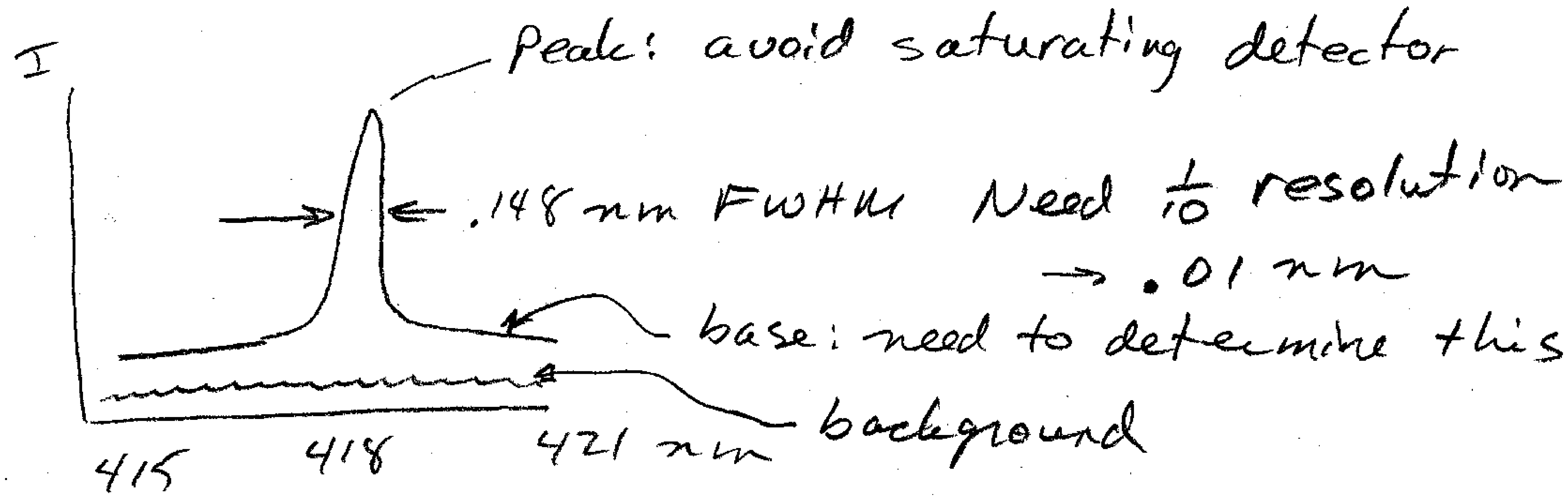
1. The task:



→ need resolution bandwidth (rbw) ≤ 0.012 nm

→ need $\frac{\text{base}}{\text{background}} > 2$

→ need $\frac{\text{peak}}{\text{base}} > 100$ or $\frac{\text{peak}}{\text{background}} > 200$ to insure FWHM can be measured accurately.

2. The grating

$$\sin\theta_i + \sin\theta_d = \frac{m\lambda}{a} \Rightarrow \left|\frac{m\lambda}{a}\right| \leq 2$$

This limits order + pitch, and is most restrictive @ long λ

@ 421 nm

P $\frac{1}{mm}$	a (nm)	m	$\frac{m\lambda}{a}$	
1200	833	3	1.51	safe
"	"	4	2.0	@ cutoff
1800	555	2	1.52	safe
"	"	3	2.27	cut off
2400	417	1	1.01	safe
"	"	2	2.02	@ cutoff
3600	278	1	1.5	safe
4750	210	1	1.99	@ cutoff

(2) Of the common candidate pitches, which gives the largest linear dispersion?

$$\lambda \cdot \frac{dL}{d\lambda} = \underbrace{\frac{m\lambda}{a}} \frac{f}{\cos\theta_d}$$

Just calculated: Hints that ~ equal dispersion for equal $m\lambda/a$

θ_i + θ_d are constrained by the geometry of a particular spectrometer. We will estimate, assuming $\theta_i \simeq \theta_d$

$$2\sin\theta_d = \frac{m\lambda}{a}$$

$$\theta_d = \sin^{-1}\left[\frac{m\lambda}{2a}\right]$$

pitch	m	θ_d	$\frac{m\lambda}{a\cos\theta_d}$
1800	2	22°	1.64
2400	1	30°	1.16
3600	1	22°	1.64

$\Rightarrow$ 1800/m=2 $\simeq$ 3600/m=1. There may be some efficiency benefits to the lower pitch

$\Rightarrow$ $\boxed{1800 \frac{l}{mm} \text{ used in } m=2 \text{ order}}$

③

3. What is the minimum linear dispersion needed?

Assume $\frac{3 \text{ pixels}}{\text{rbw}}$; Assume 6 nm spectral range

$\Rightarrow$ total # of pixels in a horizontal row

$$= \frac{6 \text{ nm}}{\frac{.012 \text{ nm}}{3 \text{ pixels}}} = 1500 \text{ pixels}$$

∴ a 1024 x 256 format is inadequate

use 2048 x 512 format, eg. PI PIXIS 2K

Pixel size 13.5μ x 13.5μ

Horizontal width = 2048 x 13.5μ = 27.6 mm

need $$\frac{dL}{d\lambda} = \frac{3 \times \text{pixel size}}{\text{rbw}} = \frac{3\,(13.5 \times 10^{-3}\text{ mm})}{.012 \text{ nm}}$$

$$\boxed{\therefore \left(\frac{dL}{d\lambda}\right)_{min} = 3.375 \ \frac{\text{mm}}{\text{nm}}}$$

The 6 nm spectrum will cover 20.25 mm

(4) What focal length?

We seek the highest NA (lowest f/#) to get the highest throughput. This usually means the shortest focal length that provides adequate disp.

$$\frac{dL}{d\lambda} = \frac{mf}{a\cos\theta_d} \Rightarrow f \geq \frac{dL/d\lambda}{\frac{m}{a\cos\theta_d}}$$

$dL/d\lambda$ → calculated on p. 3

$\frac{m}{a\cos\theta_d}$ → calculated on p. 2

$$f \geq \frac{3.375 \text{ mm/nm}}{\frac{2}{(555\text{ nm})(\cos(22°))}} = 869 \text{ mm}$$

$$\boxed{\Rightarrow \text{choose } f = 1 \text{ meter}}$$

Horiba JY 1000M f/8

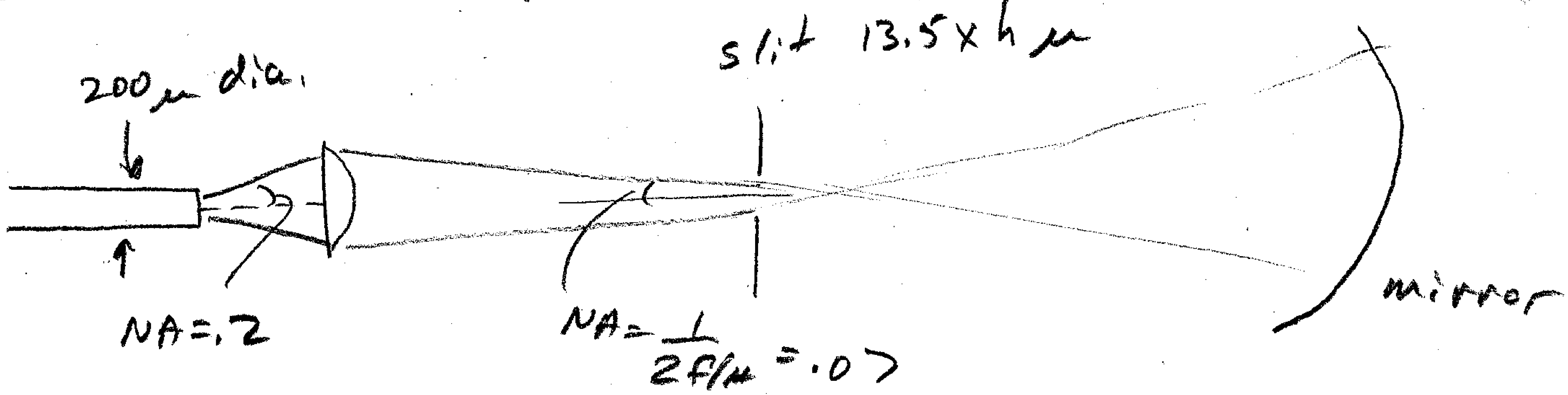
McPherson 2061A f/7 aberration-limited resol = .007 nm ✓

use this

(5) The input slit and matching optics.

To maintain pixel-limited resolution, the image of the input slit must be ≤ the pixel width, $w \le 13.5\mu$. But the image of the slit <u>height</u> can be as long as one <u>column</u> of the ccd, $512 \times 13.5\mu = 6.91$ mm. All the pixels in a column can be binned, increasing the effective pixel size.

For the spectrometer mirrors to capture as much light as possible, and avoid degrading the background level with stray light, the fiber NA must be matched to the spectrometer NA



$$Mag = \frac{.2}{.07} = 3 \text{ to match NA.}$$

∴ Image of fiber @ slit is 600μ diam.

6. Spectrometer Throughput

illuminated input slit = $13.5 \times 600\mu$

$\rightarrow$ illuminated pixels = 1×44 column

$$\text{Throughput} = \frac{(A\,\Omega)_{out}}{(A\,\Omega)_{in}}$$ but Ωs are already matched

$$\frac{A_{out}}{A_{in}} = \frac{13.5 \times 600\,\mu^2}{\pi\,(300\mu)^2} = .03$$

Now consider mirror reflectivity and grating effic.

Horiba 530-18 blazed holographic grating 55% @ $m=1$ 400 nm

Likely lower for $m=2$ (see discussion in Newport/Richardson Gratings handbook)

$\Rightarrow$ estimate grating efficiency ~ 30%

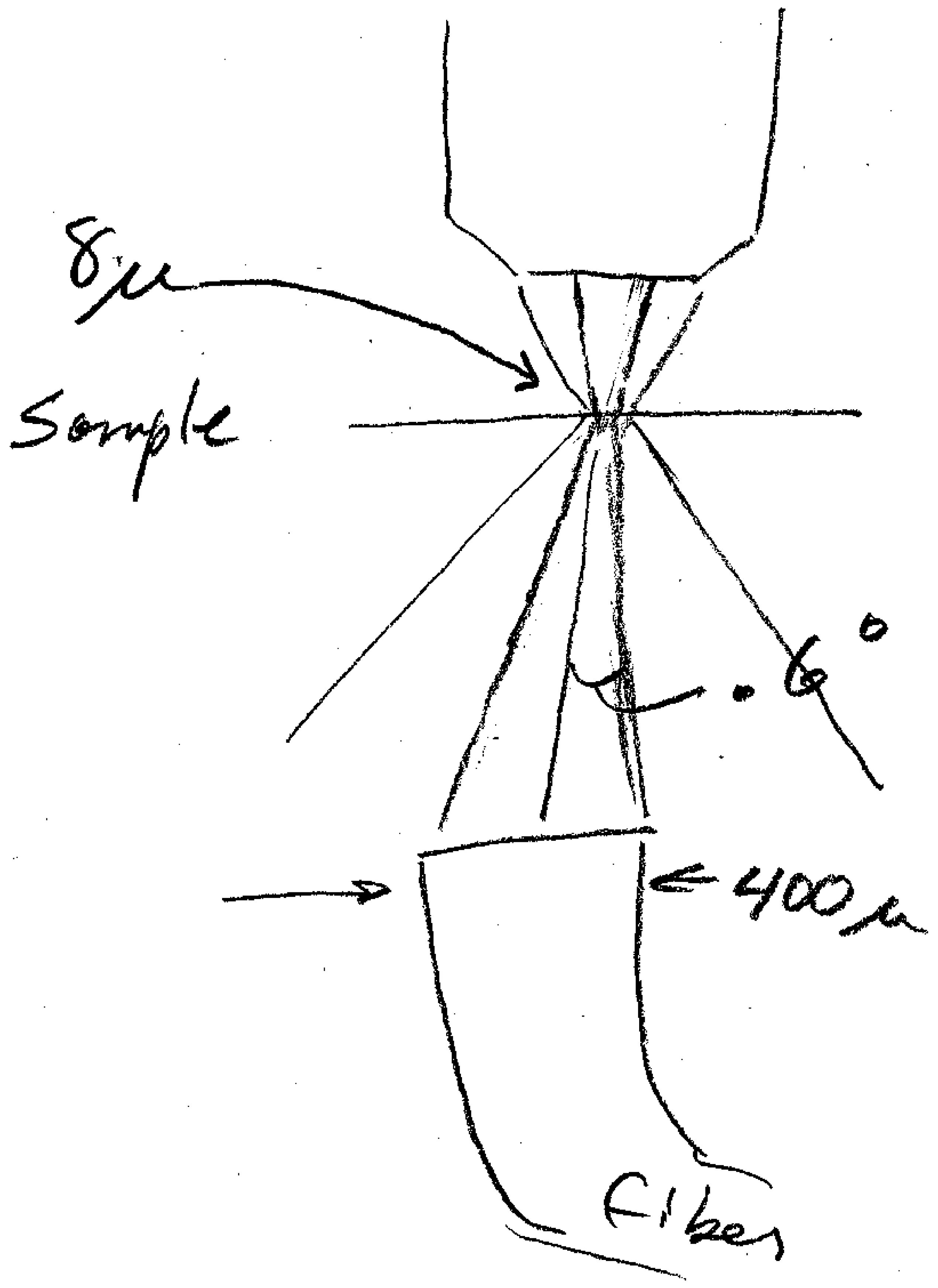
estimate mirror reflectivity ~ 90% @ 400 nm

Total spectrometer throughput, T

$$T = \frac{A_{out}}{A_{in}} \cdot \eta_{grat}\, \eta_{mirror\,1}\, \eta_{mirror\,2}$$

$$\boxed{T = .007}$$

7. Power in fiber
(Courtesy of Sudha Srinivasan)



The fiber restricts the useful NA "emitted" by the spot at the sample.

$$G = A\Omega = \pi\left(\frac{D}{2}\right)^2 \pi\, NA^2$$

$$= \pi\left(200\times10^{-6}\,m\right)^2 \pi \sin^2(0.6°)$$

$$G_{fiber} = 4\times10^{-11}\ m^2\text{-}sr$$

An ideal lens system will deliver

$$\phi = LTGB = \left(\frac{2\times10^3\,W}{m^2\ sr\ nm}\right)(1)\left(4\times10^{-11}\,m^2\text{-}sr\right) = 8\times10^{-8}\frac{W}{nm}$$

$$\boxed{\phi_{fiber} = 8\times10^{-8}\ \frac{W}{nm}}$$

for $B = .012\,nm$ (rbw),

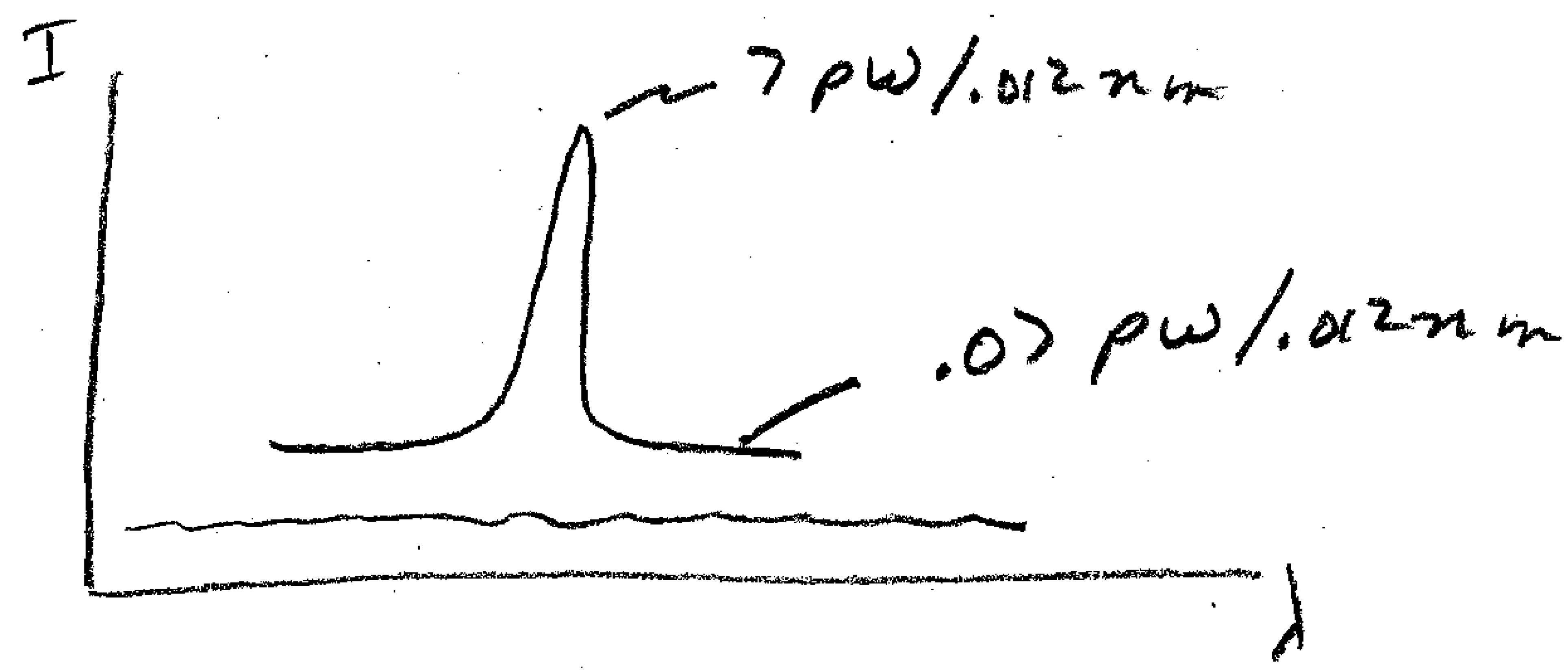
$$\boxed{\phi'_{fiber} = 1\times10^{-9}\ \frac{W}{rbw}}$$

(8.) Power @ detector array

$$P_{det} = \phi'_{fiber} T_{spec} = \left(1\times10^{-9}\,\frac{W}{rbw}\right)\left(7\times10^{-3}\right) = 7\times10^{-12}\,\frac{W}{rbw}$$

This is the power falling on <u>one column</u>, at the <u>signal peak</u> wavelength. We demand enough sensitivity to obtain $\frac{Peak}{base} \geq 100$ (in case the true spectrum provides $\frac{Peak}{base} \geq 100$)

→ the base power is $\frac{1}{100} P_{peak} = 7\times10^{-14}\,\frac{W}{rbw}$



<u>Camera specs</u> @ −70°C

QE 60%;

Dark current .006 e^-/pixel/second worst case

Read noise 6 e^- rms (worst case, @ 100 kHz readout rate)

Full well 8×10^4 e^-/pixel 8×10^5 e^-/node (column)

(9) SNR + Dynamic range

What is the maximum allowable integration time?

Peak signal, S_{peak}, @ output node (full column)

$$S_{peak} = (\Phi_{peak})(\text{Responsivity})\, QE\, \tau_{integ} \leq 8\times10^5\, e^-$$

To avoid saturation

$$8\times10^5\, e^- \geq \left(7\times10^{-12}\, \frac{J}{s}\right)\left(\frac{1\,eV}{1.6\times10^{-19}\,J}\ \frac{1\,\text{photon}}{3\,eV}\right)\left(\frac{0.6\,e^-}{\text{photon}}\right)\tau_{integ}$$

$$\boxed{\therefore\ \tau_{max} = 91\ \text{m sec to avoid saturation}}$$

The integrated dark signal is then

$$S_{dark} = \left(\frac{\text{dark current}}{\text{pixel}}\right)\left(\underset{\text{per column, used *}}{44\ \text{pixels}}\right)\left(91\times10^{-3}\ \text{sec}\right)$$

$$= 240\, e^-$$

* assumes the CCD can be "windowed" collecting only those pixels desired

The shot noise associated with this dark count is then

$$N_{shot,dark} = \sqrt{S_{dark}} \cong 15\, e^-$$

The total rms noise is

$$\boxed{N_{rms} = \sqrt{N_{shot}^2 + N_{read}^2} = \sqrt{240 + 6^2} = 22\, e^-_{rms}}$$

The peak SNR is then

$$\frac{S_{peak}}{N_{rms}} = \frac{8\times10^5\ e^-}{22\ e^-} \overset{?}{=} \boxed{3\times10^4 \quad \text{Dynamic Range for 91 ms } T_{integ.}}$$

We need the base SNR (away from the peak)

$$\boxed{\frac{S_{base}}{N_{rms}} = \frac{8\times10^3}{22} = 360}$$

Is the $\frac{S_{base}}{S_{dark}} \geq 2$, as desired?

$$\frac{8\times10^3}{240} = 33 \quad \checkmark$$

The full camera dynamic range, when operated with binned columns, is

$$\frac{8\times10^5\ e}{6e^-\ \text{read noise}} = 1.3\times10^5$$