

Homework 2 (due 1/27)

- 2.1 Rimless Wheel (RW) return map.** In this problem, you are asked to annotate Figure 1, in which the solid (blue) lines show the return map relating the post-collision angular velocity after a given step, Ω_n^+ , to the velocity after the *next* step, Ω_{n+1}^+ . (Note that Coleman02 [1] plots return maps on “Z”; Figure 1 maps Ω itself.) Hint: *Grab a ruler!* Also, download the “RW cheatsheet” from the homework website. (Assume a unit length leg ($L = 1$ [m]), point mass ($J = 0$), and 8 spokes ($2\alpha = 2\pi/8$).)

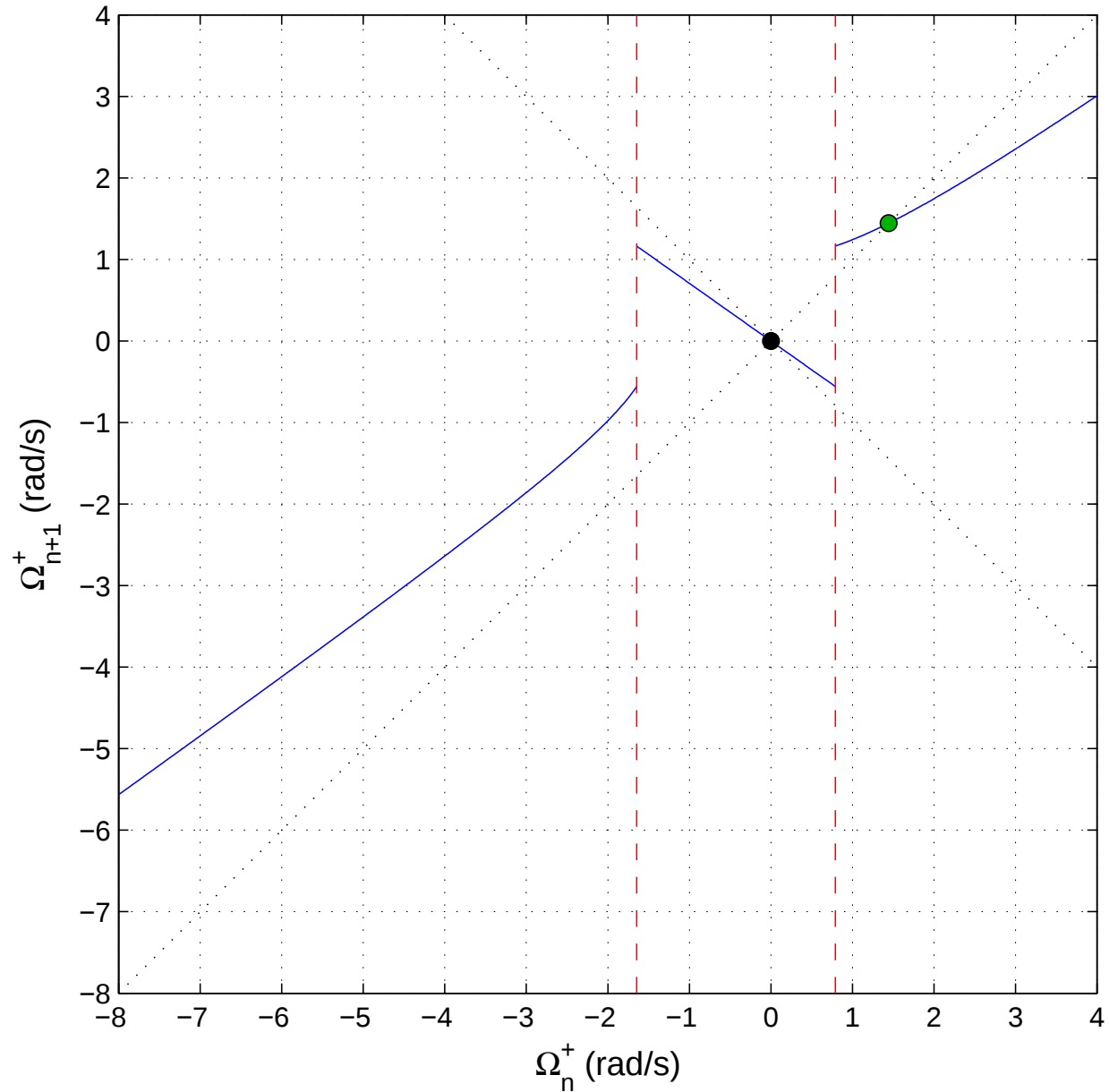


Figure 1: Rimless Wheel return map.

2.1 Rimless Wheel (RW) return map. (continued...)

- From Figure 1, what (approximately) is the value of the *rolling* fixed point, Ω_*^+ ?
- Given your estimate of Ω_*^+ (from part a), estimate the angle of the ground, γ .
- What is the slope of the return map near Ω_*^+ ?
- Given: “ $\Omega_n^+ = \Omega_*^+ + .01$ ”, estimate Ω_{n+1}^+ . *Hint: Use part c.*
- Graphically find the **basins of attraction** for the standing and rolling fixed points, using a “stair step” approach. Shade in the portions corresponding to the “rolling” fixed point. *Hint: download the m-files called “RW_return.m” and “RW_stairstep.m” from the homework webpage. Run RW_return to create a return map. Then run RW_stairstep to graphically click on the figure to create stair steps to the appropriate fixed point for this initial condition. You can continue to run RW_stairstep again and again, to overlay many stair step traces. Note that the return map shown will (intentionally) be for a different value of ground slope, γ .*

2.2 Limping Rimless Wheel. Consider a rimless wheel with 8 evenly-space spokes (i.e., same angular spacing) but where every other leg is 1.05 meters while the others are 0.95 meters. For a given range of ground slope, and for the right initial conditions, this wheel will converge to a constant, “limping” gait: every impact landing on a “long” leg will approach some value $(\Omega_*^+)_{long}$, and every impact landing on a “short” leg will approach some value $(\Omega_*^+)_{short}$.

- Derive these two values, $(\Omega_*^+)_{short}$ and $(\Omega_*^+)_{long}$ for rolling on a downhill slope of 10° . *Hint 1: Use the “RW cheatsheet” you downloaded for problem 1. Hint 2: Recall the law of cosines, to calculate any unknown-but-needed variables from Figure 2: $L_a^2 = L_b^2 + L_c^2 - 2bc \cos \theta_a$*

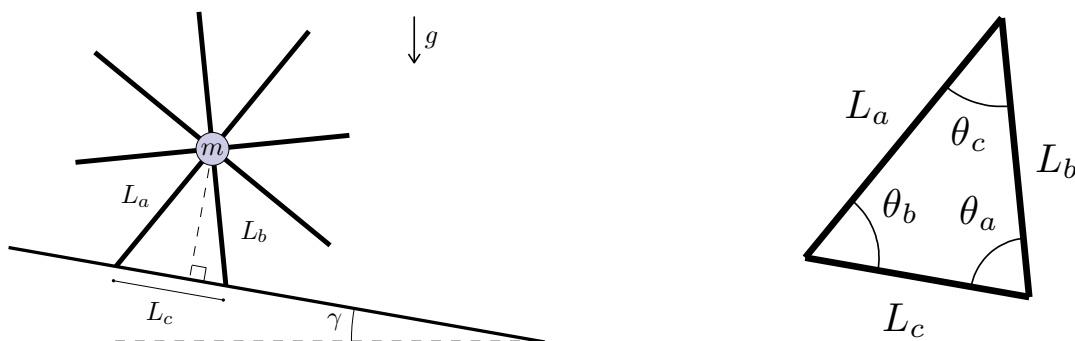


Figure 2: Limping rimless wheel geometry.

2.3 Compass Gait (CG) walker phase portrait. In this problem, you are asked to analyze a model of passive dynamic walking (PDW).

- a) From the phase portrait shown in Figure 3, estimate the state $X^* = [\theta_{ns}, \theta_s, \dot{\theta}_{ns}, \dot{\theta}_s]^T$ of the fixed point for a Poincaré section at the “post-collision” state.

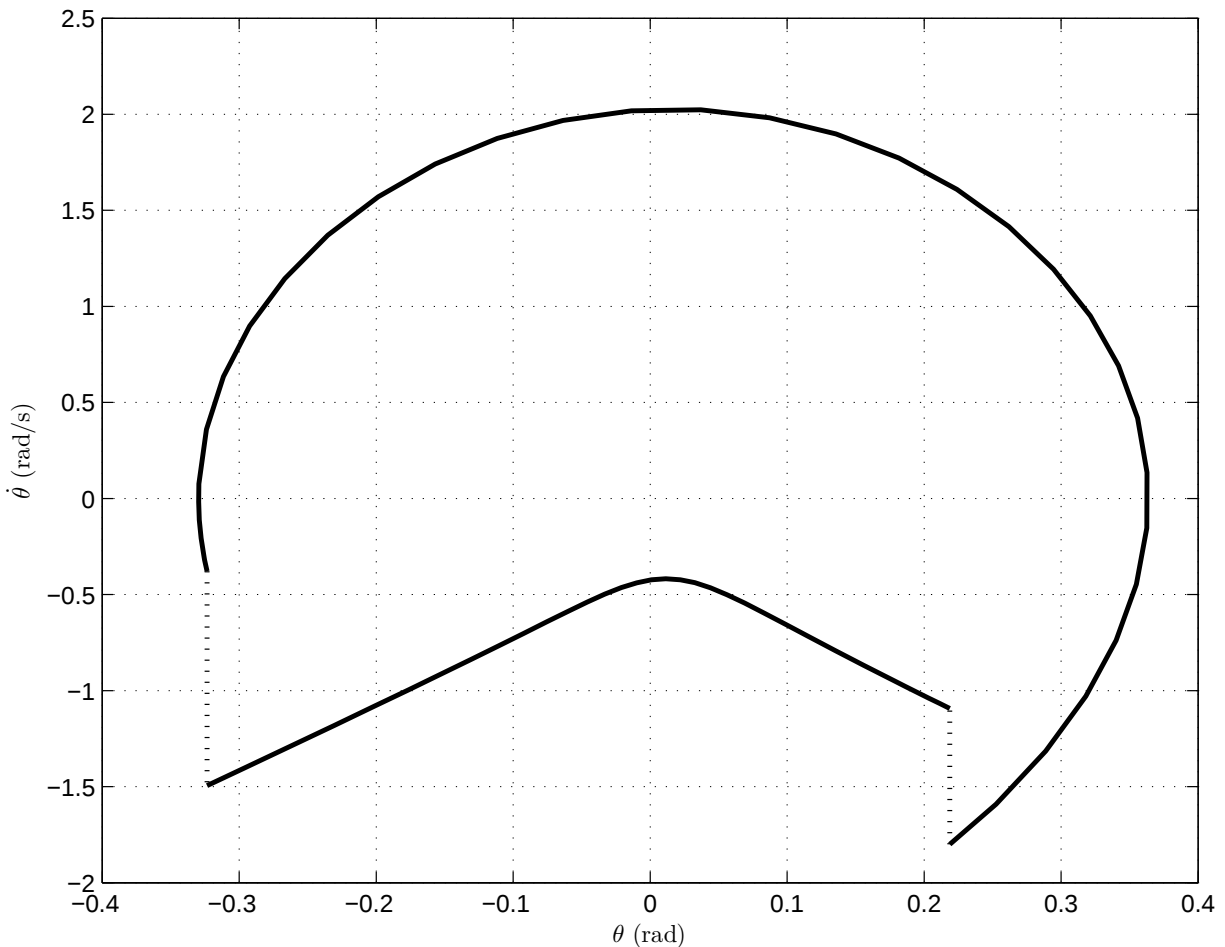


Figure 3: Compass Gait phase portrait.

- b) Download all the “compass gait simulation files” for MATLAB from the web, and use the function “cg_step.m” [which requires the other m-files to run] to create the 4×4 Jacobian.
Hint 1) Start with your estimate of the fixed point from part a, above. Use this 4×1 state as an input to the function “cg_step”, and you will get the next post-impact state as an output. e.g., create a “for loop” where “ $X_{\text{fixed_approx}} = \text{cg_step}(X_{\text{fixed_approx}})$ ”. This will simulate successive steps, each of which should asymptotically approach the true fixed point.

Hint 2) Once you have a better estimate of the fixed point, you can now use “cg_step” to generate each of the 4 columns of the Jacobian, J . See the DW2008 Tutorial Assignment and Tutorial Answers for better details:

http://www.ece.ucsb.edu/courses/ECE594/594D_W10Byl/notes/DW2008/DW2008_index.html

Also look for additional course notes on the Jacobian on the “notes” website.

- c) Find the eigenvalues of the Jacobian (which give information about the local stability of the step-to-step transitions in a neighborhood near the fixed point). Is the fixed point stable? Why (or why not)?
- d) One eigenvalue should be very close to zero (numerically). Why is this so? (*Read the DW2008 Assignment and Answers, mentioned in part a...*)
- e) Looking again at the phase plot in Figure 3, is $\theta_s(t)$ (the stance leg angle) monotonic over a given step? Is $\theta_{ns}(t)$ (the non-stance leg angle) monotonic over a step?

2.4 [EXTRA CREDIT] Compass Gait period-doubling bifurcations. Something interesting happens to the gait of the passive compass gait walker in problem 2.3 as the slope of the terrain gets steeper: It begins to “limp”! Beyond some particular slope angle, a walker which begins with appropriate initial conditions (within the basin of attraction for passive walking) will converge to a gait where every post-collision state for the “left” foot approaches one particular set of values while every footstep with the “right” foot approaches a different set.

- a) First, just find a slope angle where this occurs!
To do this, you can start with your fixed point solution from problem 2.3, increase the slope of the ground (γ) in the simulation code by *a small amount*, and then take several steps until the walker converges to a new post-collision fixed point. Repeat until you notice the post-collision states alternate with every other step. To change the value of γ , run the MATLAB function called `cg_gam_slope`. For instance, entering the command “`cg_gam_slope(3.2*pi/180)`” would reset the ground slope to 3.2° .

If you change the slope by only a small amount, you should be able to begin with your previous fixed point solution and simulate several steps. Create a loop where you increase the slope and find a new fixed point, again and again – and you should eventually notice that the post-collision state alternates between two sets of values. This is a bifurcation, and as the slope gets steeper and steeper, you should notice a gait which repeats every 4 steps, then every 8 steps, ... and eventually becomes “chaotic” (where you can no longer find any repeating pattern at all – although the walker still continues to walk on and on and on in an apparently stable way).

- b) Now that you have a slope that exhibits a “period-2” gait, find the one-step Jacobian, as you did in problem 2.3 b). What are the eigenvalues? Do they all have a magnitude less than one (indicating stability for the step-to-step transitions)?

- c) Finally, create a Jacobian on a two-step mapping. Use the same general procedure you used in problem **2.3**, but execute **two** steps instead of one step when filling in each column. (e.g., `cg_step(cg_step(Xleft))`) What are the eigenvalues now, and what do they tell us about the stability of the two-step return map?

References

- [1] Michael J. Coleman and Andy Ruina. Motions of a rimless spoked wheel: a simple 2d systems with impacts. August 2002.