HW#3: Antennas for the THz Region

1. Feedhorns: Metallic feedhorns have long been popular as antennas at mm-wave and THz frequencies. The simplest designs transform a single mode waveguide to free space through a length of linear taper, as shown in Fig. 1. The waveguide end of the feedhorn is called the *throat*, while the free-space end is called the *mouth*. If the waveguide is rectangular, the feedhorn is usually pyramidal in form. If the waveguide is circular, the feedhorn is usually conical in form. The standard types of both pyramidal and conical feedhorns have a smooth taper, and the far-field radiation pattern has sidelobes. If azimuthally-symmetric grooves are cut in the taper section of the horn, it is possible to excite a higher order spatial mode in the horn in just the right way that the sidelobes are cancelled in the far field. In this case, the far field pattern can be represented by a monotonic function of polar angle θ, often by the following Gaussian function: $F(\theta) = \exp(-a\theta^2)$, θ in units of radians. Calculate the following for a = 10:

(a) The FWHM beamwidth β
(b) The pattern solid angle Ω_B
(c) The antenna directivity D
(d) S_{max} at a range of 10 cm if the total radiated power is 1 mW.

2 Dish antennas: If carefully coupled to, aspheric reflectors such as metallic parabolic and elliptic surfaces, make very useful antennas in the THz region. Compared to the visible and infrared region, the THz wavelength is long enough that the aspherical surfaces can be fabricated with practically no aberration and "diffraction-limited" radiation performance. Suppose we have a parabolic "dish" that provides a symmetric beam having a full beam-width at half maximum of 1.5^o at 400 GHz.

(a) What is the directivity in dB? Calculate exactly and by the "pencil-beam" approximation, and compare.
(b) If the dish area is doubled, what will the new directivity and beam-width be ?
(c) If the area is kept the same as in (a), but the frequency is doubled to 800 GHz, what will the new directivity and beam-width be?
(d) Now suppose this dish is used as the primary antenna in a 400 GHz receiver coupled to a 400 GHz transmitter that is radiating 1 mW average power through the feedhorn of Problem 1. What is the received available power from the dish for an Tx-Rx separation of 1 km ?
(e) At what separation will the power from the Tx match the radiative thermal background if the Rx instantaneous bandwidth is 10 GHz ?

3. Wire-Type Integrated-Circuit Antennas. At millimeter-wave and THz frequencies it becomes advantageous to integrate resonant printed-circuit antennas monolithically on the top surface of the same substrate that support the semiconductor devices. The challenge is coupling the antennas to free space since the radiation tends to go into the lower-intrinsic-impedance, substrate side. A useful starting point for dipolar strip antennas (see Fig. 2) is the following expression for an arbitrary-length dipole in free space:

$$< \vec{S} >= \frac{I_0^2 \eta_0}{8\pi^2 r_d^2} \left\{ \frac{\cos[(kl\cos\theta_d)/2)] - \cos kl/2}{\sin\theta_d} \right\}^2 \cdot \hat{r} \tag{1}$$

where k = ω/v, v is the interface velocity, r_d is the range from the center of the dipole (i.e., the driving gap) to the measurement point, I_0 is the amplitude of a sinusoidal current drive across the driving gap, and L is the full physical length of the dipole.

a. Find the intrinsic impedance η [in ohms] in a substrate of semi-insulating (SI) GaAs. Find the average dielectric constant at the air interface as an arithmetic weighting of the two dielectric media [clue: the dielectric constant of SI GaAs at THz frequencies is practically the same as the dc value].
b. Calculate the interface velocity as the speed of light reduced by the average refractive index. Use this to design the physical length of two printed dipole, one that will approximate a half-wave circuit resonance at 600 GHz, and the second one a full-wave resonance at 600 GHz ?
c. Assuming that radiation is described by (1) but propagates only into the substrate side, find the value of θ_d that maximizes the time-averaged S for both the half-wave and the full-wave resonant dipoles.
d. Under the same assumptions, find the beam pattern function, $F(\theta_d,\phi_d)$.

4. Wire-Type Antennas (continued… a good exercise in numerical integration too): The same generic expression (1) can be used to estimate the radiated power and impedance of the printed dipole antennas if we replace η_0 with η, the intrinsic impedance of substrate material.

a. Use (1) and your favorite numerical integration tool (e.g., Matlab's quadl.m) to estimate the total radiated power into the substrate as a numerical factor times $(I_0)^2$ for the half- and full-wave dipoles at 600 GHz.
b. Use this P_{rad} to find the directivity of the half- and full-wave dipoles.
c. Find the radiation resistance R_{rad} of each antenna [clue: $R_{rad} = R_{ant} = \mathrm{Real}\{Z_{ant}\}$; $Z_{ant} = R_{ant} + jX_{ant}$, the resonance condition means that $X_{ant} \approx 0$].
d. Explain physically why the full-wave dipole has such a higher R_{rad} than the half-wave dipole in terms of the current distribution associated with I_0 on the printed conductor.

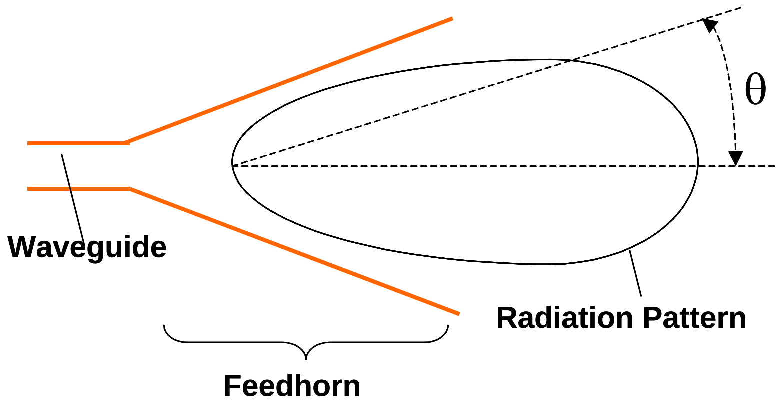
θ
Waveguide
Radiation Pattern
Feedhorn

Fig. 1.

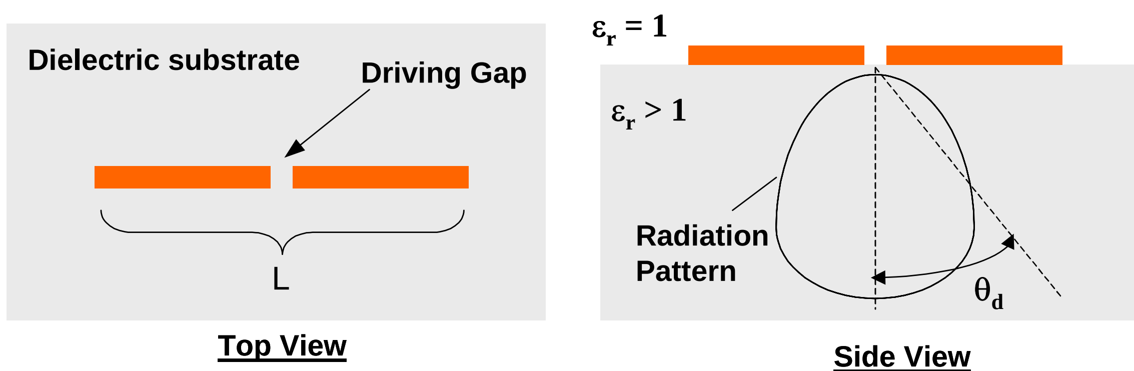
Dielectric substrate
Driving Gap
L
Top View
$\varepsilon_r = 1$
$\varepsilon_r > 1$
Radiation Pattern
θ_d
Side View

Fig. 2.