1/20/2010

"Zero Moment Point" (ZMP) [Kajita Ø3]

4 classic problems in ROBOTICS are:

- Forward KINEMATICS → by geometry, joint ∡'s set location of "end effector"
- Inverse KINEMATICS ("IK") → can have redundant (or NO) solutions
- Forward DYNAMICS → $F = ma$, so $\ddot{x} = \frac{1}{m}F$, essentially
- Inverse DYNAMICS

Problem: How to obtain desired forces?

- Specifically for ZMP PREVIEW CONTROL:
  - What is a set of prescribed motions that will achieve a desired "trajectory" of ground reaction forces over time?

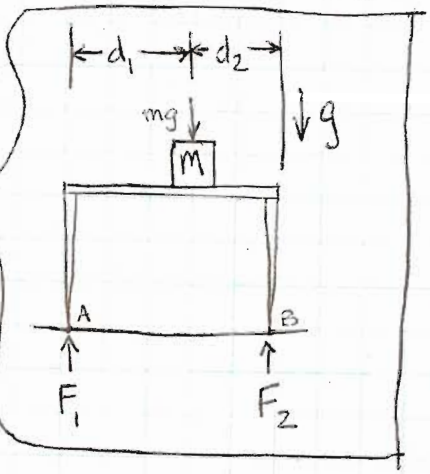
(I) "Center of Pressure" (CoP)

Given the geometry shown → (with a static mass resting on a table with "point feet"),

a) What is the force at each foot?

b) Where is the "center of pressure"?

(Assume table has zero mass...)



Answers:

a) Balance forces & moments. System is at rest, so:

$\sum M_A = 0$, where each "moment" is "$\vec{r} \times \vec{F}$".

$$0 = \sum M_A = -d_1 \cdot mg + (d_1 + d_2) \cdot F_2$$

$$\therefore \boxed{F_2 = \left(\frac{d_1}{d_1 + d_2}\right) mg}$$

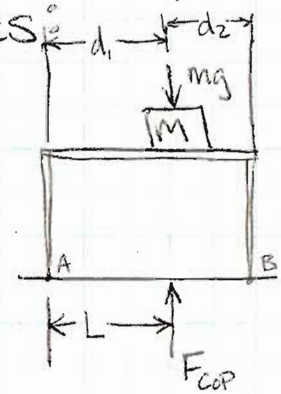
or, once F_2 is known, $F_1 = mg - F_2$

$$\boxed{\therefore F_1 = \left(\frac{d_2}{d_1 + d_2}\right) mg}$$

$$0 = \sum M_B = d_2 \cdot mg - (d_1 + d_2) \cdot F_1$$

(I) CoP, cont'd...

b) We can replace F_1 & F_2 with a <u>single</u> force at the ground, called the "Center of pressure" (CoP). It's location, magnitude (& direction, when we also have lateral forces) must <u>also</u> satisfy both <u>force</u> & <u>moment</u> balances:



$$0 = \sum F = F_{CoP} - mg$$

$$\therefore \boxed{F_{CoP} = mg}$$

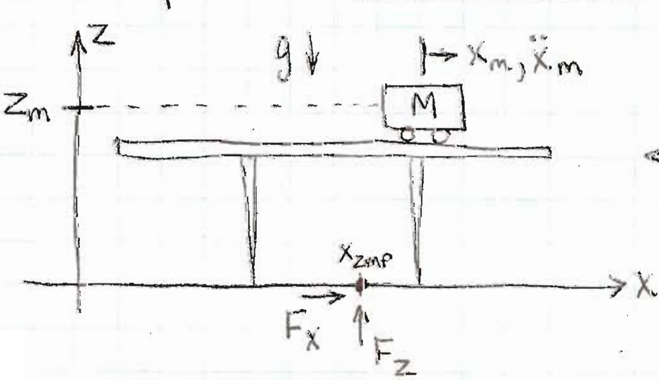
$$0 = \sum M_A = -d_1 \cdot mg + L \cdot F_{CoP}$$

$$\therefore \boxed{L = d_1}$$

(II) Zero Moment Point (ZMP)

Roboticists often refer to the center of pressure as the "zero moment point", exactly b/c it is the location on the ground where you can replace the individual "ground reaction forces" with a single, net force vector and THE <u>MOMENTS ABOUT THIS POINT</u> WILL BE <u>ZERO</u>. (as shown in (I) b)

a) Example 1: $\boxed{\text{"cart-table mode"}}$ See Fig. 3 in Kajita '03



← Here, we have 3 unknowns:

x_{zmp} (location in x of zmp)

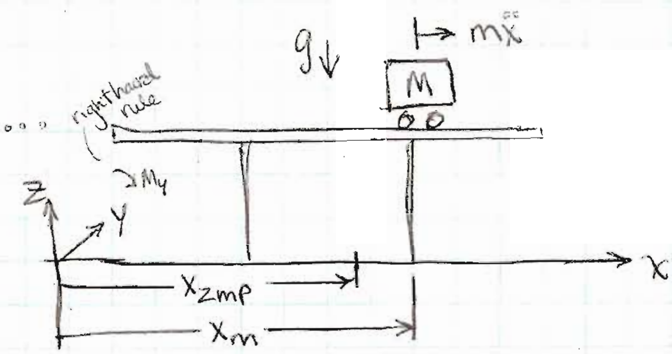
F_x (ground react. f. in x)

F_z (gr. react. force in z)

(z_{zmp} must be $\emptyset$.)

(II) ZMP, cont...

a) cart-table, cont...



Force balances in x & z require that:

$$F_x = m\ddot{x}$$

$$F_z = mg$$

A moment balance at the ZMP yields

$$\sum M_{zmp} = 0 = \vec{r}_x \times \vec{F}_z + \vec{r}_z \times \vec{F}_x$$

$$= -(x_m - x_{zmp}) \cdot mg + (z_m - \cancel{z_{zmp}}^{\,0}) \cdot m\ddot{x}$$

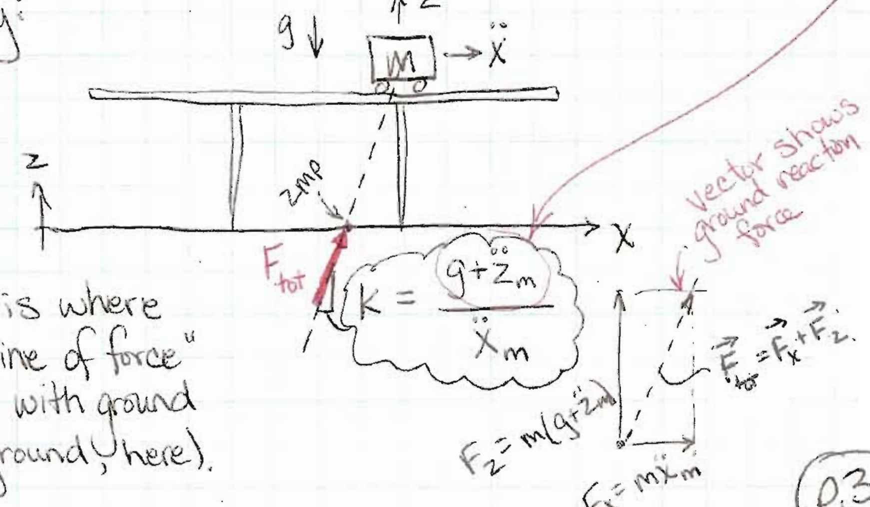
$$\vec{i}_x \times \vec{i}_z = -\vec{i}_y \qquad \vec{i}_z \times \vec{i}_x = +\vec{i}_y$$

$$\therefore X_{zmp} = X_m - Z_m \frac{\cancel{m}\ddot{x}}{\cancel{m}g}$$

$$\boxed{X_{zmp} = X_m - \left(\frac{\ddot{x}}{g}\right) Z_m}$$

Note, we can also include any accel. in Z, "$\ddot{Z}_m$": $g \to g + \ddot{z}_m$

Here is how that "moment balance" can be visualized, pictorially:

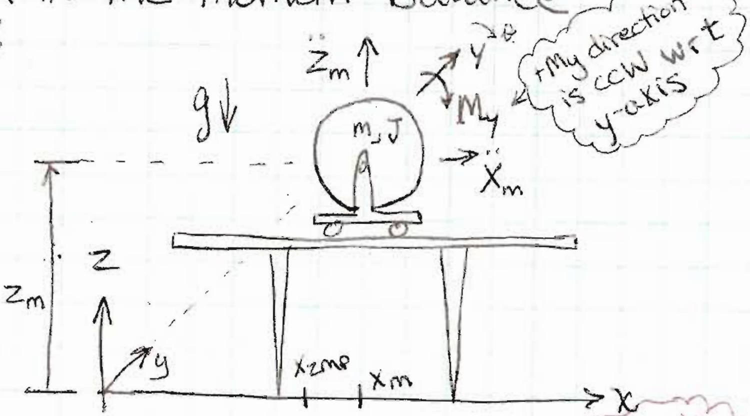


The zmp is where the net "line of force" intersects with ground (for flat ground, here).

(III) Moments - impact on ZMP location?

Imagine our "cart" now has an inertia on it ("inertia wheel") that can rotate.

- This simply enters in as an additional moment term in the moment balance at the ZMP:



$$\sum M_{zmp} = 0 = \vec{r}_x \times \vec{F}_z + \vec{r}_z \times \vec{F}_x + M_y$$

$$0 = -(x_m - x_{zmp})\,m(g+\ddot{z}_m) + z_m m \ddot{x}_m + M_y$$

moment exists if inertia (J) is accel'ing

Rearranging, to solve for the zmp location:

$$x_{zmp} = x_m - \frac{z_m \ddot{x}_m + \frac{1}{m} M_y}{(g + \ddot{z}_m)}$$

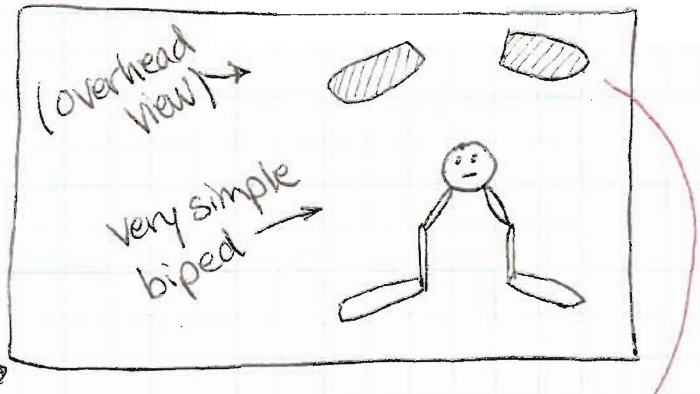
note: $M_y = \tau = J\ddot{\theta}$; so:

$$\boxed{x_{zmp} = x_m - \frac{z_m \ddot{x}_m + \frac{J}{m}\ddot{\theta}}{(g+\ddot{z}_m)}}$$

ang. accel of the reaction wheel

(IV) How/why do roboticists use "ZMP"?

Imagine a "robot" as a system with various masses & inertia moving – And with some "footprint"... (at right) →



Now, imagine a rubber band encloses the total "footprint(s)" →

The "convex hull" that the imaginary "rubber band" traces out is known in robotics as the "support polygon".

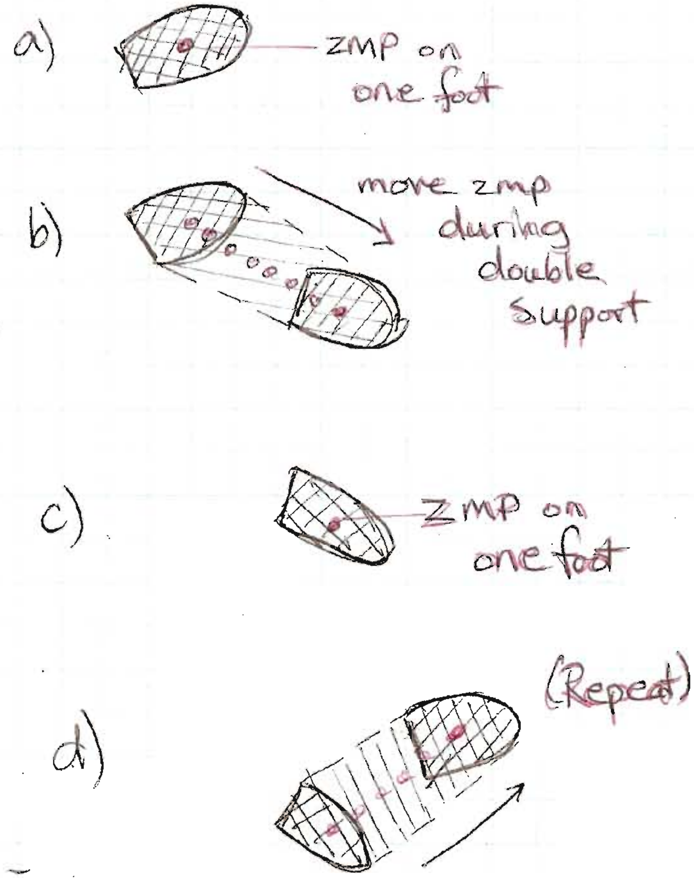
A common goal in much of (particularly in much of "humanoid") robotics (and MOST) is to plan motions such that we "guarantee" the stance feet remain on the ground & the robot does not tip over.

 → If all the planned motions result in an expected zmp that is inside the support polygon, then we can rely on the "support polygon" as a stable base – making the robot an "almost fully-actuated system", (er, until the real world & various uncertainties eventually result in being knocked over, anyway... "Murphy's Law".)

 → BUT, if the planned motions result in an expected zmp that is outside the support polygon – well, then our "plan" cannot be successfully executed – at the robot will begin to tip over the edge of the support polygon (to satisfy the REQUIRED moment balance!!!).

(V) So then: What is "Preview Control" of the ZMP?

- High degree of freedom (DOF) robots (such as ASIMO or HRP-(whatever)) are generally designed with <u>HIGH GEAR RATIOS</u>, which allow them to track desired joint angles with very high accuracy.
- If we "pre-compute" motions that satisfy the zmp constraint (i.e., zmp <u>must</u> be in the support polygon!), then we can count on high accuracy in joint angles - which (we expect) results in high accuracy for the ZMP location.



- The basic idea in "Preview Control" of the zmp is to:
  1) First plan a desired set of footholds
  2) Design a trajectory for the "<u>desired</u>" ZMP (to stay <u>inside</u> the support polygon).
  3) Solve an "inverse dynamics" problem, which finds some solution (e.g., an "optimal" one, in some sense or other) that results in an <u>actual</u> ZMP that is close to desired.

- So, the preview control really {GENERATES A MOTION PLAN} to meet <u>DYNAMIC CONSTRAINTS</u>. {Then, use your favorite flavor of control to ATTAIN this motion plan!}