Problems#6: Good THz System Problems

1. (Passive and Active Systems). Suppose you are asked to assess THz stand-off detection for determining the presence of a human body at a range of 30 ft (this could be practical in situations, such as in a smoke-filled room, where IR sensors do not work).

(a) Consider determining the presence of the body using a low-cost double-sideband heterodyne radiometer having a 50-dB gain antenna (e.g., small telescope). Assume the receiver noise temperature is 5000 K (single sideband), the center frequency is 650 GHz, the IF bandwidth is 1%, the body brightness temperature is 37 C, the emissivity is 0.8, and detection is done with an ideal square-law device. What is the before-detection SNR in this situation if the antenna is pointed directly at the body ? How long would the Rx have to integrate to get a post-detection SNR of 10 ? What would the NEΔT of the receiver be in this case ? What would the post-detection SNR and NEΔT be if the integration time was 1 s ?

(b) Now consider using direct detection with a Schottky diode rectifier having an optical NEP $1\text{x}10^{-12}$ W/Hz ^{½}, the same antenna and center frequency as above, but an instantaneous bandwidth of 20%. What would the post-detection SNR and NEΔT be with a 1-s integration time ? Again, how long would the receiver have to integrate to achieve a SNR_{PD} of 10 ?

(c) Finally, consider using a cw monostatic radar. The receiver is the same as that applied in (a) and the transmitter is a coherent sinusoid at frequency 650 GHz operating through the same antenna. What is the received power and SNR_{BD} from the body if the transmit power is 1.0 mW and the reflectivity of the body is 50% ? What is the noise-equivalent transmit power, NEP_T ? How much transmit power would be required to achieve a pre- and post-detection SNR equal to that of the radiometer in (a) with a 1-s int time ?

(d) In order to reduce the NEP_T of the radar in (c) the prudent solution is to limit the IF bandwidth greatly before the square-law detector. What would the IF bandwidth have to be to achieve a post-detection SNR of 10 with 1 mW of transmit power (easily achieved with modern SSPA technology) and 0.03 s integration time (suitable for real-time imaging) ?

[Clues: assume that the atmospheric attenuation and any clothing attenuation is zero; model the body as a specular target 1 ft in diameter. If the beam size is much smaller than 1 ft, assume a 100% fill factor).]

2. (Statistical Detection Theory) Use MATLAB (or your favorite numerical simulation tool) to get the exact ROC curves for the coherent and incoherent receivers.

(a) The coherent receiver analysis yielded the probability distribution functions:

$$P_d = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{(X_t - X_m)/(\sqrt{2\sigma^2})}^{\infty} \exp\left[-z^2\right] dz \cdot \sqrt{2\sigma^2}$$

$$= \frac{1}{2} erfc\left[(X_t - X_m)/(\sqrt{2\sigma^2} \right] \quad if \;\; X_t > X_m$$

$$= \frac{1}{2}\left[1 + erf\,(X_m - X_t)/(\sqrt{2\sigma^2} \right] \quad if \;\; X_t < X_m$$

and $$P_{fa} = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{(X_t)/(\sqrt{2\sigma^2})}^{\infty} \exp\left[-y^2\right] dz \cdot \sqrt{2\sigma^2} = \frac{1}{2} erfc\left[X_t /(\sqrt{2\sigma^2} \right]$$

Use these to derive a closed form solution for P_d in terms of P_{fa} and the SNR [$\equiv (X_m)^2/(2\sigma^2)$] {clue: in anticipation of using MATLAB (or your favorite tool), consider the functions erf^{-1} as an available library function}

(b) The incoherent receiver analysis yielded the probability distribution functions

$$P_d = \int_{r_t}^{\infty} p_1(r')dr' = \int_{r_t}^{\infty} \frac{r'}{\sigma^2} \exp\left[\frac{-(r')^2 - A^2}{2\sigma^2}\right] I_o(r'A/\sigma^2)dr'$$

$$P_{fa} = \int_{r_t}^{\infty} p_0(r')dr' = \int_{r_t}^{\infty} \frac{r'}{\sigma^2} \exp\left[\frac{-(r')^2}{2\sigma^2}\right] dr'$$

Use these to derive a closed form solution for P_d in terms of P_{fa} and the SNR [$\equiv A^2/(2\sigma^2)$] {clue: in anticipation of using MATLAB (or your favorite tool), assume that I_0 is an available library function}

(c) Compute the closed form solution in (a) for P_{fa} between 10^{-9} and 1.0 (one step every decade, at minimum) and the following values of SNR: 0.1, 0.5, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20 {clue: recast the inequalities, $X_t > X_m$, and $X_t < X_m$ in terms of SNR and P_{fa}}

(d) Compute the closed form solution in (b) for P_{fa} between 10^{-9} and 1.0 (one step every decade, at minimum) and the following values of SNR: 0.1, 0.5, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20 {clue: if using MATLAB, consider the function quadl.m, a numerical integration tool that uses the Lobatto algorithm – a fast and accurate way to do definite integrals}

(e) Plot the results of (c) and (d) together to make the Universal ROC curves – something you can put on your wall and probably not find in any book (at least Prof. Brown hasn't seen it).

3. (Matched Filter) Suppose we have a known signal *s(t)* to be detected in AWGN. The signal is pulsed-cw in form and is given as:

$$s(t) = \begin{cases} \sin\left(\frac{8\pi t}{T}\right) & 0 \le t \le T \\ 0 & elsewhere \end{cases} \tag{0.1}$$

a) Write out the physically realizable matched filter response, $h_m(t)$. (Hint a delay term is needed for realizability)

b) Write the output $g_m(t)$ for the physically realizable matched filter.

c) Now suppose instead of using a matched filter for the detector, instead we use a running time correlator. The running time correlator output, $g_c(t)$, is given as:

$$g_c(t) = \int_{t-T}^{t} h_c(u) r(u) du \tag{0.2}$$

where $h_c(u)$ is set equal to the expected signal, i.e. $h_c(u) = s(u)$ and $r(u)$ is the received signal. Show the times at which the output of the matched filter and the output of the running time correlator are equal.

d) Let the received signal, r(t), be noiseless and is equal to $s(t)$ from above. Give the expressions for the output of each detector, ie. derive $g_c(t)$ and $g_m(t)$ for the given received signal.

e) Graph the outputs, $g_c(t)$ and $g_m(t)$ results of each from $-\left(\frac{t-T}{T}\right)$ to $\left(\frac{t-T}{T}\right)$