Mat 2885 S'10
Final Ex. Soln's

1.

A) $\eta = R(\lambda)\frac{h\nu}{e}$  $R(\lambda) \simeq 0.95\,\frac{A}{W}$  $\frac{h\nu}{e} = \frac{1240\,eV\text{-}nm}{1550\,nm \cdot e} = 0.8$  $\therefore \boxed{\eta = 0.76}$

B) From fig., $I_d(-5V) \simeq 250\times10^{-12}A$

$$I_{sh,d} = \sqrt{2 e I_d B} = \sqrt{2(1.6\times10^{-19}c)(2.5\times10^{-10}\tfrac{c}{s})(\tfrac{1}{s})} = 8.9\times10^{-15}\text{ Amps}$$
@ 1 Hz BW

$$I_{thermal,d} = \sqrt{\frac{4kTB}{R_{SH}}} \qquad R_{SH} = \frac{dV}{dI} \sim 5\times10^{10}\,\Omega \text{ @ } V=-5V$$

$$= \left[\frac{4(.026\,eV)(1/s)}{5\times10^{10}\,\Omega}\;\frac{1.6\times10^{-19}c}{e}\right]^{1/2} = 5.8\times10^{-16}\,A$$
@ 1 Hz BW

$$I_{noise,total} = \sqrt{I_{sh}^2 + I_{th}^2} = \left[(8.9\times10^{-15}A)^2 + (5.8\times10^{-16}A)^2\right]^{1/2} = \boxed{8.9\times10^{-15}A \text{ @ } 1\text{ Hz BW}}$$

ie, thermal noise is negligible cf. shot noise in dark current.

c) $NEP = \frac{I_{noise}}{R(\lambda)} = \frac{8.9\times10^{-15}A}{\sqrt{Hz}}\;\frac{1}{0.95\,\frac{A}{W}} = \boxed{9\times10^{-15}\,\frac{W}{\sqrt{Hz}}}$

D) @ $V_B = 0\,V$, $R_{SHunt} = \left.\frac{dV}{dI}\right|_{0V} \cong 2\times10^{9}\,\Omega$

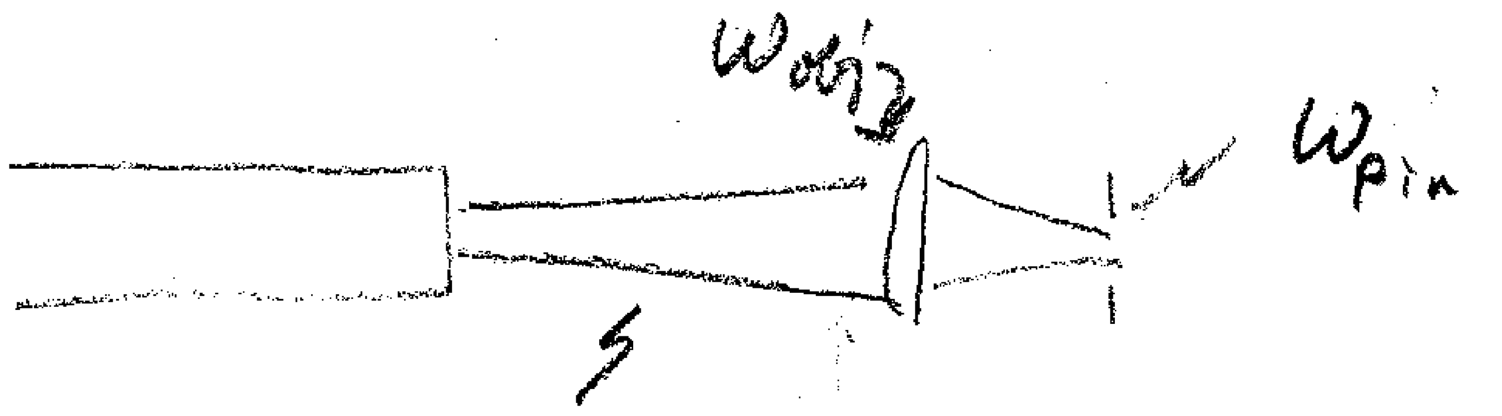
$$I_{th,d} = \sqrt{\frac{4kTB}{R_{SH}}} = \sqrt{\frac{4(.026\,eV)(1\,Hz)}{2\times10^9\,\Omega}\;\frac{1.6\times10^{-19}c}{e}} = \boxed{2.9\times10^{-15}A \text{ @ } 1\text{ Hz BW}}$$

E. $C|_{-5V} = 5\times10^{-12}F$; $R_L = 50\,\Omega$  $f_{3dB} = \frac{1}{2\pi RC} = \boxed{640\text{ MHz}}$

or, $t_r = 2.2\,RC = 5.5\times10^{-10}s$  $f_{3dB} = \frac{.35}{t_r} = 640\text{ MHz}$

2.

A) HeCd $\lambda = 325\,nm$ $\quad 2W_0 = 1.2\,mm \Rightarrow W_0 = 0.6\,mm$




Can we neglect divergence? $\Theta \sim \pi \frac{\lambda}{W} \sim \pi \frac{.3\times10^{-3}\,m}{.6\,mm} = 1.5\,mrad$

Assume ~10 cm from laser to obj, $\Rightarrow \Delta W \sim (10\,cm)(\Theta) = .01\,cm = .1\,mm$

not enitrely negligible, carry it along

$$\Rightarrow W\Big|_{obj} = .61\,mm$$

Spot size after objective, @ pinhole:

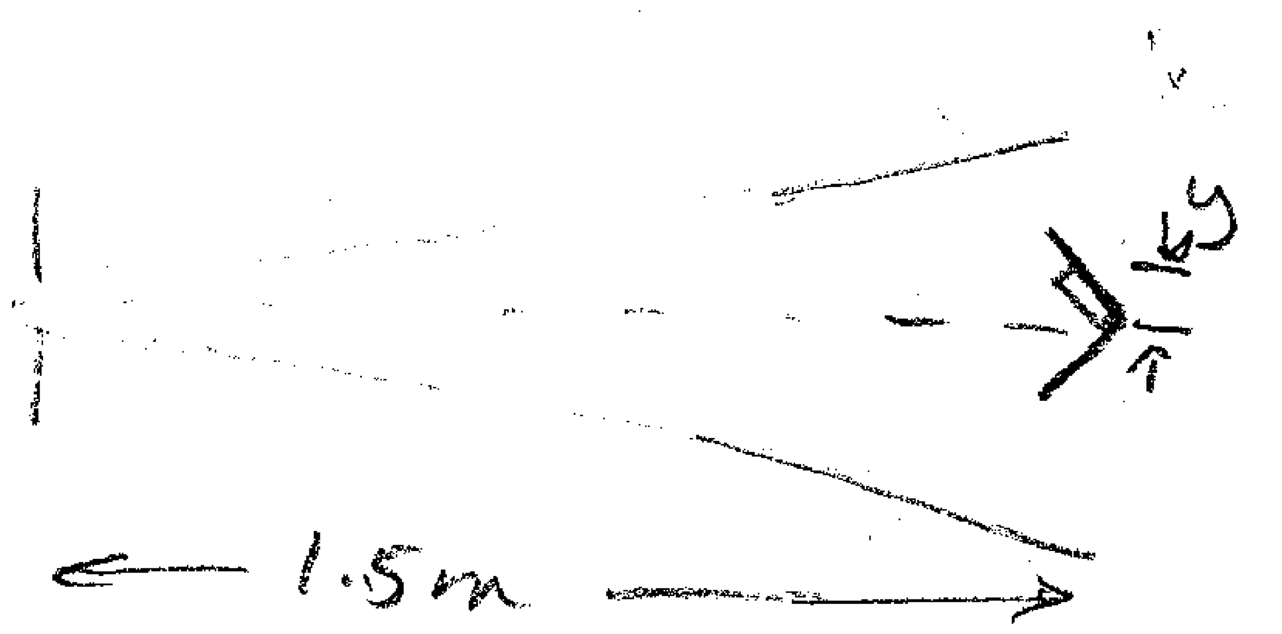
$$2W_{pin} = \frac{2\lambda f}{\pi W_{obj}} = \frac{2(.325\times10^{-3}\,mm)(11.5\,mm)}{\pi(.61\,mm)} = 3.9\,\mu$$

$$\therefore W_{pin} = 1.95\,\mu$$

$$\frac{P(r)}{P(\infty)} = 1 - \exp\left[-2\frac{r^2}{\omega^2}\right] = 1 - \exp\left[-2\left(\frac{2.5\,\mu}{1.95\,\mu}\right)^2\right] = \boxed{0.96}$$

~ full transmission through pinhole

B)

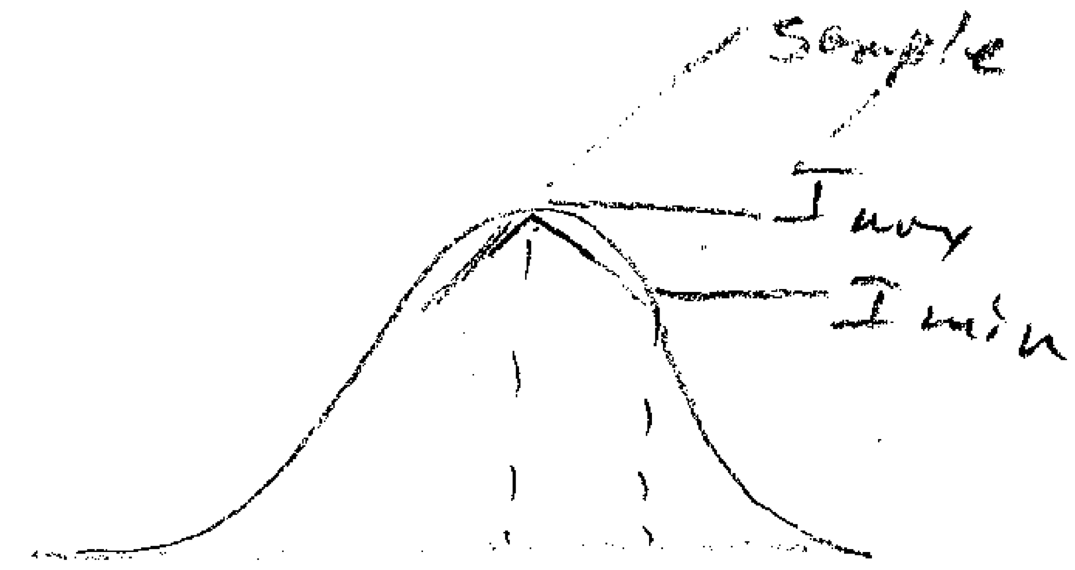



$$W(L) = \frac{\lambda L}{\pi W_{pin}} = \frac{(.325\times10^{-3}\,mm)(1.5\times10^{3}\,mm)}{\pi(1.95\times10^{-3}\,mm)}$$

$$= 79.6\,mm$$

projected width of sample, $y = \frac{l_{sample}}{\sqrt{2}} = \frac{25\,mm}{\sqrt{2}} = 17.7\,mm$

2. B) cont'd




$$\Delta I \equiv \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{1 - I_{min}/I_{max}}{1 + I_{min}/I_{max}}$$

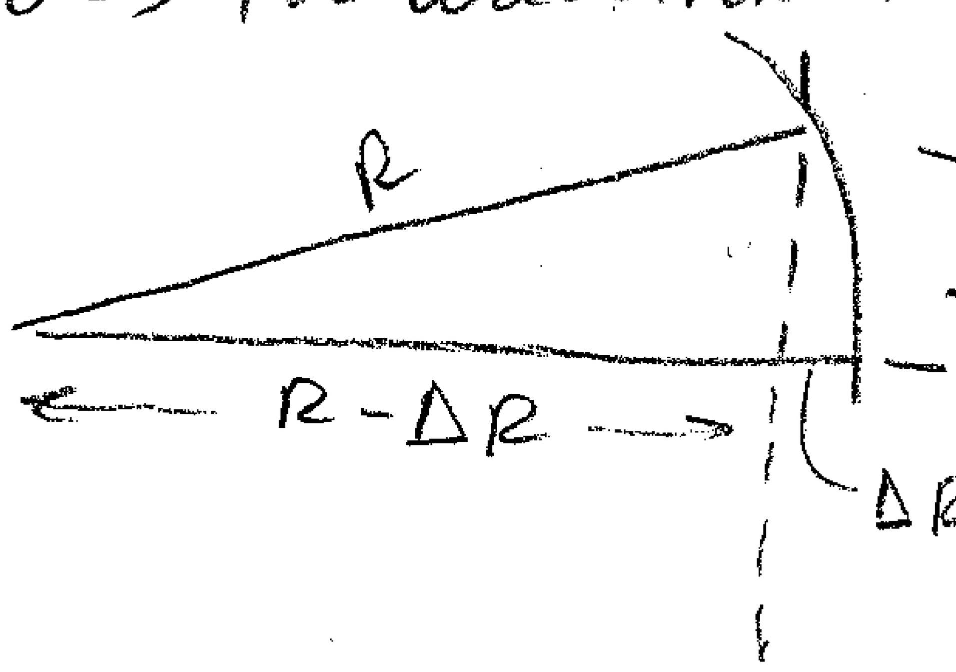
$$\frac{I_{min}}{I_{max}} = e^{-2\left(\frac{r}{W(L)}\right)^2} = e^{-2\left(\frac{17.7\,mm}{79.6\,mm}\right)^2} = 0.91$$

$$\therefore \Delta I = \frac{1 - .91}{1 + .91} = \boxed{.047}$$

c) Wavefront curvature $R(L) = L\left[1 + \frac{\pi W_{pin}^2}{\lambda L}\right] = 1.5\,m\left[1 + \underbrace{\frac{\pi(1.95\times10^{-6}\,m)^2}{(.325\times10^{-6}\,m)(1.5\,m)}}_{=2.4\times10^{-5}\ \text{negligible}}\right]$

$$\cong 1.5\,m$$

Does the wavefront differ significantly from a plane wave?




$y$ = projected sample width

$\Delta R$ = path error between plane wave & spherical wave

$$(R - \Delta R)^2 + y^2 = R^2 \Rightarrow \cancel{R^2} - 2R\Delta R + \cancel{\Delta R^2}^{\to 0} + y^2 = \cancel{R^2}$$

$$\Rightarrow \Delta R = \frac{y^2}{2R} = \frac{(17.7\,mm)^2}{2(1.5\,m)} = 0.1\,mm \text{ across } 17.7\,mm$$

This amounts to $\frac{0.1\,mm}{17.7\,mm} \cdot \frac{1\text{ wavelength}}{0.325\times10^{-3}\,mm} = \frac{18\text{ waves}}{mm}$

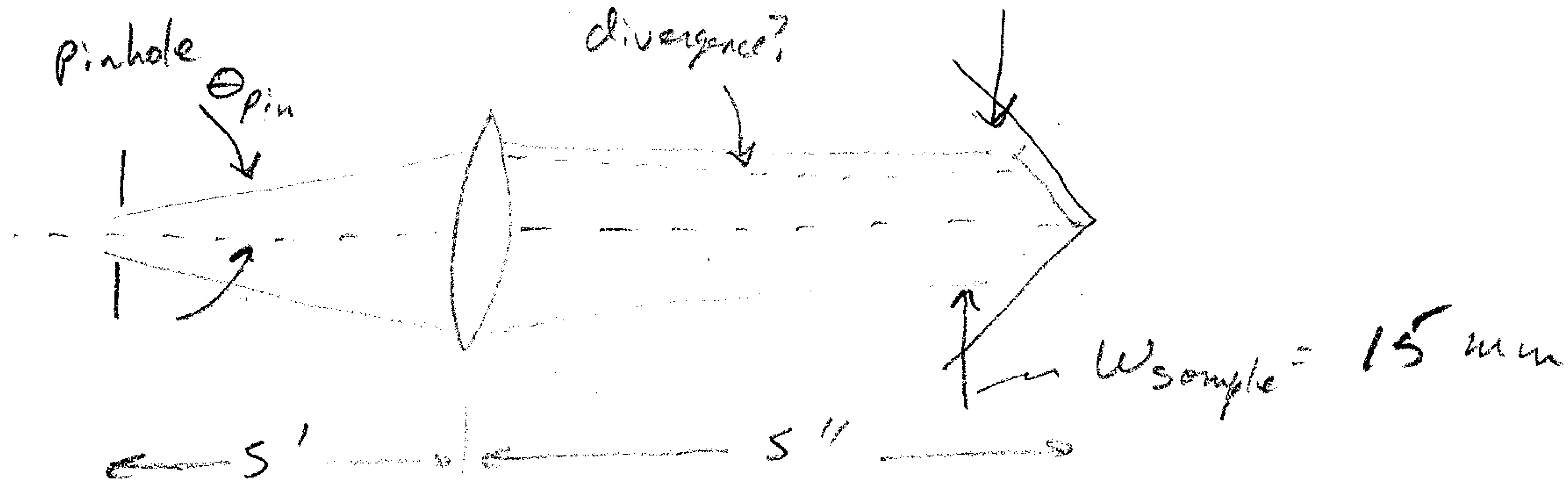
For a microfabricated grating ~ $100\mu$ long (eg. photonic crystal) the error ~ 2 waves → not negligible.

2. D) Sample size $\rightarrow$ 5 mm projected size $= \frac{5}{\sqrt{2}} = 3.5$ mm

From part (B) need $\frac{I_{min}}{I_{max}} \geq 0.9$ to get $\Delta I = .05$

$$\frac{I(r)}{I(0)} = e^{-2\frac{r^2}{w^2}} \Rightarrow \frac{1}{w^2} = \frac{-1}{2r^2} \ln\left(\frac{I(r)}{I(0)}\right)$$

"collimated" beam width $w = \left[\frac{-2r^2}{\ln\left(\frac{I(r)}{I(0)}\right)}\right]^{1/2} = \left[\frac{-2(3.5mm^2)}{\ln(0.9)}\right]^{1/2} = \boxed{15 \text{ mm}}$




Divergence of collimated beam $\theta \cong \frac{\lambda}{\pi W_{sample}} = \frac{.325 \times 10^{-3} mm}{\pi(15 \text{ mm})} = 7 \, \mu rad$

$\Rightarrow$ neglect divergence of collimated beam

From pinhole, beam must diverge to reach $W(\text{lens}) \cong W(\text{sample}) = 15$ mm

$W(s') = \theta_{pin} s' = \frac{\lambda}{\pi W_{pin}} s' = \frac{.325\mu}{\pi(2.5\mu)} s' = 15 \text{ mm} \Rightarrow \boxed{\begin{matrix} s' = 362 \text{ mm} \\ \cong f \end{matrix}}$

$\boxed{\begin{matrix} \text{lens: } f = 362 \text{ mm} \\ \text{Diam} \geq 2 W_{samp} = 30 \text{ mm} \end{matrix}}$