + \delta))}{J+mL^2}}$

negative since point!



$\Delta Z_{\max} = L(1 - \cos(\alpha - \delta))$

So, balancing K.E. = ΔP.E.,

$(\Omega_{\text{crit}}^+)_{\text{down}} = \sqrt{\frac{2mgL(1 - \cos(\alpha - \delta))}{J+mL^2}}$