Gaussian Beams

- Gaussian Beam Properties and Transformation
- Coherence
- Demo

Gaussian Waves vs. Plane Waves

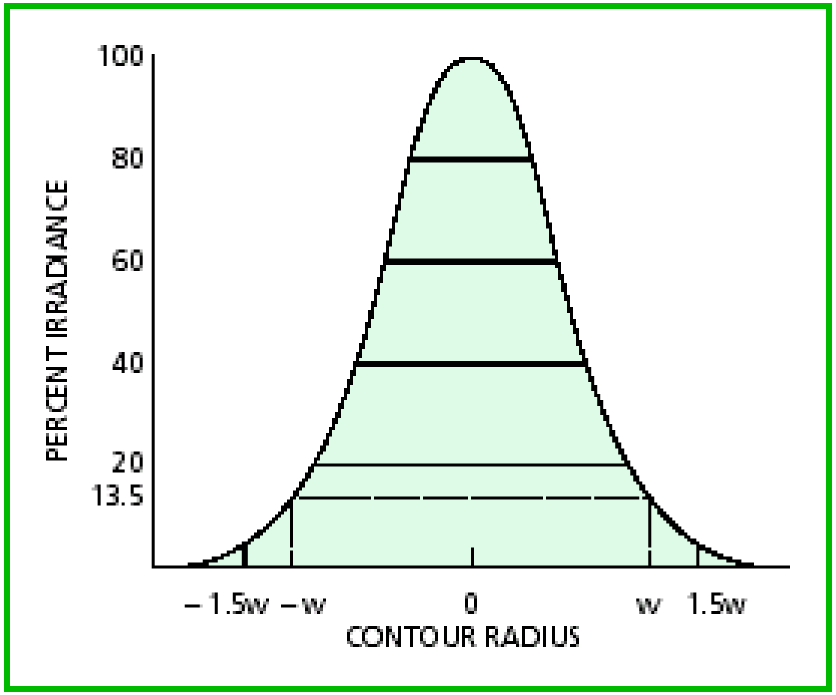
Wave Equation $\nabla^2 E + k^2(r)E = 0$

Plane Wave Solution $E(z) = \underline{E_0}\, \underline{\exp[-ikz]}$

Gaussian Wave Solution

$$E(r,z) = \underbrace{E_0 \frac{\omega_0}{\omega(z)} \exp\left[-\frac{r^2}{\omega^2(z)}\right]}_{\text{amplitude}} \underbrace{\exp\left[-i\left(kz - \tan^{-1}\left(\frac{z}{z_0}\right) + \frac{kr^2}{2R(z)}\right)\right]}_{\text{phase}}$$

Gaussian Beam Distribution

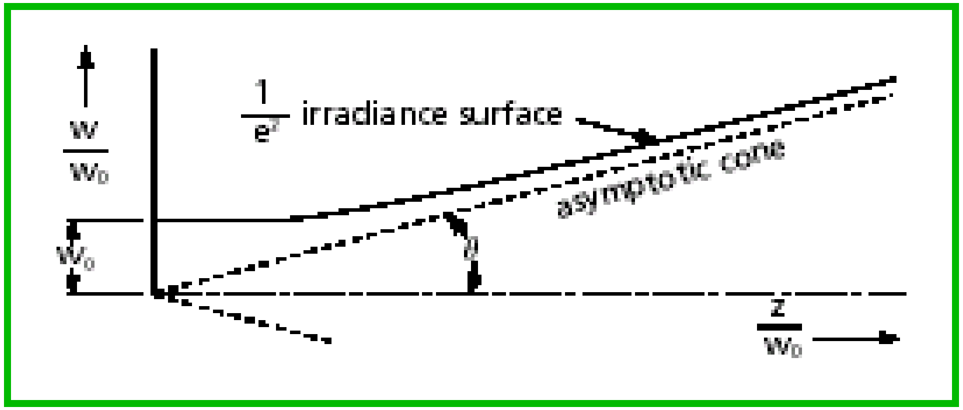


$$I(r) = I_0 \exp\left(-\frac{2r^2}{\omega^2(z)}\right)$$

Power enclosed within circle of radius r:

$$\frac{P(r)}{P(\infty)} = 1 - \exp\left[-2\frac{r^2}{\omega^2}\right]$$

Beam Waist and Divergence



$$R(z) = z\left[1 + \left(\frac{\pi\omega_0^2}{\lambda z}\right)^2\right]$$

$$\omega(z) = \omega_0\left[1 + \left(\frac{\lambda z}{\pi\omega_0^2}\right)^2\right]^{1/2}$$

For large z:

$$R(z) \cong z \qquad \omega(z) \cong \frac{\lambda z}{\pi\omega_0} \qquad \theta = \frac{\omega}{z} = \frac{\lambda}{\pi\omega_0}$$

Beam Transformation

Plane Waves

$$\frac{1}{s/f}+\frac{1}{s''/f}=1$$

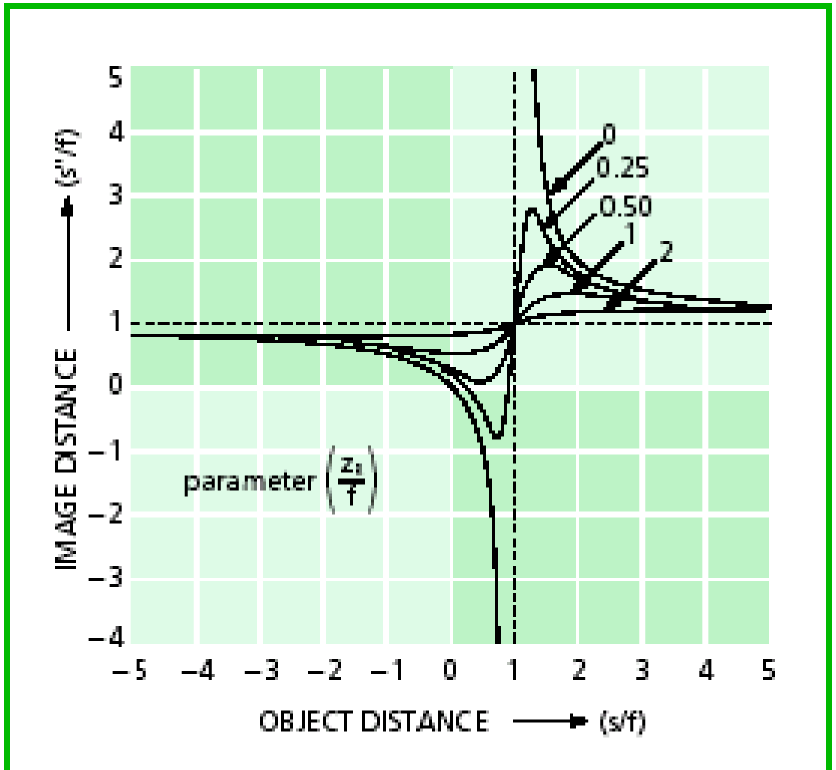
Gaussian Beams

$$\frac{1}{s/f+\left(Z_R/f\right)^2/\left(s/f-1\right)}+\frac{1}{s''/f}=1$$

Rayleigh Range

$$Z_R\equiv\frac{\pi\omega_0^2}{\lambda}$$

Beam Transformation

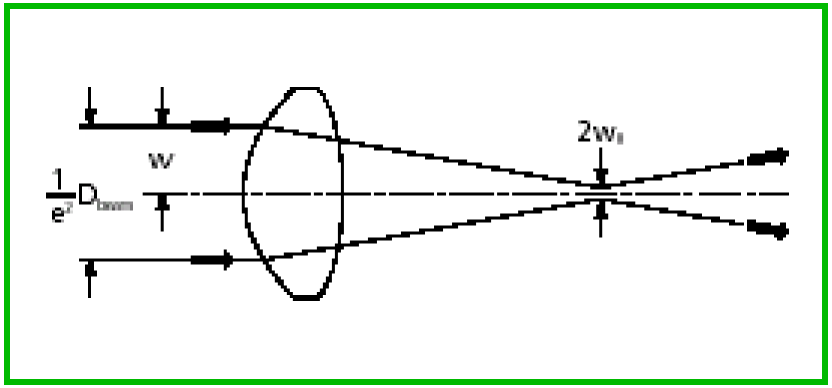


$$Z_R \equiv \frac{\pi \omega_0^2}{\lambda}$$

<u>Differences From Plane Waves</u>

- Plane wave limit at $Z_r/f=0$
- Maximum and minimum image distance exists
- "Collimation" at $s=f+Z_r$
- At s/f=1, image at s"/f =1

Beam Focusing



Spot diameter

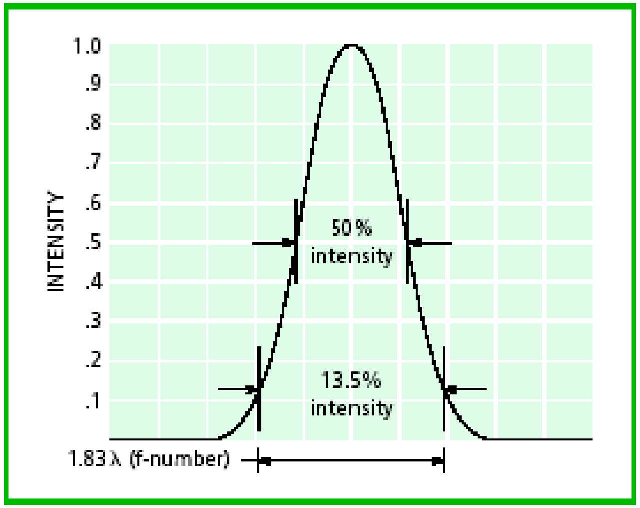
$$2\omega_0 = 2\frac{\lambda f}{\pi \omega_{lens}} \approx \frac{2}{\pi}\frac{\lambda}{NA}$$

Example

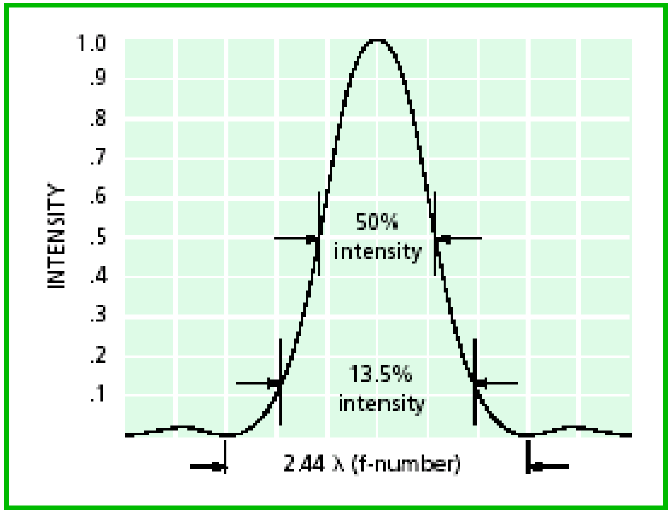
f=10 mm, $2\omega_{lens}$=5 mm, λ=1.55 μm,

spot diameter=3.9 μm

Diffraction at Apertures



Beam dia. << Lens dia.



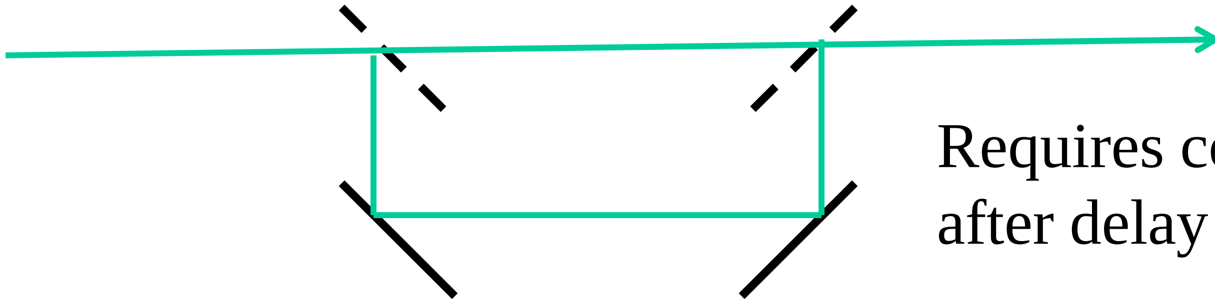
Beam dia. >> Lens dia.

Truncation Ratio =
Beam diameter / lens diameter

Truncation Ratio	d_{FWHM}	d_{1/e^2}	d_{zero}	P_L (%)
Infinity	1.03	1.64	2.44	100
2.0	1.05	1.69	—	60
1.0	1.13	1.83	—	13.5
0.5	1.54	2.51	—	0.03

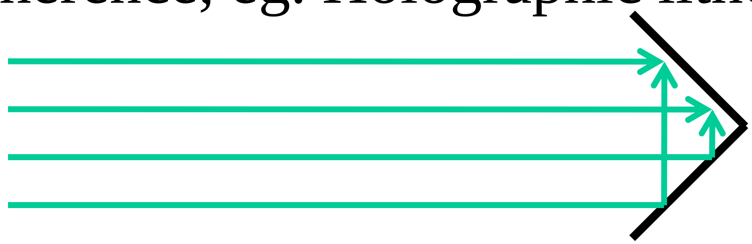
Coherence

- Temporal coherence, eg. MZ interferometer



Requires coherence after delay

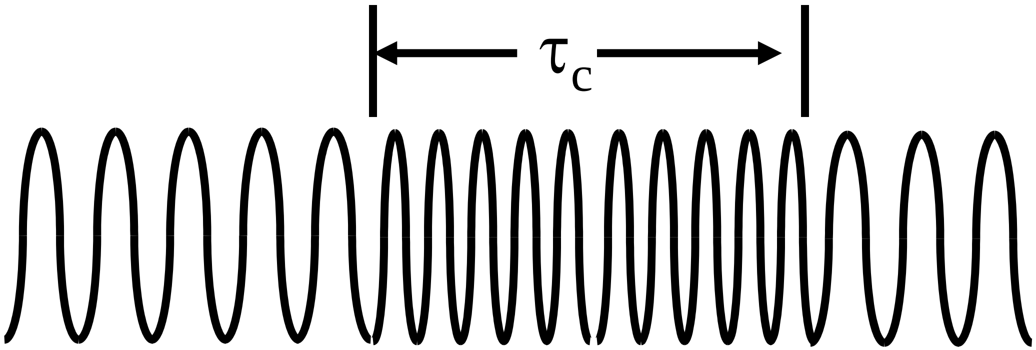
- Spatial coherence, eg. Holographic lithography



Requires coherence across wavefront

- In general, both spatial and temporal coherence is relevant
- Lasers are highly coherent, LEDs less so…

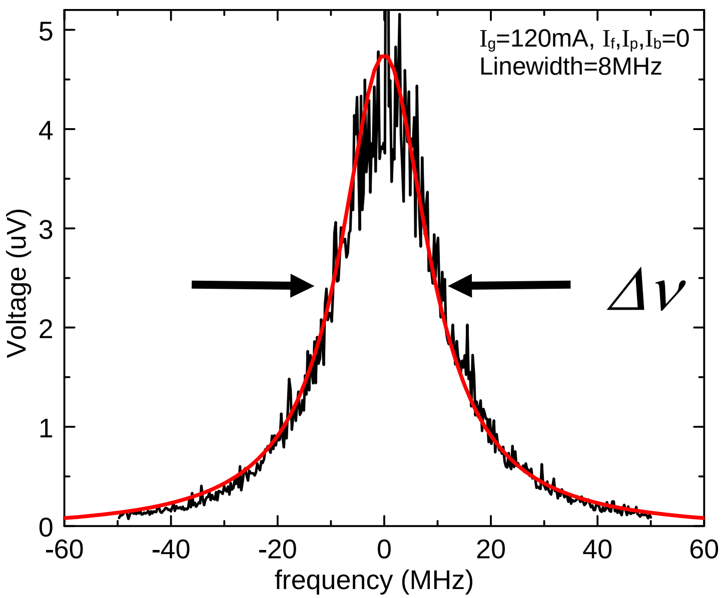
Temporal Coherence



τ_c Coherence time

$\Delta\nu$ Spectral linewidth

L_c Coherence length



$$\tau_c = \frac{1}{\pi\Delta\nu}$$

$$L_c = \frac{c}{n}\tau_c = \frac{\lambda^2}{\pi\Delta\lambda}$$

Coherence Length

Source	**Wavelength, nm**	**Spectral Width, nm**	**Coherence Length,**
Unfiltered Sun	550	300 nm	0.3 micron
LED	550	10 nm	10 micron
Blu-ray laser	405	1 nm	50 microns
HeCd (single line)	325	1 pm (3 GHz)	3 cm
HeNe (multiline)	633	1 pm (1 GHz)	10 cm
SGDBR	1550	0.04 pm (5 MHz)	19 meters

Credits

- Yariv, Optical Electronics
- www.cvimellesgriot.com/products/Documents/TechnicalGuide