# HW#4: Common Loss Mechanisms; Gaussian Beams

1. Skin Effect Losses: An important prediction of electromagnetics at THz frequencies is the skin effect whereby the component of the electric field parallel to the surface of a good conductor is confined to a progressively thinner layer as the frequency increases.

(a) Calculate the skin depth in five metals common to high-speed electronics: Cu, Au, Al, Cr, and Sn, and at four frequencies: (1) 10 GHz, 100 GHz, 1.0 THz, and 10.0 THz.

(b) Derive an expression for the specific series resistance of a metal strip on an insulating substrate of rectangular cross section 0.5 x 9.0 micron, assuming the current is confined uniformly to a skin depth around the periphery and is not affected by the substrate.

(c) Assume this strip is used as the center conductor of a miniature coaxial line designed to have a characteristic impedance of $Z_0 = 50\ \Omega$ and a specific capacitance of 1.0 pF/cm. Use the lossy TEM transmission line model of Fig. 1 to predict the absorption per unit length in this line at 100, 300, and 600 GHz (no need to derive the specific attenuation for this model; just take it from any good E&M book, such as F. Ulaby, Applied Electromagnetics).

2. Total Internal Reflection: Another universal phenomena from electromagnetics that impacts THz component design is total internal reflection, derivable from Snell's law.

(a) Look up the THz-region dielectric constant and calculate the critical angle for the following dielectric materials: (1) Teflon, (2) Rexolite, (3) Mylar, (4) single crystal quartz, (5) single crystal alumina (sapphire), (6) high-resistivity silicon, and (7) semi-insulating GaAs.

(b) Given the ideal dipole antenna radiation pattern of HW#3, and assuming it is fabricated on a dielectric slab what fraction of its radiation would be trapped in the slab for each of the 7 materials of (a) if it is operated at its full-wave resonance ?

3. Gaussian Beams: One of the useful applications of the $TEM_{00}$ Gaussian-beam formalism is in predicting the propagation of radiation through optical components, such as lenses. A common scenario is the lens-focusing problem shown in Fig. 2. A $TEM_{00}$ Gaussian beam having beam waist $\omega_{01}$ is transformed by a thin dielectric lens of focal length f. The distance between the waist location and the lens location is $z = d_1$.

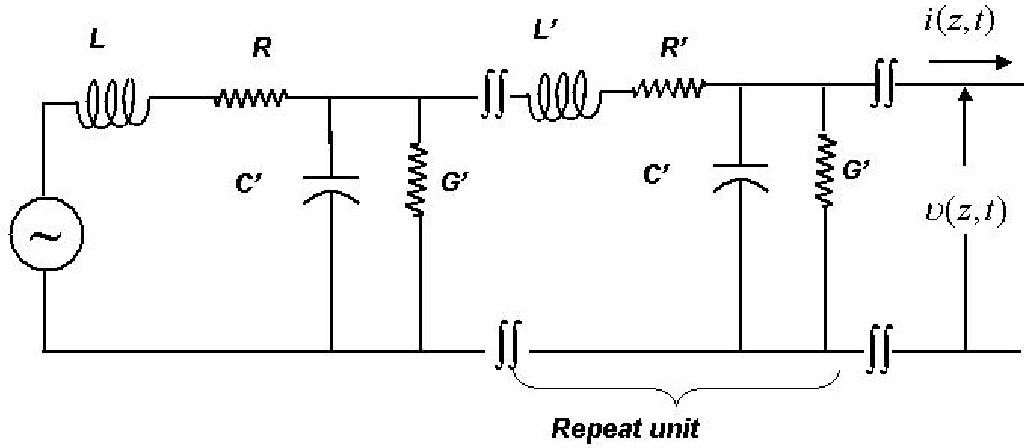
(a) Find the condition on f in terms of $\lambda$, $\omega_{01}$, and $d_1$, which guarantees that a second beam waist $\omega_{02}$ occurs somewhere to the right side of the thin lens.

(b) Using you favorite graphical tool, plot the distance $d_2$ vs f for $\nu = 1$ THz, $\omega_{01} = 2$ mm, and $d_1 =20$ cm. How many values of f can create a given $d_2$ ?

(c) Solve for the possible values of f when $d_2 = 10$ cm ? Of the possible values, which is the most practical one, and why ?

(d) A practical question, often faced in THz experiments and systems, is how to form the smallest possible $\omega_{02}$ when $\omega_{01}$ is given (i.e., you already have a source), f is given (i.e., you already have a lens) but $d_1$ and $d_2$ are variable. Start by finding an expression for $\omega_{02}$ in terms of $\omega_{01}$, $d_1$, $d_2$, and f. Qualitatively, what does this solution suggest for the best values of $d_1$ and $d_2$ relative to f ? According to this solution, how small can $\omega_{01}$ be ?

(e) In practice $d_1$ can not be made arbitrarily large because of "spillover" of the source Gaussian beam at the lens. And $d_2$ can not be made arbitrarily large because of limits on the size of the experiment or instrument. This leads to the common "constrained" problem whereby $d_1 + d_2$ is limited to a maximum value of D. Fortunately, the relatively long wavelengths in the THz region (compared to the visible or infrared) allow one to fabricate lenses rather easily, so the choice of f is great. Using elementary calculus, determine the values of $d_1$ and d2 ιν τερμσ οφ φ ανδ Δ τηατ μινιμιζε $\omega_{02}$. And write the expression for the minimum $\omega_{02}$.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R
L'
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Fig. 1

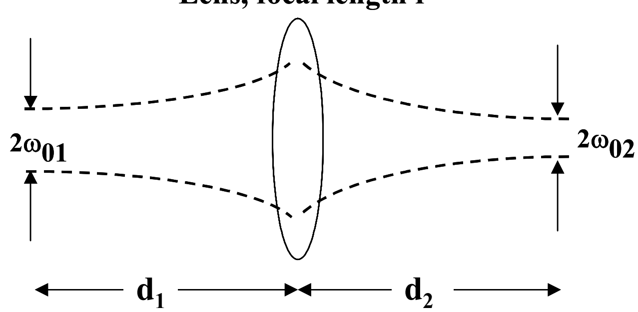
Lens, focal length f
2ω01
2ω02
d1
d2

Fig. 2.