

Solutions#6

1. Comparison of radar and radiometry.

- (a) For the double-sideband receiver at 650 GHz, the antenna gain $G = 50 \text{ dB} = 10^5 \approx 16/\beta^2$, or $\beta = 0.0127 \text{ rad} = 0.7 \text{ deg}$. The angle subtended by 1-ft-wide body at 30 ft is $\alpha \approx 2 \cdot \arctan(0.5/30) = 0.033 \text{ rad} = 1.9 \text{ deg}$. So assuming the receiver beam is pointed correctly, its beam is “filled” by the body so the apparent brightness temperature is equal to the body temperature (37 C) times the effective emissivity or $T_B = eT_s = 0.8 \times 310 \text{ K} = 248 \text{ K}$. This is to be contrasted to the double-sideband receiver noise temperature, which is often close to one half the single-sideband value (assuming that the IF bandwidth is much less than the LO frequency, 1% in the present case). So assuming a single spatial mode is received and the RJ limit, we estimate a pre-detection $\text{SNR}_{PD} = T_B / (T_B + T_R) = 248 / (2500 + 248) = 0.090$. Square law detection and integration improves this $\text{SNR}_{AD} = \text{SNR}_{PD}^2 (\Delta\nu / 2\Delta f) \approx \text{SNR}_{PD}^2 (\Delta\nu \cdot t_i)$ where $\Delta\nu$ is the IF bandwidth and t_i is the integration time. Substitution of $\Delta\nu = 0.01 \times 650 \text{ GHz} = 6.5 \text{ GHz}$ and setting $\text{SNR}_{AD} = 10$ yields $t_i \approx 2.3 \times 10^{-6} \text{ s}$. From the definition given in class, the $\text{NE}\Delta T = \text{NEP}_{AD} / (k_B \Delta\nu)$ (assuming a single spatial mode received), and $\text{NEP}_{AD} \approx k_B T_R \Delta\nu / (\Delta\nu t_i)^{1/2} = 1.8 \times 10^{-12} \text{ W}$, so $\text{NE}\Delta T = 20 \text{ K}$. If the integration time is raised to 1 s, then the SNR_{AD} goes up by $1/(2.3 \times 10^{-6})$ from 10 to a value of 4.3×10^6 . And the $\text{NE}\Delta T$ falls by a factor of $(2.3 \times 10^{-6})^{1/2}$ to 31 mK.
- (b) For the 650-GHz “direct” receiver and 20% THz bandwidth, we again assume the body “fills” the antenna beam. The absolute optical NEP_{AD} is just the specific NEP_{AD} times the square root of post detection bandwidth $\Delta f = 1/(2t_i)$. So for a 1 s integration time, $\text{NEP}_{AD} = (\text{NEP}_{AD})' \cdot (1/2t_i)^{1/2} = 7.07 \times 10^{-13} \text{ W}$. For the unimodal antenna, the $\text{SNR}_{AD} = k_B T_B \Delta\nu / (\text{NEP}_{AD}) = 629$ for $T_B = 248 \text{ K}$ and $\Delta\nu = 130 \text{ GHz}$ (20% BW). The $\text{NE}\Delta T$ under this condition (assuming R-J limit) is $\text{NEP}_{AD} / (k_B \Delta\nu) = 0.39 \text{ K}$. To achieve $\text{SNR}_{AD} = 10$, the required integration time would only be $1 \text{ s} \cdot (10/629)^2 = 0.25 \text{ ms}$.
- (c) For the monostatic radar at 650 GHz, we apply the modified radar range equation (for a specular target) to get a pre-detection $\text{SNR}_{PD} (= \text{SNR}_{BD}) = P_{\text{rec}} / [k_B (T_s + T_R) \Delta\nu] = (P_t G^2 \Gamma^2 \lambda^2 \epsilon_p) / [(4\pi)^2 (2R)^2] / [k_B (T_s + T_R) \Delta\nu]$, where Γ^2 is the power reflection (0.5), R is the range (30 ft or 9.14 m), T_R is the single-sideband receiver noise temperature (5000 K), and ϵ_p is the polarization matching factor which we will assume to be unity. Under the stated system conditions, $P_{\text{rec}} = 20 \times 10^{-6} \text{ W}$ for $P_t = 1 \text{ mW}$ and all other parameters as stated. And for $\Delta\nu = 6.5 \text{ GHz}$, the $\text{SNR}_{PD} = 4.2 \times 10^4$. The noise-equivalent transmit power is just $\text{NEP}_T = [(4\pi)^2 (2R)^2] \cdot [k_B (T_s + T_R) \Delta\nu] / [G^2 \Gamma^2 \lambda^2 \epsilon_p] = 0.47 \mu\text{W}$! From (a) above, the $\text{SNR}_{PD} = 0.09$ and $\text{SNR}_{AD} = 4.3 \times 10^6$ for a 1-s integration time in the double-sideband radiometer. To achieve these with the radar using a square-law detector and 1 s integration time, we again apply $\text{SNR}_{AD} = \text{SNR}_{PD}^2 (\Delta\nu / 2\Delta f) \approx \text{SNR}_{PD}^2 (\Delta\nu \cdot t_i)$. Setting $\Delta\nu = 6.5 \text{ GHz}$ again, we get $\text{SNR}_{PD} = 0.026$. The required transmit power in this case is just $(0.026/4.2 \times 10^4) \cdot 1 \text{ mW} = 0.6 \text{ nW}$. Although these solutions are overly optimistic, they make the point that active systems are often superior to passive ones when the target is reasonably close. And things get even better when the target is quasi-specular, provided that the reflected power is aligned with the receiver main beam. This is the well-known “pointing” challenge of all THz systems (and RF systems too) with high-gain antennas.
- (d) Not required.

2. Since MATLAB offers the inverse error function, the two desired expressions for P_d vs P_{fa} and SNR for coherent detection are as follows:

$$(a) \quad P_d = \frac{1}{2} \operatorname{erfc} \left[\operatorname{erf}^{-1}(1 - 2P_{fa}) - \sqrt{\operatorname{SNR}} \right] \quad \text{if} \quad \operatorname{SNR} < \left[\operatorname{erf}^{-1}(1 - 2P_{fa}) \right]^2$$

$$P_d = \frac{1}{2} \{ 1 + \operatorname{erf} \left[\sqrt{\operatorname{SNR}} - \operatorname{erf}^{-1}(1 - 2P_{fa}) \right] \} \quad \text{if} \quad \operatorname{SNR} > \left[\operatorname{erf}^{-1}(1 - 2P_{fa}) \right]^2$$

(b) With MATLAB as the tool, the ROC analysis for incoherent detection yields:

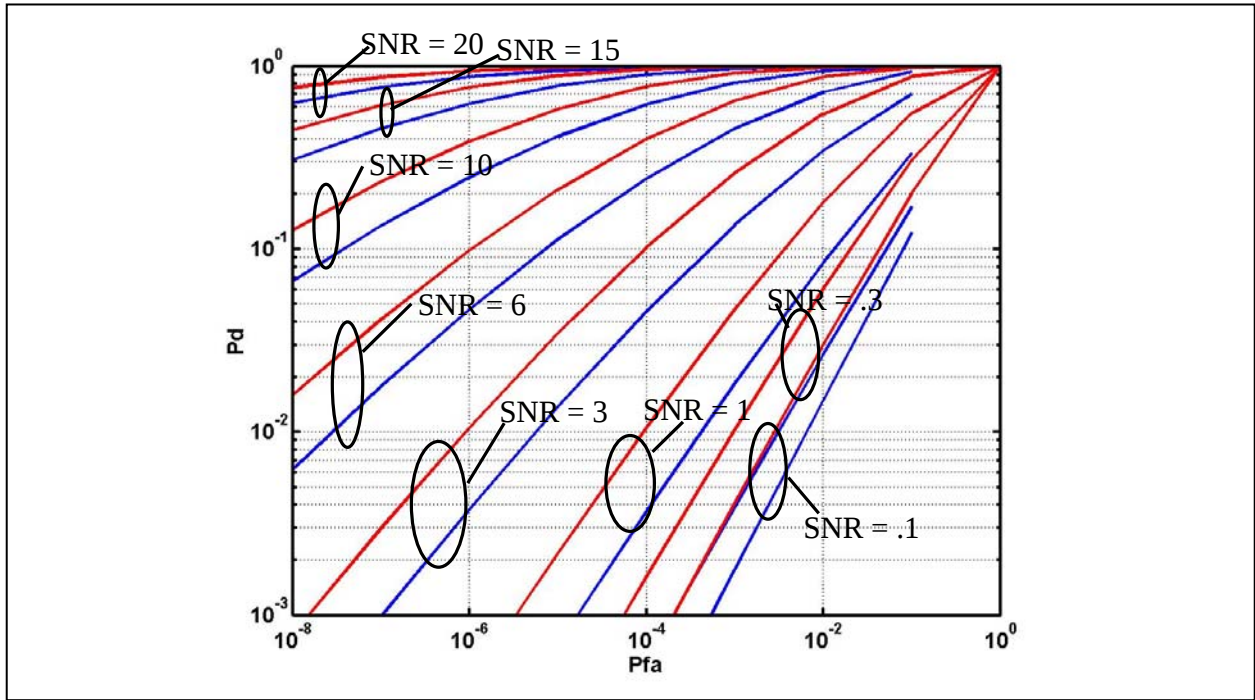
$$P_d = \frac{\exp(-\operatorname{SNR})}{2\operatorname{SNR}} \int_{2\sqrt{-\operatorname{SNR} \ln P_{fa}}}^{\infty} u \exp \left[\frac{-u^2}{4\operatorname{SNR}} \right] I_0(u) du$$

(c) and (d) The Matlab code used is as shown here (there are certainly more efficient ways to do the computation, but the given way is very straightforward:

```
%MATLAB code to compute Pd vs Pfa parameterized by SNR for coherent
%and incoherent receiver cases. Written by Prof. E. R. Brown
numberSNR=input('what is the number of the signal to noise ratios?');
startingPfa=input('what is the starting PFA?');
numberPfa=input(' what is the number of PFAs , assuming order of magnitude
increments?');
for i = 1:numberSNR
    SNR=input('what is the SNR?');
    for j = 1:numberPfa
        Pfa(j) = startingPfa*10.^(j-1);
        lowerlimit=2.*sqrt(-SNR.*log(Pfa(j)));

        f=inline('x.*exp(-(x.^2)./(4.*SNR)).*besseli(0,x)','x','SNR');
        y= quad8(f,lowerlimit,lowerlimit*10,1e-6,0,SNR);
        PdI(j)=(exp(-SNR)/(2*SNR))*y;
        if SNR < (erfinv(1 - 2.*Pfa)).^2
            PdC(j)=.5.*erfc(erfinv(1 - 2.*Pfa(j)) - sqrt(SNR));
        else
            PdC(j) =.5.*(1 +erf( (sqrt(SNR) - erfinv(1 - 2.*Pfa(j)))));
        end
    end
end
loglog(Pfa,PdI)
hold on
loglog(Pfa,PdC,'color','red')
hold on
grid on
end
```

(e) See “Universal ROC” plot below



3. Matched filter problem

a) Physically unrealizable:
$$h'_m(t) = \begin{cases} \sin\left(-\frac{8\pi t}{T}\right) & -T \leq t \leq 0 \\ 0 & \text{elsewhere} \end{cases}$$

Physically realizable:
$$h_m(t) = \begin{cases} \sin\left(\frac{8\pi(T-t)}{T}\right) & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases}$$

b)
$$g_m(t) = \int_t^{t+T} \sin\left(\frac{8\pi(u-t+T)}{T}\right) r(u) du$$

- c) $t=T$; there are other times in this particular case that the outputs are equal but for the illustrative purposes of this problem, $t=T$ is of interest. Moreover $t=T$ is the time which the observation interval time ends.

$$g_c(t) = \frac{(T-|t-T|)}{2} + \frac{T}{36\pi} \sin\left(\frac{16\pi(|t-T|)}{T}\right)$$

d)
$$g_m(t) = \frac{(T-|t-T|)}{2} \cos\left(\frac{8\pi(|t-T|)}{T}\right) + \frac{T}{16\pi} \sin\left(\frac{8\pi(|t-T|)}{T}\right)$$

e) see plot below

