

HW#7: Good THz System Problems

1. (Passive and Active Systems). Suppose you are asked to assess THz stand-off detection for determining the presence of a human body at a range of 30 ft (this could be practical in situations, such as in a smoke-filled room, where IR sensors do not work).
 - (a) Consider determining the presence of the body using a low-cost double-sideband heterodyne radiometer having a 50-dB gain antenna (e.g., small telescope). Assume the receiver noise temperature is 5000 K (single sideband), the center frequency is 650 GHz, the IF bandwidth is 1%, the body brightness temperature is 37 C, the emissivity is 0.8, and detection is done with an ideal square-law device. What is the before-detection SNR in this situation if the antenna is pointed directly at the body ? How long would the Rx have to integrate to get a post-detection SNR of 10 ? What would the NE Δ T of the receiver be in this case ? What would the post-detection SNR and NE Δ T be if the integration time was 1 s ?
 - (b) Now consider using direct detection with a Schottky diode rectifier having an optical NEP 1×10^{-12} W/Hz^{1/2}, the same antenna and center frequency as above, but an instantaneous bandwidth of 20%. What would the post-detection SNR and NE Δ T be with a 1-s integration time ? Again, how long would the receiver have to integrate to achieve a SNR_{PD} of 10 ?
 - (c) Finally, consider using a cw monostatic radar. The receiver is the same as that applied in (a) and the transmitter is a coherent sinusoid at frequency 650 GHz operating through the same antenna. What is the received power and SNR_{BD} from the body if the transmit power is 1.0 mW and the reflectivity of the body is 50% ? What is the noise-equivalent transmit power, NEP_T ? How much transmit power would be required to achieve a pre- and post-detection SNR equal to that of the radiometer in (a) with a 1-s int time ?
 - (d) In order to reduce the NEP_T of the radar in (c) the prudent solution is to limit the IF bandwidth greatly before the square-law detector. What would the IF bandwidth have to be to achieve a post-detection SNR of 10 with 1 mW of transmit power (easily achieved with modern SSPA technology) and 0.03 s integration time (suitable for real-time imaging) ?

[Clues: assume that the atmospheric attenuation and any clothing attenuation is zero; model the body as a specular target 1 ft in diameter. If the beam size is much smaller than 1 ft, assume a 100% fill factor].
2. (Analog Matched Filter) Suppose we have a known signal $s(t)$ to be detected in AWGN. The signal is pulsed-cw in form and is given as:

$$s(t) = \begin{cases} \sin\left(\frac{8\pi t}{T}\right) & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases} \quad (0.1)$$

- a) Write out the physically realizable matched filter response, $h_m(t)$. (Hint a delay term is needed for realizability)
- b) Write the output $g_m(t)$ for the physically realizable matched filter.
- c) Now suppose instead of using a matched filter for the detector, instead we use a running time correlator. The running time correlator output, $g_c(t)$, is given as:

$$g_c(t) = \int_{t-T}^t h_c(u) r(u) du \quad (0.2)$$

where $h_c(u)$ is set equal to the expected signal, i.e. $h_c(u) = s(u)$ and $r(u)$ is the received signal. Show the times at which the output of the matched filter and the output of the running time correlator are equal.

- d) Let the received signal, $r(t)$, be noiseless and is equal to $s(t)$ from above. Give the expressions for the output of each detector, ie. derive $g_c(t)$ and $g_m(t)$ for the given received signal.

- e) Graph the outputs, $g_c(t)$ and $g_m(t)$ results of each from $-\left(\frac{t-T}{T}\right)$ to $\left(\frac{t-T}{T}\right)$

3. (Introductory Statistical Detection) Use MATLAB (or your favorite numerical simulation tool) to get the exact ROC curves for the coherent and incoherent receivers.

- (a) The coherent receiver analysis yielded the probability distribution functions:

$$\begin{aligned} P_d &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{(X_t - X_m)/(\sqrt{2\sigma^2})}^{\infty} \exp[-z^2] dz \cdot \sqrt{2\sigma^2} \\ &= \frac{1}{2} \operatorname{erfc} \left[(X_t - X_m)/(\sqrt{2\sigma^2}) \right] \quad \text{if } X_t > X_m \\ &= \frac{1}{2} \left[1 + \operatorname{erf} (X_m - X_t)/(\sqrt{2\sigma^2}) \right] \quad \text{if } X_t < X_m \end{aligned}$$

$$\text{and } P_{fa} = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{(X_t)/(\sqrt{2\sigma^2})}^{\infty} \exp[-y^2] dz \cdot \sqrt{2\sigma^2} = \frac{1}{2} \operatorname{erfc} \left[X_t/(\sqrt{2\sigma^2}) \right]$$

Use these to derive a closed form solution for P_d in terms of P_{fa} and the SNR $[\equiv (X_m)^2/(2\sigma^2)]$ {clue: in MATLAB (or your favorite computational tool), consider the functions erf^{-1} as an available library function}

- (b) Compute the closed form solution in (a) for P_{fa} between 10^{-9} and 1.0 (one step every decade, at minimum) and the following values of SNR: 0.1, 0.5, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20 {clue: recast the inequalities, $X_t > X_m$, and $X_t < X_m$ in terms of SNR and P_{fa} }
- (c) Plot the results of (b) to make a set of Universal ROC curves – something you can put on your wall and not easily find in any book. Being that the detection is assumed to be fully coherent, these curves represent the “best-case” scenario for active sensing in the presence of band-limited Gaussian noise.