Quiz#1

1. THz Radiation

(a) For T_b = 4200 K, we have $h\nu/k_BT = 0.011$ so the RJ approximation is good. Since the antenna is unimodal, the received power in a 100 GHz bandwidth is P = 5.8x10^{-9} W. For the exponential (i.e., Boltzmann) statistics applicable to thermal radiation, the rms fluctuation is equal to the mean, or P_{rms} = 5.8x10^{-9} W.

(b) For T_b = 2.7 K, $h\nu/k_BT = 17.8$ so the RJ approximation is NOT valid. The correct expression for the mean photon energy is $h\nu/[\exp(h\nu/k_BT) - 1]$, which evaluates to 1.25x10^{-29} J. The RJ approximation yields 3.7x10^{-23}. The error is about 3x10^{6} !

2. THz Phenomenology.

(a) For a diatomic rotor, $I = \dfrac{m_1 m_2 d_{12}^2}{m_1 + m_2}$ where d_{12} is the interatomic distance. Substitution of m_1 = m_H = 1.0 m_p m_2 = m_{Cl} = 35.4 m_p, and d_{12} = 1.274 Ang yields I = 2.66x10^{-47} KG-m^2. Substitution into the formula given yields $h\nu$ = 4.186x10^{-22} J, or ν = 632 GHz.

(b) ν_r is the Debye relaxation frequency. For pure water, it is between 10 and 100 GHz. The absolute value of the imaginary part of ε is (after some simple complex algebra) is $|\mathrm{Im}\{\varepsilon\}| = (\varepsilon_0 - \varepsilon_\infty)(\nu/\nu_r)/[1 + (\nu/\nu_r)^2]$. At $\nu = 0$, this equals zero. As ν increases from 0, it linearly increases. As ν approaches ∞, it goes back to zero. Therefore, there IS a maximum of $|\mathrm{Im}\{\varepsilon\}|$ at some frequency, and the absorption should be very large near this frequency.

2. THz Antennas and the Friis Formula

(a). β = 1 inch/12 inch = 0.083 rad or 4.8^o. Given the fact that β [radian] << 1, we have $D \approx 16/\beta^2$ = 2304 (33.6 dB). The effective aperture is $A_{eff} = \lambda^2 D/4\pi$ = 3.9x10^{-5} m^2. With reflection R and attenuation A, we have G = D(1-R)(1-A). For D = 2304, R = 0.2, and A = 0.3, this evaluates to G = 1451 (31.6 dB).

(b) Specular reflectors act like mirrors so do not change the shape or diffraction angle of beams that strike them… just the propagation direction. Therefore, we can apply the Friis' transmission formula given in the clue (applicable to communications links) simply by replacing r by 2R, setting τ = 1 (no atmospheric losses) and ε = 1 (perfect polarization match), yielding $P_{out} = P_{in} \cdot (\lambda/8\pi R)^2 G^2$, Given the input power of 1 mW, R = 12 inch (0.305 m) and G = 1451, we find P_{out} = 7.65x10^{-6} W.