ECE594I, Prof. Brown, Fall Quarter 2008

HW#5: Noise Mechanisms, Noise Figure, Detection, and Dynamic Range

1. (Device Noise) So many devices or parts of devices in RF and THz electronics display the canonical junction rectification I-V curve, $I = I_0 [\exp(eV/k_BT) - 1]$. Suppose the device is a one-port that displays generalized Johnson-Nyquist noise and full shot noise, and operates at 300 K into a load resistance R_L .
(a) For $R_L = 0$ (short-circuit case) and zero bias, what is the RMS current fluctuation through the device in terms of I_0 ? Interpret the answer in terms of the necessary device physics behind I_0
(b) If $R_L = 0$ (short-circuit case), at what bias voltage does the RMS shot-noise current equal the RMS thermal noise current ?
(c) Suppose this device is connected to an $R_L = 50\text{-}\Omega$ load. At what bias voltage does the noise power generated in the load from the rectifier equal the noise power from the load itself ?
(d) Express the noise power in (c) as a power spectral density of current fluctuations.

2. (System Noise Figure) Every linear component in a receiver chain can be represented by a noise figure, $F_i = (SNR_{in}/SNR_{out})i = 1 + (N_{add})_i/(G_i\, k_BT_{300}\, B_i)$, where $(N_{add})_i$ is the noise added by the ith component, G_i is its gain, and B_i is its equivalent noise bandwidth.
(a) Use these definitions to derive Friis' formula for total noise figure, F_T.
(b) Suppose a receiver chain consists of a bandpass filter (BPF) having G = -3.0 dB and F = 0.5 dB, an LNA having G = Y dB and F = 2.0 dB, a mixer having G = -5 dB and NF = 6 dB, and a power amp having G = 30 dB and F = 10 dB. What is the total noise figure if Y = 30 dB ? [clue: easy and fun with an Excel spreadsheet]
(c) In the limit of very large Y, the noise figure approaches asymptotically a "front-end" limited value. What is this value ? And to what value can Y be reduced so that the actual F is within 1 dB of the "front-end" limit ?
(d) Suppose the BPF and LNA are interchanged. What is the new F_T if Y = 30 dB ?

3. (Power-Law Detection) A common detector type used in THz sensors, particularly at low signal levels, is the square-law device represented by $X_{out}(t) = A[X_{in}(t)]^2 \equiv \Re P_{in}$.
(a) Of all the possible power-law detectors $X_{out} = A(X_{in})^n$, show that the square-law is the most effective at maximizing the ratio of baseband (dc) power to input (THz) power.
(b) Assume a nonlinear diode is biased into a region where the I-V curve can be approximated to have the form $I = BV^2$. Use Taylor's theorem to write a general expression for the rectified current $\delta I = I - I_0$ in terms of the voltage difference δV induced at some bias point V_0. Re-express this result as $\delta I = \Re\, P$, where $P = (\delta V)^2/R$ and $\Re$ is the short-circuit responsivity. Write an expression for $\Re$ in terms of known derivatives.
(c) One of the most useful devices through the history of THz sensors has been the metal-semiconductor, or Schottky, diode. A convenient model I-V curve for Schottky diodes is $I = I_S\{\exp(eV/nk_BT) - 1\}$, where n ("ideality" factor) > 1. Formulate the square-law short-circuit $\Re$ for a Schottky diode. Evaluate this responsivity at 300 K assuming n = 1.0 – the best possible case.

4. The *linear dynamic range* is an important metric in THz and all RF sensors, defined as the difference in input power between the Rx noise power and the level that makes the receiver output compress by 1 dB (from nonlinearities in the electronics).

Suppose a receiver chain consists of a bandpass filter having G = -3.0 dB and F = 0.5 dB, an LNA having G = 30 dB and F = 2.0 dB, a mixer having G = -5 dB and NF = 6 dB, and a power amp having G = 30 dB and F = 10 dB.

a) If the LNA has the transfer function $P_{out} = G_0P_{in}$ /[$1+(G_0P_{in}/P_{sat})$] where G_0 is the small-signal gain and all other components are assumed to be perfectly linear, what values of P_{in} set the 3- and 1-dB compression points of the receiver when expressed in terms of P_{sat} and G_0 ? (note this is a useful gain-compression characteristic in many first-pass system calculations) Evaluate these Pin values for P_{sat} = +20 dBm and G_0 = 30 dB.

b) Make a plot of P_{out} vs P_{in} for the receiver on a log-log axis showing clearly the noise floor and 1-dB compression level if the bandwidth is assumed to be 1 MHz.

c) From this plot, or from the calculations leading to the plot, evaluate the linear dynamic range of this receiver.