HW#4: Common Loss Mechanisms; Gaussian Beams

1. Skin Effect Losses: An important prediction of electromagnetics at THz frequencies is the skin effect whereby the component of the electric field parallel to the surface of a good conductor is confined to a progressively thinner layer as the frequency increases.

(a) Calculate the skin depth in five metals common to high-speed electronics: Cu, Au, Al, Cr, and Sn, and at four frequencies: (1) 10 GHz, 100 GHz, 1.0 THz, and 10.0 THz.

(b) Derive an expression for the specific series resistance of a metal strip on an insulating substrate of rectangular cross section 0.5 x 9.0 micron, assuming the current is confined uniformly to a skin depth around the periphery and is not affected by the substrate.

(c) Assume this strip is used as the center conductor of a miniature coaxial line designed to have a characteristic impedance of $Z_0 = 50\ \Omega$ and a specific capacitance of 1.0 pF/cm. Use the lossy TEM transmission line model of Fig. 1 to predict the absorption per unit length in this line at 100, 300, and 600 GHz (no need to derive the specific attenuation for this model; just take it from any good E&M book, such as F. Ulaby, Applied Electromagnetics).

2. Total Internal Reflection: Another universal phenomena from electromagnetics that impacts THz component design is total internal reflection, derivable from Snell's law.

(a) Look up the THz-region dielectric constant and calculate the critical angle for the following dielectric materials: (1) Teflon, (2) Rexolite, (3) Mylar, (4) single crystal quartz, (5) single crystal alumina (sapphire), (6) high-resistivity silicon, and (7) semi-insulating GaAs.

(b) Given the ideal dipole antenna radiation pattern of HW#3, and assuming it is fabricated on a dielectric slab what fraction of its radiation would be trapped in the slab for each of the 7 materials of (a) if it is operated at its full-wave resonance ?

3. Gaussian Beams: One of the useful applications of the TEM_{00} Gaussian-beam formalism is in predicting the propagation of radiation through optical components, such as lenses. A common scenario is the lens-focusing problem shown in Fig. 2. A TEM_{00} Gaussian beam having beam waist ω_{01} is transformed by a thin dielectric lens of focal length f. The distance between the waist location and the lens location is $z = d_1$.

(a) Find the condition on f in terms of λ, ω_{01}, and d_1, which guarantees that a second beam waist ω_{02} occurs somewhere to the right side of the thin lens.

(b) Using you favorite graphical tool, plot the distance d_2 vs f for $\nu = 1$ THz, $\omega_{01} = 2$ mm, and $d_1 = 20$ cm. How many values of f can create a given d_2 ?

(c) Solve for the possible values of f when $d_2 = 10$ cm ? Of the possible values, which is the most practical one, and why ?

(d) A practical question, often faced in THz experiments and systems, is how to form the smallest possible ω_{02} when ω_{01} is given (i.e., you already have a source), f is given (i.e., you already have a lens) but d_1 and d_2 are variable. Start by finding an expression for ω_{02} in terms of ω_{01}, d_1, d_2, and f. Qualitatively, what does this solution suggest for the best values of d_1 and d_2 relative to f ? According to this solution, how small can ω_{01} be ?

(e) In practice d_1 can not be made arbitrarily large because of "spillover" of the source Gaussian beam at the lens. And d_2 can not be made arbitrarily large because of limits on the size of the experiment or instrument. This leads to the common "constrained" problem whereby $d_1 + d_2$ is limited to a maximum value, D. Fortunately, the relatively long wavelengths in the THz region (compared to the visible or infrared) allow one to fabricate lenses rather easily, so the choice of f is great. Using elementary calculus, determine the values of d_1 and d_2 in terms of f and D that minimize ω_{02}. And write the expression for the minimum ω_{02}.

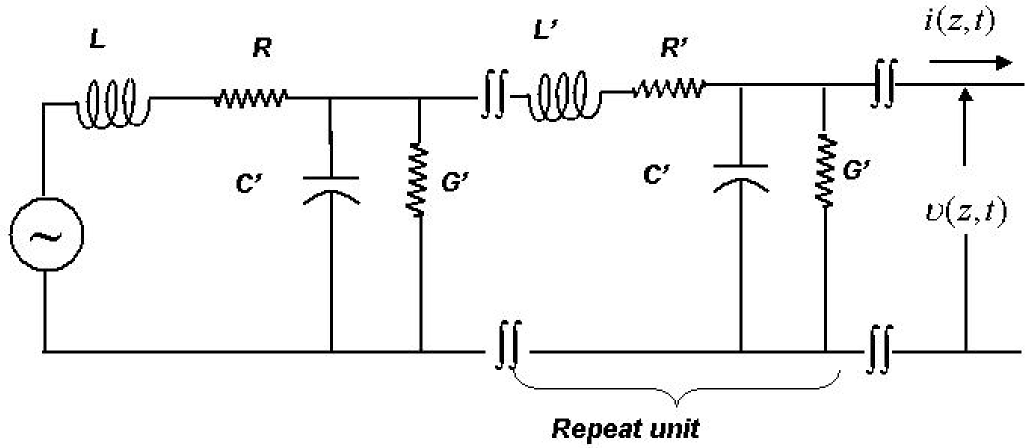

Fig. 1

Lens, focal length f

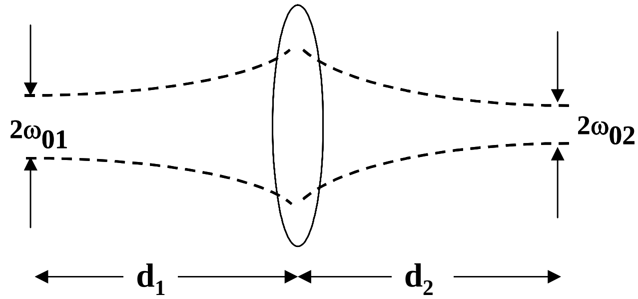

Fig. 2.